Cumulus Parameterization and Rainfall Rates I

T. N. KRISHNAMURTI, Y. RAMANATHAN, HUA-LU PAN,
RICHARD J. PASCH AND JOHN MOLINARI

Department of Meteorology, Florida State University, Tallahassee 32306

(Manuscript received 24 August 1979, in final form 23 December 1979)

ABSTRACT

Modeling of convective rainfall rates is a central problem in tropical meteorology. Toward numerical weather prediction efforts the semi-prognostic approach (i.e., a one time-step prediction of rainfall rates) provides a relevant test of cumulus parameterization methods. In this paper we compare five currently available cumulus parameterization schemes using the semi-prognostic approach. The calculated rainfall rates are compared with observed estimates provided in the recent publication of Hudlow and Patterson (1979). Among these the scheme proposed by Kuo (1974) provides the least root-mean-square error between the calculated and the observed estimates, slightly better than that of Arakawa and Schubert (1974), which was used by Lord (1978a). The simplicity of the approach holds promise for numerical weather prediction. Unlike some of the other schemes this method is not sensitive to and does not require computation of internal parameters such as profiles of cloud mass flux updrafts and downdrafts, detrainment of cloud matter and entrainment of environmental air. The present paper does not address the prognostic evolution and verification of the vertical distribution of temperature, humidity or momentum. These will be compared for the different methods in more detail separately.

1. Introduction

The specification of convective rainfall rates as a function of large-scale meteorological variables is an important part of the cumulus parameterization problem. Successful demonstrations of its feasibility were recently presented by Thompson *et al.* (1979), Lord (1978a), Yanai *et al.* (1973), Ogura and Cho (1973) and several others. This class of studies had as a common ingredient the availability of upper air data over a local polygonal array (over an area ~500 km^2) and somewhat reliable measures of the observed rainfall rates (e.g., Hudlow and Patterson, 1979). The observations from field projects such as GATE (the GARP Atlantic Tropical Experiment) have provided some of the most promising tests for this problem.

Historically, two approaches have been followed: the diagnostic approach and the prognostic approach. The diagnostic approach assumes the tendencies of large-scale meteorological variables are known and the cumulus-scale contributions such as vertical eddy fluxes of heat, moisture, momentum and the rainfall rates are found as residuals. Diagnostic models usually describe the interactions between a cloud ensemble and the large-scale variables. The prognostic approach, more relevant to numerical weather prediction (NWP), is described by a different type of model. The closure for a prognostic model requires additional physical statements because the tendencies of the large-scale variables are unknown.

A series of studies on numerical weather prediction toward the central objectives of GATE are now being undertaken by our group (Krishnamurti *et al.*, 1979, 1980). This paper is an evaluation of the rainfall rates, utilizing the GATE A/B scale network data, during Phase III (a list of symbols and definition of acronyms is presented in Table 1). Here we compare five schemes that are currently used in numerical weather prediction models. The five schemes are (i) hard convective adjustment; (ii) soft convective adjustment; (iii) the Arakawa-Schubert cumulus parameterization scheme; (iv) Kuo's scheme based on his 1965 study; and (v) Kuo's scheme based on his 1974 study. The convective rainfall rates are calculated in each series of experiments utilizing the so-called semi-prognostic (Silva-Dias *et al.*, 1977) approach, i.e., a one time-step prediction. Data for every 6 h are processed and analyzed over the GATE A/B scale, with a vertical resolution of 25 mb following Thompson *et al.* (1979).

2. The GATE A/B scale observed rainfall and upper air data

The observed rainfall rates used in the present study are adapted from the studies of Hudlow and Patterson (1979), Thompson *et al.* (1979) and Lord

0027-0644/80/040465-08$06.00

TABLE 1. List of useful symbols and acronyms.

Symbol	Meaning
GATE	GARP Atlantic Tropical Experiment
PHASE III	the period 1–18 September 1974
E_s	moist static energy
q_s	saturation specific humidity
z_s	geopotential height (subscript s indicates values after hard adjustment)
T_s	absolute temperature
q_I	initial specific humidity
e_s	saturation vapor pressure
σ	fraction of grid scale area in which hard adjustment occurs
η	relative humidity; also normalized cloud mass flux distribution
$\eta'(n,\lambda)$	detraining rate of precipitation
R	rainfall rate
λ	parameter for the fractional entrainment rates to distinguish cloud types
m_B	cloud base mass flux
p_B	cloud base pressure level
l	liquid water content
C_0	conversion rate between liquid water and rain drops
b	moistening coefficient
I	available moisture supply

(1978a). Here the radar reflectivities are first calibrated against the GATE ship raingage data sets. The final product undergoes a space-average over ¼° latitude/longitude squares, and time smoothing using a 1-2-1 filter. The time smoother acts as a filter on the $2\Delta\tau = 12$ h rainfall fluctuations, the sampling interval $\Delta\tau$ being 6 h. The rainfall data sets obtained in this manner appear to be well suited for tests on cumulus parameterization procedures, which require a smoothing of the higher frequency oscillations from squall line systems. The time history of the GATE B-scale observed rainfall rates for an extended period are shown in Figs. 1–5 (dashed lines). The upper air data sets used in this study were extracted by Murakami (1979).

3. Hard convective adjustment

The solution of the hard convective adjustment is obtained from the following four equations (Krishnamurti and Moxim, 1971):

Conservation of moist static energy

$$gz_s + c_pT_s + Lq_s = E_S \tag{1}$$

Hydrostatic law

$$\frac{\partial}{\partial p} gz_s = \frac{RT_s}{p}(1 + 0.61q_s) \tag{2}$$

Tetens law

$$e_s = 6.11 \exp\left[\frac{a(T_s - 273.16)}{(T_s - b)}\right] \tag{3}$$

For saturation over water $a = 17.27$; $b = 35.86$.
For saturation over ice $a = 21.87$; $b = 7.66$.

Relation between saturation vapor pressure and specific humidity

$$q_s = \frac{0.622e_s}{p - 0.378e_s}. \tag{4}$$

The subscript s refers to the saturated sounding present after the hard adjustment. Eqs. (1)–(4) are solved by an iterative procedure over the part of the sounding for which $-\partial\theta_e/\partial p < 0$. The value of the moist static energy E_s of the adjusted sounding is the average moist static energy of the initial sounding over the unstable layer.

The drastic consequence of a hard adjustment in NWP was discussed by Krishnamurti and Moxim (1971). The rainfall rates are expressed by,

$$R_{\text{HARD}} = \frac{1}{g}\int_{p_T}^{p_B} (q_I - q_s)\frac{dp}{\Delta\tau}, \tag{5}$$

where $\Delta\tau$ is a time scale of convective adjustment usually taken equal to a representative live time of a tall cumulonimbus, i.e., ~30 min. The adjustment produces a cooling and as a consequence a substantial reduction of moisture ($q_I > q_s$) in the lower troposphere, while the converse is the case in the upper troposphere. Since q_I and q_s decrease almost exponentially with height in the atmosphere, the loss of moisture in lower layers far exceeds the gain in the upper levels, and the implied rainfall rates at a point are large. Furthermore, since the hard adjustment, by definition, is invoked on the entire grid scale, the grid-scale rainfall rates come out quite large. Fig. 1 illustrates the rainfall rates from the hard adjustment for the entire third phase of GATE. The gross overestimation of rainfall rates is apparent. In actual NWP practice, if hard adjustment is invoked continuously, the largest errors occur in the first time step (Krishnamurti and Moxim, 1971). From then on the thermal and moisture structure of weak tropical disturbances are usually destroyed beyond any hope of recovery. Although subsequent rainfall rates are usually much smaller, the loss of a reasonable vertical structure of temperature and moisture in disturbances makes it unsuitable for NWP. Several authors (Manabe *et al.*, 1965; Miyakoda, 1969; Kurihara, 1973) have proposed soft adjustment as a way out of this predicament. Here the large-scale conditional instability is preserved during the adjustment process and there is hope for maintaining the weak thermal and humidity gradients of tropical waves. Next we shall examine a formulation of a soft adjustment for the GATE rainfall rate problem, which differs somewhat in formulation from the schemes mentioned above.

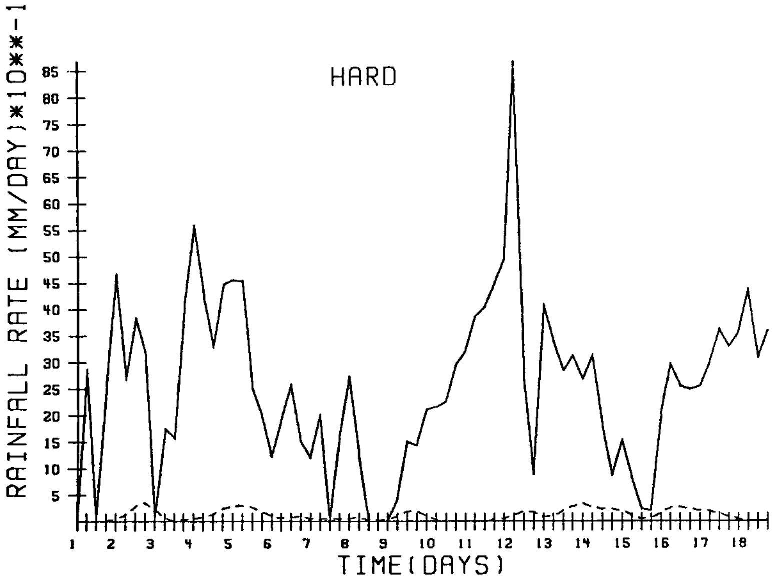

FIG. 1. Comparison of observed (dashed line) and predicted (solid line) rainfall rates (mm day^{-1}) using hard convective adjustment. Days 1 to 18 correspond to the third phase of GATE between 1 September and 18 September 1974. Data are for 6 h intervals beginning with 0000 GMT 1 September.

4. Soft convective adjustment

The soft adjustment can provide reasonable rainfall rates. The formulation used here assumes that hard adjustment occurs over a fraction σ of the grid-scale area. Over the remaining area $(1 - \sigma)$ it is assumed that the vertical profiles of temperature and humidity remain invariant during one time step due to various, unstated, physical processes. The final sounding (z_s, T_s, q_s) in the σ region is determined from the construction of a moist adiabat. At this stage, σ is still an unknown. This is determined from specified relative humidity criteria. The final temperatures and specific humidity on the grid scale may be written as

$$T_F = \sigma T_s + (1 - \sigma)T_I, \tag{6}$$

$$q_F = \sigma q_s + (1 - \sigma)q_I, \tag{7}$$

where T_I, q_I are the initial large-scale values.

The relative humidity η is given by

$$\eta = q_F/q_s(T_F), \tag{8}$$

where $q_s(T_F)$ is the saturation specific humidity at the final temperature.

The mean relative humidity is given by

$$\bar{\eta} = (p_B - p_T)^{-1} \int_{p_T}^{p_B} \eta dp. \tag{9}$$

In our calculations we have always found a single value of σ for a given $\bar{\eta}$ in the range $0 \leqslant \sigma \leqslant 1$ and $0 \leqslant \bar{\eta} \leqslant 1$. The value of σ is determined by a successive correction method by assigning a sequence of values of σ toward a convergence on the assigned value of $\bar{\eta}$. The rainfall rate for soft convective adjustment is given by

$$R_{\text{SOFT}} = \frac{1}{g} \int_{p_T}^{p_B} (q_I - q_F)dp/\Delta\tau, \tag{10}$$

where $\Delta\tau$ is a time scale of convective adjustment and is assigned a value of 30 min, as before.

A number of values of $\bar{\eta}$ were assigned and the computed rainfall rates were compared with the observed magnitudes for Phase III of GATE. For values of $\bar{\eta} = 82.4\%$ and $\sigma = 0.037$, the agreement between observations and calculations was the best (in the rms sense) and the results are illustrated in Fig. 2. The rms value for the soft convective adjustment for the 82.4% criterion is 11.5 mm day^{-1}.

Observations generally show that prior to the arrival of African waves, the conditional instability reaches a maximum. This takes place some 1–2 days prior to the occurrence of heavy rainfall. An undesirable aspect of hard and soft convective adjustment is that by removing this conditional instability they tend to produce the heaviest rainfall amounts much earlier. A phase lag is evident in Fig. 2. In view of this feature, the use of this scheme for the prediction of tropical waves seems undesirable.

5. The Arakawa-Schubert parameterization and Lord's estimates of rainfall rates

The Arakawa and Schubert (1974) cumulus parameterization scheme has been used by Lord

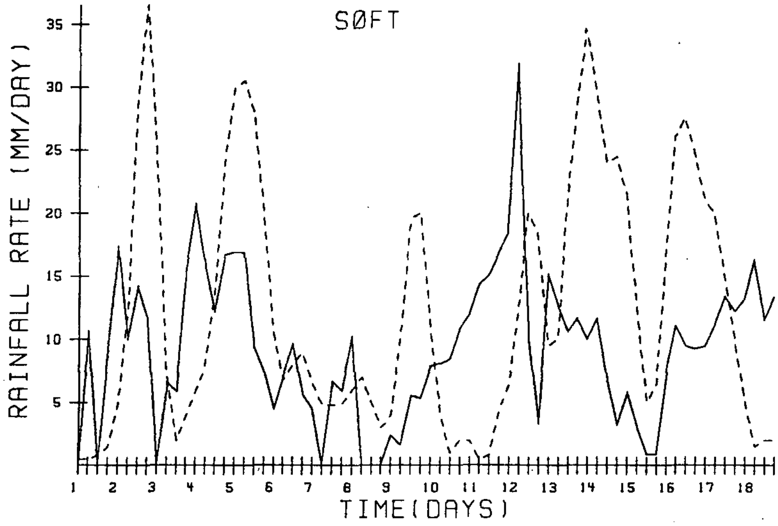

FIG. 2. As in Fig. 1 except for using soft convective adjustment.

(1978a) to predict precipitation rates as well as cumulus-scale warming and drying over the synoptic-scale grid. Here a single positive parameter λ, which is a measure of the fractional rate of entrainment, characterizes a cloud type within an ensemble. The scheme is based on a quasi-equilibrium hypothesis which states a quasi-balance between the generation of moist convective instability by large-scale dynamical processes and its stabilization by clouds. The cycle of events may be schematically outlined in the following manner: Large-scale advective and radiative processes provide a forcing for the thermodynamic large-scale variables in the environment. They increase the moist convective instability of the atmospheric column. Cumulus clouds will then develop. Buoyancy forces, due to release of latent heat, generate kinetic energy of updrafts within the cumulus ensemble. Modification of the environment by compensating subsidence and associated warming and drying will produce stabilization, a decreased buoyancy force in the clouds and kinetic energy dissipation. The properties of the cumulus ensemble required for the prediction of the large-scale changes are 1) the mass flux distribution function at the top of the mixed layer, $m_B(\lambda)$; 2) a normalized mass flux distribution $\eta(p,\lambda) = [m_B(\lambda)]^{-1} \times$ mass flux at a pressure level; and 3) the rain rate $R(p,\lambda)$ which is assessed from a conversion rate of liquid water into raindrops.

A simple model of a cumulus cloud ensemble is considered for n cloud types each of which detrains at the top of the n respective vertical levels. In this model, the storage term is neglected after summation over all subensemble members in the budget equations. Implicit in the equations and computational procedure are the following assumptions. The cloud ensemble occupies a fraction $\sigma \ll 1$ of the total area of the clouds and environment; clouds are saturated; they detrain only at the level at which they lose their buoyancy while entrainment is assumed to take place at all levels; no overshooting by Cb clouds is considered; downdrafts, multilevel detrainment and ice phase transitions are usually ignored; evaporation of cloud water occurs only when it detrains; rainfall rate is proportional to the liquid water content; and interaction between cloud types is not direct but only through their modification of the environment.

Though Arakawa and Schubert specifically include the effect of cumulus subsidence on the planetary boundary layer depth, cloud base p_B is assumed for simplicity to be 950 mb. Lord (1978a) included the effects of multilayer detrainment in some of his experiments and the effect of ice phase transitions in his primary results. Rainfall rates (R) are estimated from

$$R\Delta t = \sum_{\lambda=\lambda_{\max}}^{0} \sum_{n=1}^{N} k(n,\lambda) m_B(\lambda) \Delta t, \qquad (11)$$

where $k(n,\lambda)$ is the rain-production term for a layer n penetrated by cloud type λ, and N is the total number of such layers. Here $k(n,\lambda) = \eta'(n,\lambda) C_0 l(n,z) \Delta z(n)$, where $\eta'(n,\lambda)$ is the detraining rate of precipitation per $m_B(\lambda)$ and $C_0 = 2 \times 10^{-3}$ m^{-1} is the conversion rate between liquid water and raindrops. It should be noted that unlike the schemes of cumulus parameterization that depend on moisture convergence, here the magnitude of rainfall rates is mostly determined by $m_B(\lambda)$ which in turn depends largely on the thermal forcing.

Fig. 3, adapted from the study of Lord (1978a), illustrates the calculated versus observed rainfall

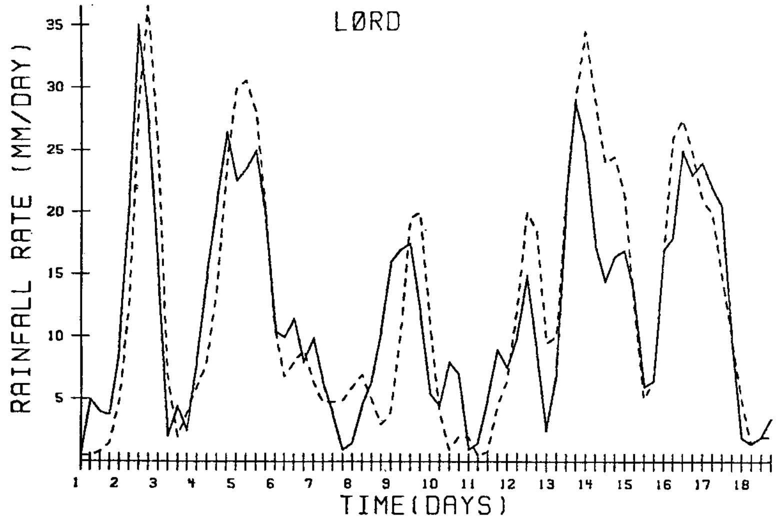

FIG. 3. As in Fig. 1 except for using Lord's (1978a) scheme.

rates during the third phase of GATE. The correspondence between the two curves is very close and provides the first most promising results toward the central objectives for GATE. The rms error of the computed values is ~5.1 mm day^{-1}.

6. Kuo's scheme I with a disproportionate partitioning of moistening and heating

Kuo's (1965) well-known scheme for the parameterization of cumulus convection has been remarkably successful in hurricane simulation studies (Rosenthal, 1970; Mathur, 1974; and many others). However, it was also recognized that it underestimated the convective rainfall (and heating) rates in large-scale tropical applications (Krishnamurti *et al.*, 1976; Anthes, 1977; Carr and Bosart, 1978; and several others). According to this theory, the total large-scale supply of moisture I is partitioned into two parts, one for the moistening of the vertical column $(q_c - q)/\Delta\tau$ and the other toward the heating of the column $(c_p/L)(T_c - T)/\Delta\tau$, the proportions being determined by the relative magnitudes of the above expressions. The subscript c refers to values on a moist adiabat. Lord (1978b) examined the rainfall rates (during the third phase of GATE) implied by the partitioning and quite correctly concluded that the rainfall rates were in poor agreement with the observed rainfall rates presented by Hudlow and Patterson (1979). In Fig. 4 we present the comparison of observed versus the calculated rainfall rates for this scheme for the

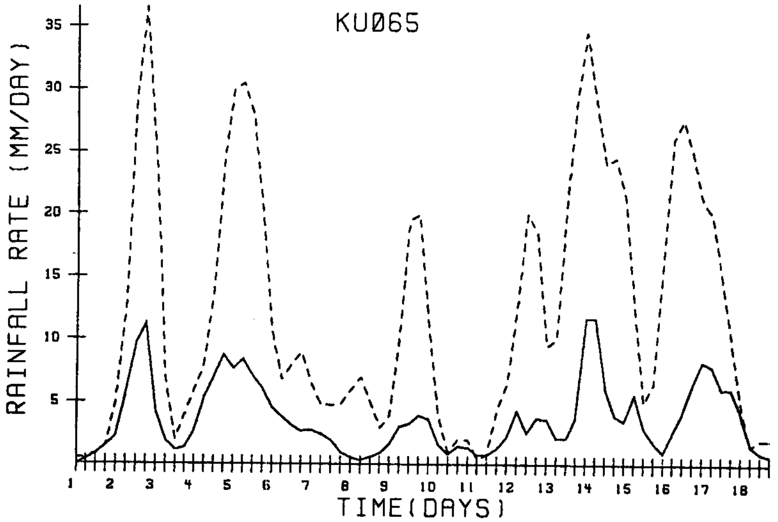

FIG. 4. As in Fig. 1 except for using Kuo's (1965) scheme.

entire third phase of GATE. We attribute this discrepancy to the disproportionate partitioning of the moisture supply (Kuo, 1974; Krishnamurti *et al.*, 1976; Kanamitsu, 1975; Anthes, 1977). In this formulation an excessive amount of the large-scale supply of moisture goes toward moistening of the columns so that the amount precipitated is too small. Kuo (1974) was among the first to recognize this shortcoming and he emphasized the need for a more reasonable subdivision of this moisture supply.

7. Kuo's scheme II with a flexible partitioning of the available moisture supply

Using the well known observations of Reed and Recker (1971), both Kuo (1974) and Cho and Ogura (1974) found that only a very small amount of the available moisture supply I is used for the moistening of the column. The expression $I = R + Ib$ [or alternatively $R = (1 - b)I$] is used in these diagnostic calculations. Here R is the rainfall rate and b the percentage of the moisture supply which is not precipitated. They noted a value of b in the range 0.23–0.36 in the region west of the trough of the composited ITCZ wave of Reed and Recker (1971).

The GATE A/B scale observations during the third phase of GATE were subjected to an analysis similar to that of Kuo (1974) and Cho and Ogura (1974). In the search for a value of the moistening parameter b, rms errors were calculated between the observed rainfall rates and various components of the available moisture supply I. Here I is defined by

$$I = -\frac{1}{g}\int_{p_T}^{p_B}\left(\nabla\cdot q\mathbf{V} + \frac{\partial}{\partial p}q\omega\right)dp. \tag{12}$$

The components of I were defined as $I = I1 + I2 = I3 + I4$, where

$$I1 = -\frac{1}{g}\int_{p_T}^{p_B}\nabla\cdot q\mathbf{V}dp, \tag{13}$$

$$I2 = -\frac{1}{g}\int_{p_T}^{p_B}\frac{\partial}{\partial p}q\omega dp, \tag{14}$$

$$I3 = -\frac{1}{g}\int_{p_T}^{p_B}\mathbf{V}\cdot\nabla q dp, \tag{15}$$

$$I4 = -\frac{1}{g}\int_{p_T}^{p_B}\omega\frac{\partial q}{\partial p}dp. \tag{16}$$

Functions $I1$, $I2$, $I3$ and $I4$ were calculated with the 3 h data sets using the same computational methods as in Thompson *et al.* (1979). Among these a surprisingly close agreement between the rainfall rates (R) and the function $I4$ was noted. This suggested that in the relation

$$I = bI + R$$
$$\text{Moistening} = \text{rain} + \text{fallout},$$

we define

$$b = -\frac{1}{g}\int_{p_T}^{p_B}\mathbf{V}\cdot\nabla q dp/I, \tag{17}$$

$$R = -\frac{1}{g}\int_{p_T}^{p_B}\omega\frac{\partial q}{\partial p}dp. \tag{18}$$

These relations are used to provide the necessary closure for the Kuo's (1974) parameterization scheme (see also Cho and Ogura, 1974; Krishnamurti *et al.*, 1976). The closure is not intended to imply a physical process, since we are not sure of its validity in other regions or at different time periods. We subdivide the total available large-scale moisture supply into two parts, contributions 1) by the horizontal advection and 2) by the vertical advection. The GATE A/B scale data sets during the third phase (18 days) show that the storage and rainfall rates are close to the two parts, respectively. Fig. 5 illustrates the correspondence between the observed and the calculated rainfall rates for the proposed closure of Kuo's (1974) scheme. The agreement between the two curves is extremely good. The construction of the remaining aspects of the parameterization scheme for NWP is analogous to that presented in Krishnamurti *et al.* (1976). Since the objective of this study is on the semi-prognostic calculations of rainfall rates, we shall not go into further details on the parameterization theory here.

8. Discussion

In Table 2 the rms errors for the five different parameterization schemes are shown. The largest errors occur for the hard convective adjustment where a portion of the sounding is replaced by a local moist adiabat by raising a parcel from the earth's surface. The reasons for the excessively large rainfall rates calculated by this scheme have been discussed in Section 3. The soft adjustment provides some improvement in the estimates of the rainfall rates.

As discussed earlier, a major defect of the adjustment schemes is related to the occurrence of maximum conditional instability 1–2 days prior to the heavy rainfall events. Convective adjustment is invoked if conditional instability exists and upward motion is found at the top of the planetary boundary layer (in these calculations, 900 mb). These criteria seem to be satisfied prior to rainfall occurrence and thus a phase shift of the convectively adjusted rainfall rate is noted. Lord (1978a) and Thompson *et al.* (1979) have also alluded to a phase shift

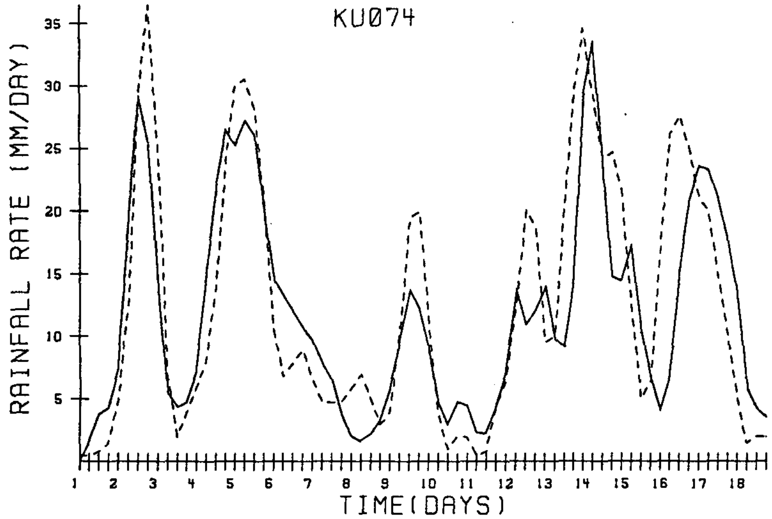

FIG. 5. As in Fig. 1 except for using Kuo's (1974) scheme.

between the moisture convergence and the occurrence of heavy rainfall. This may be related to the somewhat delayed response of the cloud and anvil buildups to the field of convergence. This slight lag is evident in Kuo's schemes as well as in Lord's computations.

The work of Lord (1978a), summarized here, incorporates the Arakawa and Schubert (1974) parameterization scheme. This is a major contribution to the central objectives of GATE. The rms errors between the predicted (semi-prognostic one time-step prediction) and the observed rainfall rate is approximately 5.1 mm day^{-1}. What is astounding about this calculation is the manner in which the calculated rainfall rates increase and decrease, describing quite closely the observed rainfall rates for all the tropical wave passages. This, no doubt, also includes heavy rainfall rates arising from the passage of squall systems. This may imply that the rainfall from squall systems, in fact, is expressible as a function of the large-scale variables, i.e., the two scales somehow seem to know the existence of each other even though the squall lines seem to move westward at about twice the speed of the wave. This is an interesting, unsolved problem.

Among Kuo's techniques discussed here, the first scheme places a stringent restriction on the partitioning of the available moisture supply by the large-scale motions. The semi-prognostic rainfall rates by the first scheme underestimate the observed magnitudes considerably. The second scheme provides the smallest rms error of those presented here, slightly less than that of the Arakawa-Schubert-Lord method. The flexible partitioning of available moisture supply is used here (with the GATE A/B observations) to determine a moistening parameter *b*. Although the aforementioned tests were carried out for the GATE A/B scale network, we do not feel that the problem of specification of rainfall rates as a function of large-scale variables is necessarily solved for other regions or for other periods.

The advantage of Kuo's schemes has long been recognized in NWP practice. The method is appealing in that, as a parameterization procedure, it provides immediate measures of the cumulus-scale heat and moisture fluxes in terms of the measurable large-scale variables, without having to compute cloud dynamical processes (such as entrainment, detrainment and downdrafts) and cloud microphysical processes.

The semi-prognostic estimation of reasonable *rainfall rates* is only a small step toward the solution of the parameterization problem. As one examines the complete problem, the need for a careful reassessment of research goals becomes clearer. For example, the following parameters frequently appearing in parameterization theory are not well known from observations:

1) LIQUID WATER. Only a few incomplete vertical and/or horizontal profiles are available from research aircraft probes.

2) DETRAINMENT OF LIQUID WATER. The many vertical profiles that have appeared in diagnostic

TABLE 2. Root mean square errors of calculated rainfall rates. (Third phase of GATE.)

Parameterization scheme	rms error (mm day^{-1})
1. Hard convective adjustment	288
2. Soft convective adjustment	11.5
3. Arakawa-Schubert-Lord method	5.1
4. Kuo's scheme I	11.8
5. Kuo's scheme II	4.9

or semi-prognostic budget studies have not been observationally verified.

3) ENTRAINMENT OF ENVIRONMENT AIR. No adequate observations exist on the appropriate scale or define the vertical variations which are usually portrayed in budget studies.

4) VERTICAL EDDY FLUXES OF MOIST STATIC ENERGY, MOISTURE AND DRY STATIC ENERGY. No direct observational verification of these detailed profiles has been possible.

5) VERTICAL DISTRIBUTION OF CONDENSATION AND EVAPORATION. These again appear in most budget studies and whether or not they are parameterized adequately cannot be directly verified against available observations. The radar reflectivity gives some quantitative measures. However, its interpretation does not seem very straightforward.

6) VERTICAL MASS FLUX DISTRIBUTIONS (UPDRAFTS AND DOWNDRAFTS). Although much emphasis has appeared in the literature on determining detailed profiles in budget studies, only isolated research aircraft probes have given some indication of what they look like at some levels and over some parts of a cloud cluster.

The use of parameterization procedures in prognostic models does provide an avenue for further observational verification of the effects of convection on the evolution of the vertical profiles of momentum, temperature and humidity, rainfall rates and pressure. The drawback in the use of prognostic models for testing a parameterization of a physical process is that the models usually contain many errors arising from horizontal, vertical and time-differencing schemes, other parameterized processes such as those of the planetary boundary layer, incorporation of inadequate mountains, initialization and boundary conditions. However, in spite of these shortcomings, the full impact of parameterization will only be realized with the actual prediction of weather events. Results for the fully predictive approach are in preparation and will be reported later.

Acknowledgments. Helpful comments on this work were received from J. Smagorinsky, S. Manabe, Y. Kurihara and K. Miyakoda at a presentation of this work at Princeton University last spring. This work is supported by the National Science Foundation under Grant ATM 78-19363. The computations were performed at the National Center for Atmospheric Research, which is sponsored by the National Science Foundation.

REFERENCES

Anthes, R. A., 1977: A cumulus parameterization scheme utilizing a one-dimensional cloud model. *Mon. Wea. Rev.*, **105**, 270–286.

Arakawa, A., and W. H. Schubert, 1974: Interaction of a cumulus cloud ensemble with the large-scale environment, Part I. *J. Atmos. Sci.*, **31**, 674–701.

Carr, F. H., and L. F. Bosart, 1978: A diagnostic evaluation of rainfall predictability for tropical storm Agnes, June 1972. *Mon. Wea. Rev.*, **106**, 363–374.

Cho, H. R., and Y. Ogura, 1974: A relationship between cloud activity and the low-level convergence as observed in Reed-Recker's composite easterly waves. *J. Atmos. Sci.*, **31**, 2058–2065.

Hudlow, M. D., and V. L. Patterson, 1979: GATE Radar Rainfall Atlas. Published as a NOAA special report by U.S. Dept. of Commerce, 155 pp. [U.S. Govt. Printing Office.]

Kanamitsu, M., 1975: On numerical prediction over a global tropical belt. Rep. No. 75-1, based on Ph.D. dissertation, Department of Meteorology, Florida State University, 282 pp.

Krishnamurti, T. N., and W. J. Moxim, 1971: On parameterization of convective and nonconvective latent heat release. *J. Appl. Meteor.*, **10**, 3–13.

——, R. J. Pasch and P. Ardanuy, 1980: Prediction of African waves and specification of squall lines. *Tellus*, **32** (in press).

——, H. Pan, C. B. Chang, J. Ploshay and W. Oodaly, 1979: Numerical weather prediction for GATE. *Quart. J. Roy. Meteor. Soc.*, **105**, 979–1010.

——, M. Kanamitsu, R. Godbole, C. B. Chang, F. Carr and J. H. Chow, 1976: Study of monsoon depression II, dynamical structure. *J. Meteor. Soc. Japan*, **54**, 208–226.

Kuo, H. L., 1965: On formation and intensification of tropical cyclones through latent heat release by cumulus convection. *J. Atmos. Sci.*, **22**, 40–63.

——, 1974: Further studies of the parameterization of the influence of cumulus convection on large-scale flow. *J. Atmos. Sci.*, **31**, 1232–1240.

Kurihara, Y., 1973: A scheme of moist convective adjustment. *Mon. Wea. Rev.*, **101**, 547–553.

Lord, S. J., 1978a: Development and observational verification of a cumulus cloud parameterization. Report based on Ph.D. dissertation, Department of Atmospheric Science, University of California, Los Angeles, 359 pp.

——, 1978b: An observational verification of the Arakawa-Schubert cumulus parameterization. *Workshop on the Parameterization of Cumulus Convection*, 23–25 October, European Centre for Medium Range Forecasts, 104–150.

Mathur, M. B., 1974: A multiple-grid primitive equation model to simulate the development of an asymmetric hurricane (Isbell, 1964). *J. Atmos. Sci.*, **31**, 371–393.

Manabe, S., J. Smagorinsky and R. F. Strickler, 1965: Simulated climatology of a general circulation model with a hydrologic cycle. *Mon. Wea. Rev.*, **93**, 769–798.

Miyakoda, K. J., Smagorinsky, R. F. Strickler and G. D. Hembree, 1969: Experimental extended predictions with a nine-level hemispheric model. *Mon. Wea. Rev.*, **97**, 1–76.

Murakami, M., 1979: Large-scale aspects of deep convective activity over the GATE area. *Mon. Wea. Rev.*, **107**, 994–1013.

Ogura, Y., and H. R. Cho, 1973: Diagnostic determination of cumulus cloud populations from observed large-scale variables. *J. Atmos. Sci.*, **30**, 1276–1286.

Reed, R. J., and E. E. Recker, 1971: Structure and properties of synoptic-scale disturbances in the equatorial Western Pacific. *J. Atmos. Sci.*, **28**, 1117–1133.

Rosenthal, S. L., 1970: A circularly symmetric primitive equation model of tropical cyclone development containing an explicit water vapor cycle. *Mon. Wea. Rev.*, **98**, 643–663.

Silva-Dias, P., S. Lord and W. Schubert, 1977: Studies using spectral parameterization theory. U.S. GATE Central Workshop Report, NCAR, 569–574.

Thompson, R. M., Jr., S. W. Payne, E. E. Recker and R. J. Reed, 1979: Structure and properties of synoptic-scale wave disturbances in the Intertropical Convergence Zone of the eastern Atlantic. *J. Atmos. Sci.*, **36**, 53–72.

Yanai, M., S. Esbensen and J. -H. Chu, 1973: Determination of bulk properties of tropical cloud clusters from large-scale heat and moisture budgets. *J. Atmos. Sci.*, **30**, 611–627.