Estimates of the Natural Variability of Time-Averaged Temperatures over the United States

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ABSTRACT

Estimates of the natural variability of monthly mean temperature data from 107 U.S. stations are made. The natural variability of monthly means is defined as those interannual fluctuations that can be attributed to the effects of statistical sampling alone. It is variability resulting from the variance and autocorrelation associated with daily weather fluctuations. It does not indicate "climate change" but rather it is the variability within an "unchanging climate"; as such it is a measure of unpredictable "climatic noise". Comparisons between the natural and actual interannual variability are discussed in the context of potential long-range predictability. The natural variability is proposed as a lower limit for the standard error of estimate for any long-range prediction. A characteristic time between independent estimates is computed.

1. Introduction

Climatic states are typically defined in terms of finite time averages of meteorological variables, and as such are subject to fluctuations of statistical origin which we refer to as natural variability or "climatic noise". This natural variability of climatic states is variability that would be present even within an "unchanging climate". To define an unchanging climate we proceed in a manner analogous to the discussion of Leith (1975). We first consider that each occurring season represents a single realization of an imagined ensemble of realizations. We assume that the physical laws governing the behavior of the atmosphere allow a single, unique set of statistics in the presence of constant external conditions such as sea surface temperature, ice cover, solar radiation, etc. If external conditions were constant, we would conclude that each seasonally or monthly averaged realization is from an ensemble of realizations having constant statistical properties or an "unchanging climate". These individual realizations would, however, exhibit a certain variability due to the nature of statistical sampling which we refer to as natural variability.

Here we estimate the natural variability of time-averaged temperature at 107 stations in the United States. Similar estimates of the natural variability of time-averaged sea level pressure were presented in an earlier paper (Madden, 1976, hereafter referred to as A). The rationale behind the method that we use to estimate this natural variability and its relevance to the detection and usefulness of potentially predictable climate variability are discussed in A and in Madden (1977, hereafter referred to as B). For the present we point out that Leith (1975) has argued that the daily weather fluctuations of meteorological variables such as temperature may be reasonably well modeled by a first-order autoregressive or Markov process. The spectrum of such a process approaches a constant ("white noise"), nonzero value at frequencies small relative to those of the weather fluctuations. This low-frequency, white noise (LFWN) extension of weather fluctuations would then contribute to the variability of time-averaged data on all time scales longer than that of the weather fluctuations themselves. In his discussion of the spectrum of climatic variability, Mitchell (1976) considers this contribution to the variance of long-time scale variations as a sort of "shelf" produced by shorter time scale variations. These long-time scale variations would occur even in an unchanging climate. With these ideas in mind we determine the natural variability of time-averaged temperatures by attempting to estimate a LFWN extension in the spectrum produced by relatively fast varying weather variations.

In defining an unchanging climate we have considered sea surface temperature and ice cover as external conditions. They are not external in the sense that they are independent of the atmosphere itself as are Mitchell's (1976) external processes or Lorenz's (1976) environmental parameters. Here, we conceptually view all processes whose time scales are long relative to daily weather fluctuations

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to be external. It is in this context that we attempt to estimate the natural variability or variability within an unchanging climate under the influence of "constant" external conditions. The spectra of daily temperature variations are estimated for separate seasonal segments or realizations. These spectra provide information on variations whose time scales are equal to or less than that of a season. Interannual variations are then approximated by a LFWN extension of these seasonal spectra and the resulting spectra, which then extend to zero frequency, are used to compute the natural interannual variability of time-averaged temperatures. Clearly, the task of separating the influence of changing external conditions from that of daily weather fluctuations is not a simple one since, for example, some sea surface temperatures may change enough within a single season to alter both mean air temperatures and the character of daily weather. However, since the seasonal mean is subtracted from each seasonal realizations, information about interannual changes in the mean that might reflect the effects of longer period changes (a year or more) in external conditions is excluded from the seasonal spectra. Undoubtedly changing external conditions can also alter the character of daily weather in an area and thereby change the spectra of daily temperatures themselves. This is a problem that, for the present, we only examine in a preliminary way by comparing estimates of the natural variability based on the first and second halves of our data.

Estimates of the natural variability reflect fluctuations in daily weather whose memory is on the order of days or at most one or two weeks (see Lorenz, 1973; Leith, 1973; A). At time scales of a year or more the natural variability would have zero autocorrelation and a white spectrum implying a lack of any statistical predictability. Furthermore, since we cannot deterministically predict weather fluctuations beyond a few weeks the natural variability gives a measure of what is for certain unpredictable in the climate. On the other hand, variability over and above the natural variability may be at least potentially predictable. While it is likely that this natural variability associated with weather fluctuations is small relative to climate variations on time scales exceeding hundreds of years, we will argue that it is big relative to variations evident in the instrumental record, and, as such, it imposes important limitations on our ability to predict and detect climate variability.

Finally, the above introduction has assumed that with constant external conditions there would be no change in climate. Lorenz (1968, 1976) has pointed out that this is not necessarily the case, for the atmosphere may be intransitive or almost intransitive. This possibility need not concern us here for we do not discuss the nature of climate change but only estimate the natural variability that we might expect without any climate change.

2. Data

Daily mean temperatures (determined by averaging daily maximum and minimum temperatures) at 107 U.S. stations are examined in this study. Most stations have 22–24 years of data. Specifically the data spanned the following time periods: January 1948–June 1963 and July 1969–September 1976. The former consisted of tapes from the National Climatic Center which were edited and filled in at the National Meteorological Center, and the latter consisted of tapes from the Techniques Development Laboratory of NOAA. Each set was carefully checked to minimize the number of erroneous reports. The entire data set is now available at the National Center for Atmospheric Research (Jenne, 1975). Some monthly mean temperature data were taken from the World Weather Records (Smithsonian Institution, 1927, 1934, 1947; U.S. Weather Bureau, 1959, 1965).

3. Standardizing the data

The data were standardized in such a way as to minimize effects of seasonal variations. The standardization procedure is described in detail in A. It will suffice to state here that a mean temperature was computed for each day of the year. These daily means were smoothed and an appropriate smoothed mean was subtracted from each daily value. The standardized data should be approximately stationary in their mean during any one year. Because the shape of the spectrum of the daily data is of fundamental importance in our estimates of the natural variability, we attempt to account for its variations within a year by estimating spectra separately for each season. Estimating the spectra for each season also minimizes the effects of any seasonal variations in variance that may be present.[2]

4. Estimating the natural variability

a. Method

Our basis for estimating the natural variability of monthly means is the relationship between the input and output spectral density functions of a linear sys-

[2] In A, all values were divided by a smoothed estimate of the standard deviation for the appropriate day of the year in order to minimize effects of seasonal variations of variance. The analysis proceeded with the data in "standardized units". There are then some uncertainties involved in recovering the original units for presentation of results. Here, the analyses were done both with and without such standardized units. The results were similar and therefore we present only those of the simplest method, which does not involve dividing data by a smoothed estimate of the standard deviation.

tem (Blackman and Tukey, 1958, p. 25 and p. 112; Bendat and Piersol, 1971, p. 136), i.e.,

$$s(f) = S(f)|H(f)|^2,$$

where $s(f)$ is in the present case the power spectral density function of monthly average temperatures at a frequency f, $H(f)$ is the amplitude response of a 30- or 31-day average, and $S(f)$ is the power spectral density function of daily mean temperatures.

The variance σ_T^2 of monthly averages, where T represents the monthly averaging time, is given by

$$\sigma_T^2 = \int_{-\infty}^{\infty} s(f)df$$

or

$$\sigma_T^2 = \int_{-\infty}^{\infty} S(f)|H(f)_T|^2 df. \quad (1)$$

$H(f)_T$ is easily determined for various averaging times T and we need only estimate $S(f)$ from the daily mean temperatures.

b. *Estimates of the spectra* S(f) *of daily data*

The entire record of standardized data at each station was broken up into individual seasonal series (realizations). Each seasonal series consisted of 96 days with spring, summer, fall and winter beginning 1 March, 1 June, 1 September and 1 December, respectively. Linear interpolations in time were used to fill in gaps for missing data. Data were missing on only a few days.

The mean of each seasonal series was subtracted, nine values at the beginning and end of the series were tapered with a cosine bell as suggested by Bingham *et al.* (1967), and a fast Fourier transform was used to determine the coefficients of the 48 harmonics. Seasonal spectra were then estimated by averaging the squared Fourier coefficients or periodograms over all available years. The winter season spectrum computed in this way for North Platte, Nebraska, is shown in Fig. 1 as an example. At North Platte, 23 winter seasons were averaged so that each spectral estimate has about 46 degrees of freedom (df) assuming each periodogram value has 2 df. There is some small loss of df due to the standardization and the tapering procedures (Julian, 1971). All spectra $[S(f)]$ have been normalized so that

$$\int_0^{0.5} S(f)df = \sigma^2 \text{ (total variance of daily data within a season).} \quad (2)$$

c. *Estimates of the natural variability* σ_T^2

The natural variability of monthly means is estimated by evaluating the discrete form of (1), i.e.,

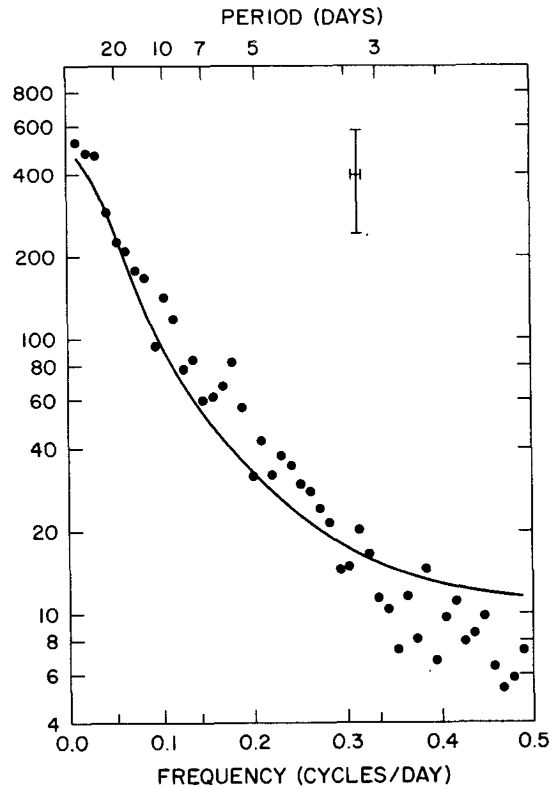

FIG. 1. Winter season spectrum for daily temperature data at North Platte, Nebraska (dots). The 95% confidence limits, assuming at least 40 df, and the band width of the analyses are indicated by the cross. The ordinate is °C² × day. The spectrum for a Markov process with a lag 1 autocorrelation appropriate for North Platte (0.72) is indicated by the smooth line.

$$\sigma_T^2 = \frac{S(0)H^2(0)_T}{192} + \frac{1}{96}\sum_{f=1/96}^{f=47/96} S(f)H^2(f)_T + \frac{S(0.5)H^2(0.5)_T}{192}. \quad (3)$$

The first and last spectral estimates apply to a fre-

TABLE 1. Tabulation of data used to evaluate Eq. (3) for a 31-day winter average at North Platte. The frequency in cycles per day, the spectral estimates (from Fig. 1) and the response squared are given by f, $S(f)$, and $H^2(f)_{31}$. The frequency interval, the product $S(f)H^2(f)_{31}$ times the frequency interval and the partial sum of (3) are given by Δf, $s(f)\Delta f$ and (3), respectively. $S(f = 0)$ is assumed to be equal to $S(f = 1/96)$.

f	$S(f)$	$H^2(f)_{31}$	Δf	$s(f)\Delta f$	.(3)
0	522	1.000	1/192	2.72	2.72
1/96	522	0.701	1/96	3.81	6.53
2/96	477	0.196	1/96	0.97	7.50
3/96	465	0.001	1/96	0.01	7.51
4/96	289	0.038	1/96	0.11	7.62
5/96	224	0.034	1/96	0.08	7.70
·	·	·	·	·	·
·	·	·	·	·	·
48/96	·	·	1/192	·	7.76

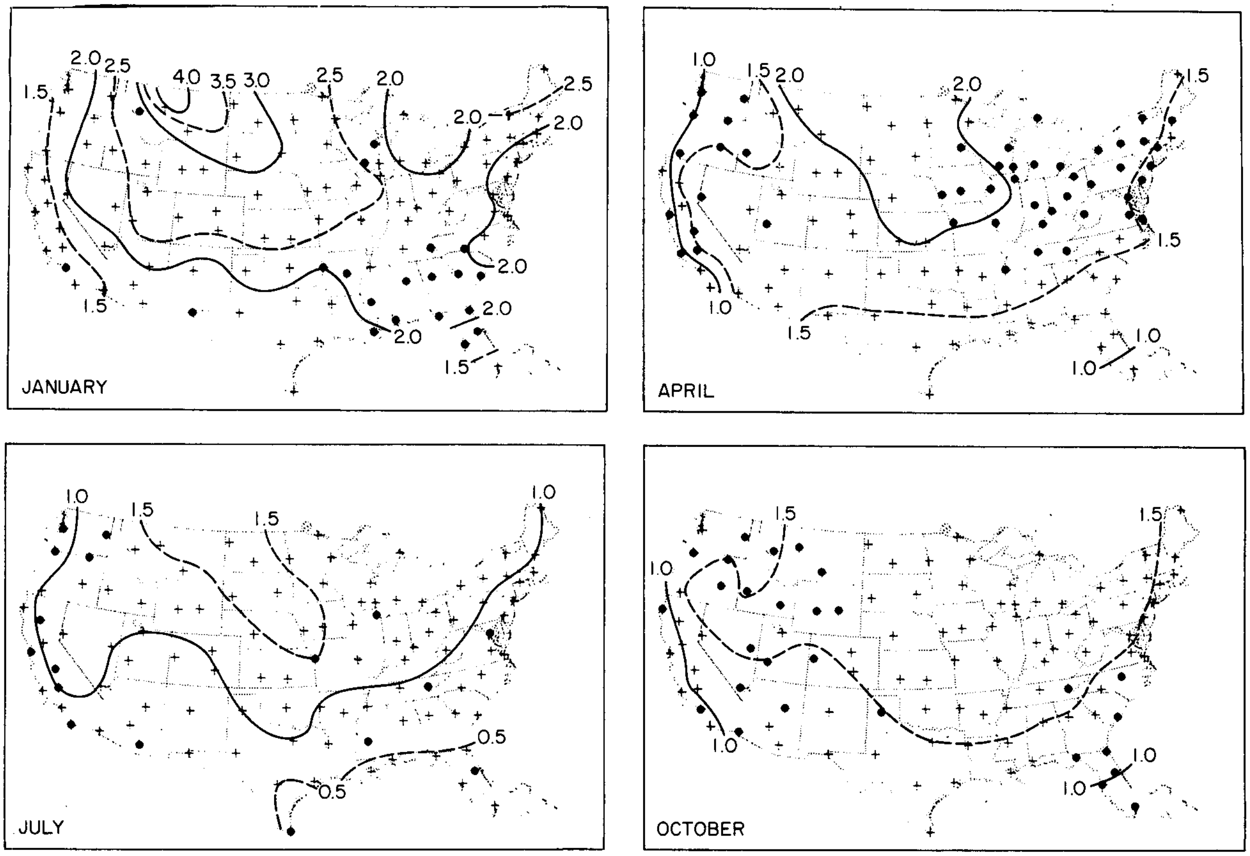

FIG. 2. Standard deviation (°C) associated with the estimated natural variability for January, April, July and October. Analysis based on data from stations indicated by the cross. Circles indicate stations where the F-ratio of σ_T^2's computed for the first and second half of the record exceeds 1.6 (see text).

quency interval of 1/192 cycles per day (cpd). All other spectral estimates apply to frequency intervals of 1/96 cpd.

Subtracting each 96-day mean from each seasonal realization removes all low-frequency (<1/96 cpd) information from the resulting spectra. The first nonzero spectral estimate is at a frequency of 1/96 cpd, or a period of 96 days, the order of a season. As discussed in the Introduction, the low-frequency ends of the spectra are approximated by white noise. This is done by assuming that the contribution to the total variance at near-zero frequencies (long periods) by relatively fast varying weather variations is best approximated by the spectral estimate at 1/96 cpd. This LFWN extension of the daily spectra is similar to modeling the near-zero frequency spectral estimates with a Markov process. This similarity is discussed further in A. The smooth curve in Fig. 1 is the spectrum of a Markov process for the 1 day lag autocorrelation estimated for North Platte in winter (0.72).

The procedure for evaluating (3) is illustrated by the values presented in Table 1. Note that $S(f = 0)$ is assumed to be equal to $S(f = 1/96)$, and $H(f)$ is that for a 31-day average. In winter at North Platte, Nebraska, the variance associated with natural variability for a 31-day average is estimated to be approximately 7.8°C². It is important to note that the natural variability depends primarily on the spectral density at the lowest frequencies, a point that was made by Jones (1975). The variance of 31-day averaged winter temperatures were similarly estimated for 107 stations in the United States. Their square roots, which we assume to be estimates of the standard deviation associated with the natural variability of January means, are presented in Fig. 2.

The reliability of these estimates of the natural variability depend on the validity of the LFWN model and on the representativeness of the estimated spectra and variance of daily data. With regard to the validity of the model we can see from Table 1 that what is assumed about the spectrum near $f = 0$ has a substantial impact on the final estimate of the natural variability. At North Platte in winter the near-zero frequency contribution (~2.7) is more than one-third of the estimate of σ_T^2 (~7.8). We cannot prove that the LFWN model is valid but can only argue that it is not an unreason-

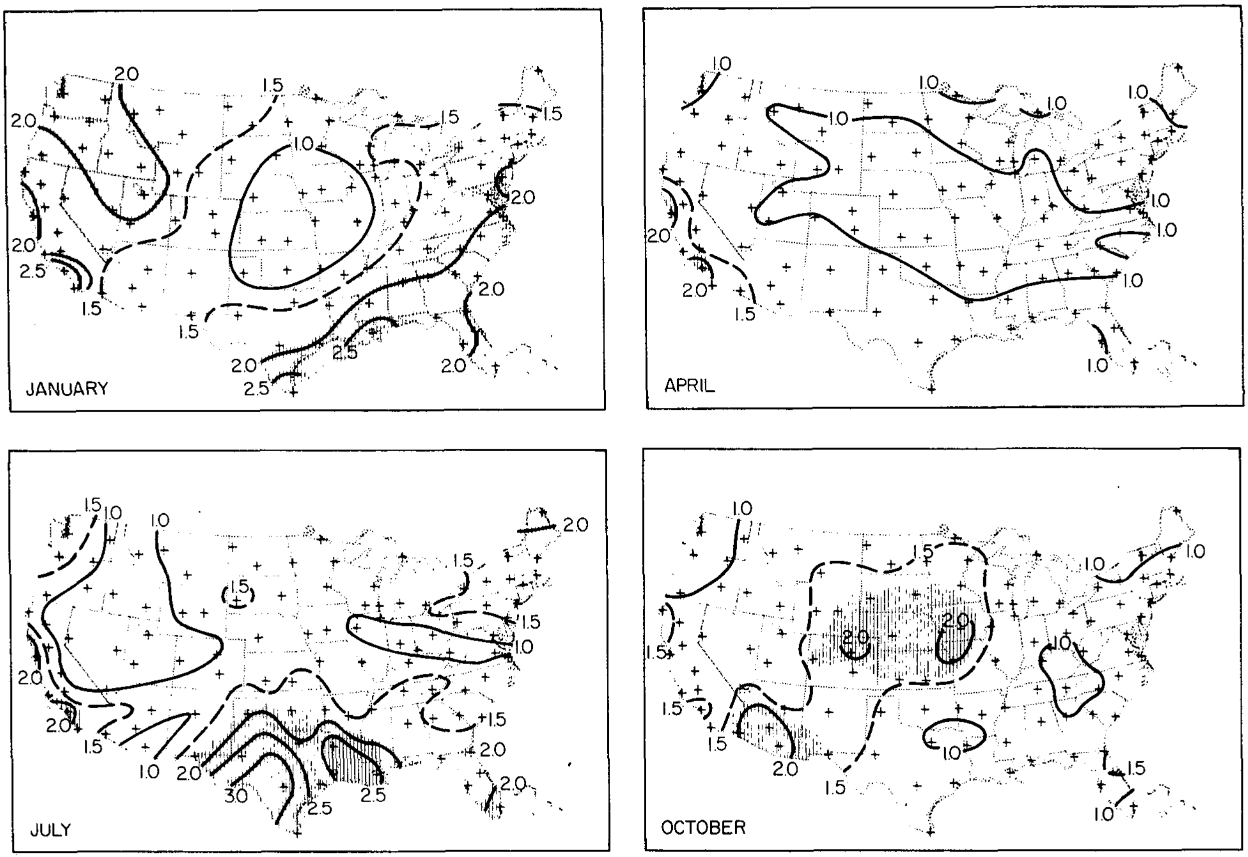

FIG. 3. Ratio of actual interannual variance σ_A^2 of monthly means to the variance σ_T^2 associated with the estimated natural variability. σ_A^2 is estimated from the 22–24 year records. The analysis is based on data from stations indicated by the crosses. Ratios greater than 1.7 are indicated by vertical hatching.

able one. It gives results similar to those of a Markov process,[3] and several authors have modeled daily weather variations with such a model with some success (Klein, 1951; Jenkinson, 1957; Leith, 1973).

The representativeness of the estimated seasonal spectra and variance of daily data were tested by computing σ_T^2 for the first and second halves of the record separately. The ratio of these two estimates of σ_T^2 would be distributed like an F-distribution if their differences were the result of sampling variations. At stations where 24 years of data were available there are at last 12 periodograms averaged together in forming seasonal spectra for half the record. The df of each spectral estimate is then about 24. To estimate df for the estimates of σ_T^2 we follow the procedure of Blackman and Tukey (1958, p. 24 and 111) which is discussed further in A.[4] From the values in Table 1, this procedure indicates that the df of estimates of σ_T^2 should be about 40% larger than those of the individual spectral results themselves. This increase will differ slightly from station to station and from season to season depending on the shape of the individual spectra. To get some idea how big differences are likely to be, we estimated the increase in df for spectra similar to Markov spectra for a 1-day lag autocorrelation ranging from 0.5 to 0.8 which is about that of the observed range. Corresponding increases in df range from slightly more than 40% to about 25%.

Assuming then that there is at least a 25% increase in df, σ_T^2 estimated from half the records would have about 30 df ($2 \times 12 \times 1.25$). In such a case the ratio of the largest to the smallest σ_T^2 should exceed 1.6 only about 10% of the time if each represents an estimate of the same variance. The percentages of stations where this F-ratio exceeds 1.6 are 19, 45, 16 and 26% for January, April, July and October, respectively. Since the stations are not independent of each other, rigorous statistical inference is not possible. It seems likely, however, that over the years of record there may be some change in the

[3] σ_T^2's were computed at all stations assuming Markov processes based on a 1-day lag autocorrelation estimated at each station for each season and they differed by less than 10% from those estimated from the LFWN model.

[4] Eq. (7) in A is incorrect and should be written

$$\mathrm{df} = k[\sum_{f=0}^{0.5} s(f)\Delta f]^2 / \sum_{f=0}^{0.5} [s(f)\Delta f]^2.$$

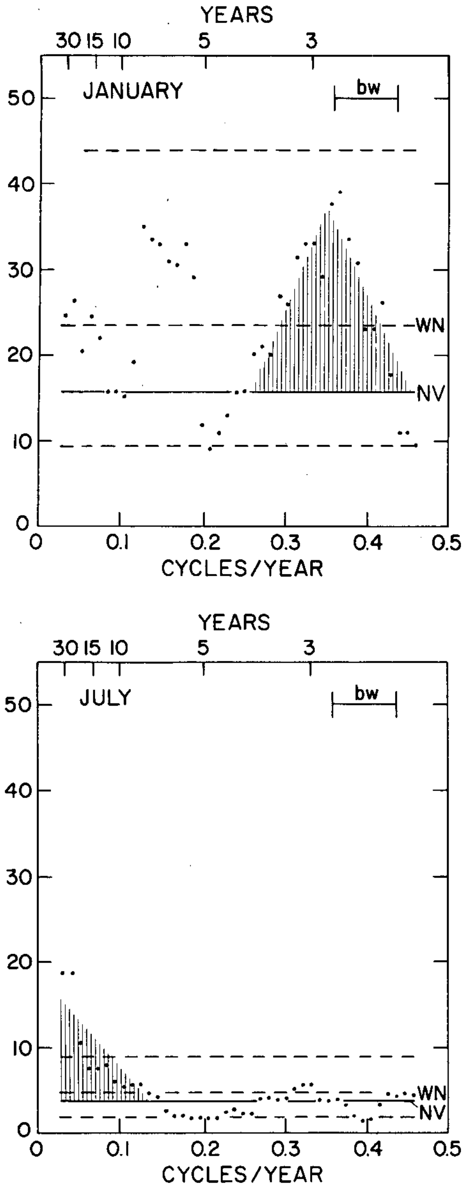

FIG. 4. Spectra of January and July temperatures at North Platte, Nebraska, based on estimates from 96 years of data (dots). The ordinate is units of °C² × year. Areas are proportional to variance. A white noise null hypothesis is indicated by the dashed lines marked WN. The 95% confidence limits, assuming 14 df, are indicated by remaining dashed lines. The contribution to the total variance that is ascribed to natural variability as determined from Fig. 2 is the solid horizontal line marked NV. The band width (bw) of the analysis is shown in upper right. Shaded areas are "signals" discussed in text.

spectra of the daily data. This is particularly true in April and October. Such changes introduce uncertainty in the estimates of natural variability over and above the uncertainty regarding the validity of the LFWN model. For the majority of the stations where the F-ratio of the $\sigma_T{}^2$'s estimated for each half of the record is less than 1.6, changes in σ_T itself are less than about ±26%. Stations where this F-ratio exceeds 1.6 are indicated in Fig. 2.

5. Potential long-range predictability

We argue that the natural variability estimated here is variability that we can expect even without climate fluctuations. The standard deviations of monthly means that are shown in Fig. 2 are measures of this natural variability. Since the natural variability results from high frequency weather fluctuations it is unpredictable on time scales as long as a year. We can compute the actual variance $\sigma_A{}^2$ of monthly means over the available years and compare it with the variance ascribed to this unpredictable, natural variability $\sigma_T{}^2$. Potential for long-range climate predictability should be greatest where the actual interannual variability is larger than that predicted to be due to natural variability.

The ratio of $\sigma_A{}^2/\sigma_T{}^2$ is shown in Fig. 3. Assuming at least 23 years of data, df of $\sigma_A{}^2$ is about 22 and that of $\sigma_T{}^2$ is about 58 (2 × 23 × 1.25). We set the null hypothesis that $\sigma_T{}^2 = \sigma_A{}^2$, which, based on the arguments made here, is equivalent to assuming that there is no potential long-range predictability evident in the data. From the F-distribution the ratio would exceed 1.7 only about 5% of the time if this hypothesis were true.

There are reasonably large changes in the ratio $\sigma_A{}^2/\sigma_T{}^2$ from season to season. For example, it exceeds 1.7 at only two stations, Los Angeles and San Francisco, in April while in July it exceeds 1.7 over most of the south, the extreme Northeast and at five stations near the West Coast. In January the ratio is approximately 1 over the Great Plains and the Mississippi Valley. In contrast, it exceeds 1.7 at 10 different stations in those regions in October. October differs from the three other months in that the ratio is a minimum over the Far West stations in that month. We conclude that over the 22–24 years studied here evidence for potential long-range predictability varies considerably from season to season and from place to place. In April there is little such evidence over most of the United States. It is similarly small over the midsection of the country in January, between the Rocky Mountains and the Sierra and Cascade Ranges in July, and in the Far West and East in October. These areas are ones where there is correspondingly least evidence for climate fluctuations during the 22–24 years studied.

Since we argue that the standard deviations shown in Fig. 2 are the result of statistical sampling variations, they may serve as lower limits for approximating the standard errors of estimate for any long-range prediction. We can discuss this implication further by considering the possible effects of natural variability on the spectrum of monthly mean temperatures.

The spectra of January and July mean temperatures at North Platte are presented in Fig. 4. The spectral estimates are based on 96-years of monthly mean temperatures, and were determined by first subtracting the 96-year mean from the data, tapering nine values at the beginning and end of the series with a cosine bell, and computing a smoothed periodogram by averaging seven adjacent periodogram estimates. The area under the dots in Fig. 4 is equal to the total variance of the monthly mean data and is 11.8 and 2.4°C² for January and July, respectively. In order that an integration from 0.0 to 0.5 cycles per year give the total variance the white noise null hypotheses must appear as the horizontal lines marked WN at 23.6 (2 × 11.8) and 4.8°C² × year (2 × 2.4) in Fig. 4. The areas under the dots and that under the white noise line are, of course, equal. Except for a period near five years the January spectrum falls within the χ^2/df 95% limits for the white noise null hypothesis. In contrast, the July spectrum falls below the 95% limits at high frequencies and above at low frequencies. This result is consistent with the findings of B which indicate that few spectra based on winter season temperatures differ at the 5% level from a white noise model while many based on summer season temperatures do.[5]

Using Mitchell's terminology, the natural variability due to weather variations would produce a variance shelf in the spectrum of monthly means. From Fig. 2, σ_T's for January and July at North Platte are 2.8 and 1.4°C, respectively. Variations responsible for σ_T are essentially uncorrelated at time scales exceeding a year so that the variance shelf due to natural variability would appear in the spectra of January and July means as white noise at levels of 15.7 (2.8 × 2.8 × 2) and 3.9 (1.4 × 1.4 × 2)°C² × year indicated by the horizontal lines marked NV in Fig. 4. The ratio of the area under the dots or the WN line to that under the NV line in Fig. 4 is equal to $\sigma_A{}^2/\sigma_T{}^2$, where $\sigma_A{}^2$ is now estimated from the 96 years of data. For North Platte, January and July ratios are approximately 1.5 and 1.2. These ratios are not significant at the 5% level; however, the fact that the January ratio is somewhat larger than that shown in Fig. 3 might possibly indicate real climate fluctuations that are manifest in the 96-year record but not in the shorter record.

For argument's sake, we assume that the spectrum of January means at North Platte really consists of a white noise continuum due to natural variability at the NV level and that the variance above that level reflects predictable "signals". We further assume that the spectral estimates at 2.2, 5 and 10 years that fall below the NV line are simply sampling variations and that the true spectrum is at NV at those periods. One "signal" is present between about 2.5–4 year periods. It is possible that this "signal" is part of the quasi-biennial oscillation evident in many temperature records (Landsberg, 1962). For simplicity we will neglect "signals" between 5 and 10 years and also that at periods longer than 15 years. The predictable January variance associated with the 2.5–4 year signal is then indicated by the area of the shaded triangle shown in the January spectrum of Fig. 4, and is approximately equal to 2.2°C². If we assume that the variations associated with this variance can be approximated by a simple sine wave then this wave would have an amplitude of 2.1°C.[6] Accordingly one could predict shifts from the long-term mean of ±2.1°C. January temperatures would still have a noise or unpredictable component σ_T of 2.8°C. In this most optimistic case the ratio of the signal, or change in the climate mean, to the noise, or unpredictable component, is 0.75 (2.1/2.8). Following Leith (1973), a signal-to-noise ratio of +0.75 (−0.75) would change probabilities that the January mean temperature would be below, near or above the long-term mean from 1/3, 1/3 and 1/3 to approximately 0.12(0.61), 0.27(0.27) and 0.61(0.12). Similar arguments can be made about July temperatures. For example, the variance associated with the shaded triangle at the low-frequency end of the July spectrum in Fig. 4 is about 0.6°C². If we were to approximate the variations associated with this variance by a sine wave we could predict shifts in the mean of 1.1°C and have a resulting signal-to-noise ratio of approximately 0.79 (1.1/1.4). The July "signals" are smaller than those in January but so is the noise.

Although there are uncertainties in estimating the natural variability of time-averaged temperatures, we propose that the σ_T's of Fig. 2 provide reasonable lower limits for the standard error of estimates of any long-range predictions.

6. Characteristic time between independent estimates

From sampling theory one can state that

$$\sigma_T{}^2 = \sigma^2/(T/T_0), \tag{4}$$

where $\sigma_T{}^2$ is the variance of the averages of T values, σ^2 the variance of the individual values, and T_0 a "characteristic time between independent estimates" (Leith, 1973). We can evaluate T_0 for various averaging times T by taking $\sigma_T{}^2$ from (3) and σ^2 from (2). The resulting characteristic times between in-

[5] The spectra in Fig. 4 differ somewhat from those shown in Fig. 2 of B because Fig. 4 is based on 96 monthly means, while those of Fig. 2 in B are based on 64 seasonal means. In addition, a linear trend was removed from the 64-year records studied in B.

[6] The variance of a sine wave is the amplitude squared divided by 2.

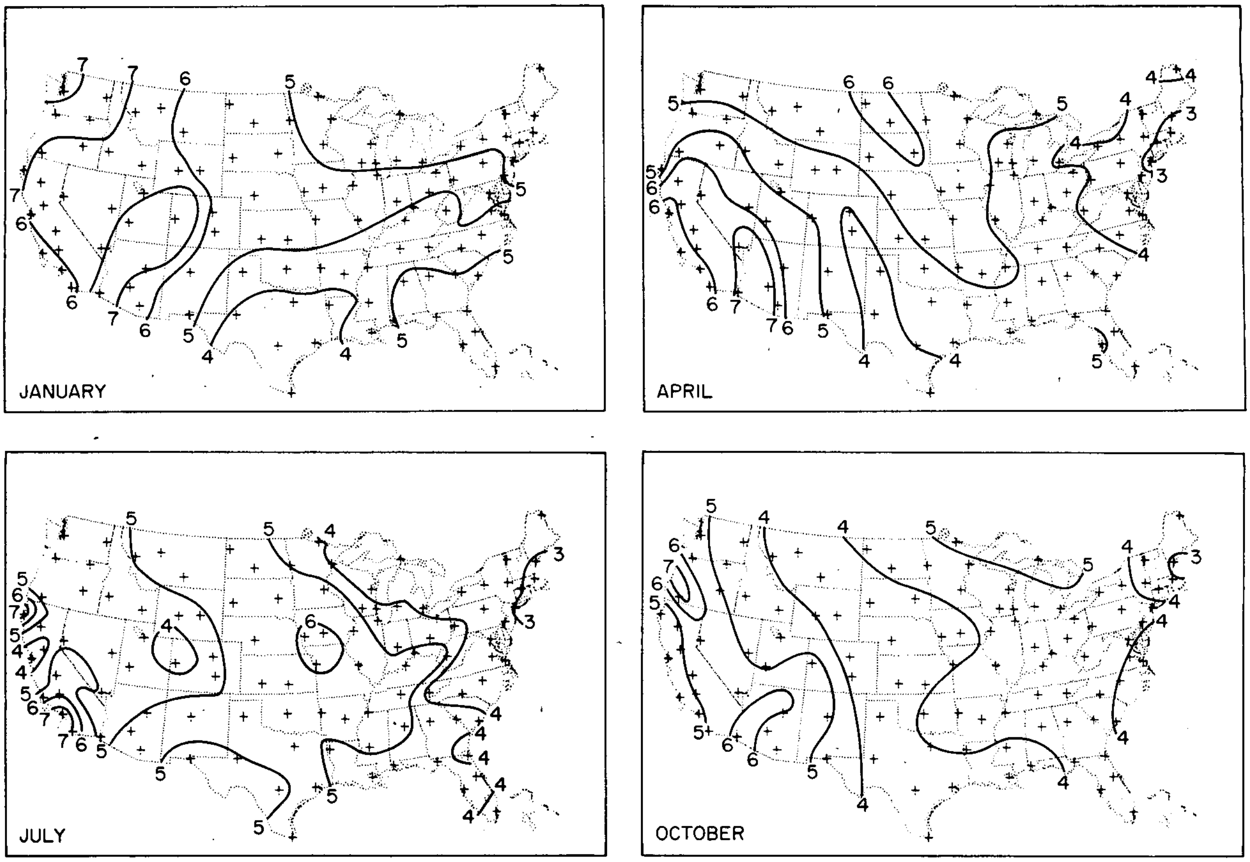

FIG. 5. Characteristic time in days between effectively independent sample values.

dependent estimates range from less than three days at some stations in the Northeast during April, July and October to more than seven days at stations in Southwest and Far West during all months (Fig. 5). T_0 for temperature is two to three times bigger than that reported for sea level pressure in A.

7. Discussion

Estimates of the natural variability of time-averaged temperature have been made. We argue that this variability is essentially unpredictable and places important limitations on our ability to make long-range predictions and even to detect possible climate change.

Actual interannual variability exceeds the natural variability at several locations. This additional variability may be similarly unpredictable, but for the present we have considered it to be potentially predictable. The ratios σ_A^2/σ_T^2 over the United States tend to be bigger for temperature data than those reported earlier for sea level pressure data suggesting that the potential for long-range predictions of temperatures over the United States is somewhat greater than that for pressure.

We propose that the natural variability provides lower limits for the standard error of estimates of any long-range prediction. While it is important for us to gain understanding of temperature variations on all time scales, their amplitude will have to be large relative to this standard error if such understanding is to provide immediate, practical improvement in long-range prediction.

Finally we have found that the characteristic time between independent estimates is two to three times bigger for temperature data than for sea-level pressure data. This reflects that fact that there is more day-to-day persistence in temperature.

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