Objective Determination of Hurricane Tracks from Aircraft Observations

HUGH E. WILLOUGHBY AND MARCY B. CHELMOW[1]

National Hurricane Research Laboratory, NOAA/AOML, Coral Gables, FL 33146

(Manuscript received 29 September 1981, in final form 26 May 1982)

ABSTRACT

An algorithm for location of hurricane centers by least squares using aircraft data has been developed. As the aircraft traverses the eye, lines of position normal to the wind are constructed each 100 m along its track. An additional line of position is constructed normal to the track at the closest point of approach to the center. The center coordinates are then chosen such that the sum of the squares of the normal distances from the center to the lines of position is minimized. A cubic spline storm track is first constructed using centers based on winds in a coordinate system fixed to the earth. A track based upon winds in moving, storm-centered coordinates may be obtained by transformation of the winds into such a coordinate system and iterative redetermination of the centers.

For intense hurricanes, the centers can be located with an accuracy of 3 km and the mean motion over a period of four to six hours determined to within 4° of direction and 0.5 m s^{-1} of speed. The details of the track oscillations analyzed by this techique agree with those observed simultaneously by land-based radar. The technique is used routinely to prepare storm-centered composites and has potential for real-time operational application.

1. Introduction

Buys Ballot's or Farrell's law [see Kutzbach (1979) for an informative historical account] is essentially a statement of the gradient wind relation. It states that, in the Northern Hemisphere, the center of a cyclonic circulation lies to the observer's left as he faces downwind. This rule has wide application for the location of vortex centers in navigation and weather analysis. One application is in aircraft reconnaissance of tropical cyclones. During penetration of the eye, the aircraft track is directed 90° to the right of the direction from which the wind blows. Since the effect of cross-isobar flow is neglected, the aircraft may spiral into the center rather than penetrate along a radius. Eventually, calm winds followed by a reversal of wind direction mark the vortex center. The position where the wind speed is zero is reported as the "center fix." Although radar is also used during penetration and determination of the center, Buys Ballot's law, in conjunction with position and wind information from modern inertial navigation equipment (INE), remains the basis of much operational aircraft fix data.

On research aircraft equipped with digital data systems, data are available to the observer at intervals of 1 s. At typical airspeeds the spatial interval between observations is ~100 m. Aircrew members claim that the time of closest point of approach (CPA) to the center can be determined in real time to within 5 or 10 s, or ~0.5 to 1 km. Total position errors (including navigational errors) of 2–4 km (1–2 n mi) are commonly reported when fixes are transmitted to warning centers.

Neumann and Pelissier (1981) describe the sensitivity of the National Hurricane Center's (NHC) hurricane track forecast models to initial position data. Inaccurate initial positions contribute to errors in the Climatology and Persistence (CLIPER) model in two ways: uncertainties in the initial position itself account for a small percentage of the operational 24 h forecast error, whereas the additional uncertainties in the initial motion, combined with the position errors, account for more than a quarter (C. J. Neumann, personal communication, 1982) of the 24 h error. Since the output of the CLIPER model is an input to the statistical synoptic and statistical-dynamical models, improved methods for determination of position and, more importantly, motion seem to offer potential for improved short-term forecasts.

The remainder of this paper attempts to evaluate the accuracy and repeatability with which the dynamic center of the vortex, as distinct from the geometric center of the eye, can be tracked by aircraft. To this end we formulate an objective, least-squares algorithm for center location and present examples of tracks constructed using the algorithm and research aircraft observations. If the objective results support the accuracies claimed in operational fixes, it may well be that either further improvement of accuracy or more frequent observations can correct some of

---

[1] *Current affiliation:* Metropolitan Life Insurance Co., New York, NY 10010.

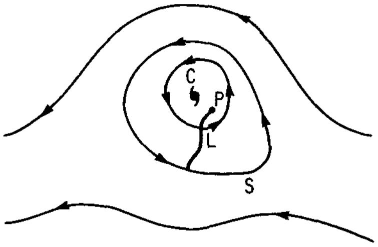

FIG. 1. The field of the streamfunction ψ that characterizes the rotational flow in a closed vortex with outermost closed streamline S and center C. The circulating flux for any point P is the flow of air across an arbitrary curve L joining P with S.

the track forecast errors due to inaccurate inputs of initial position or motion.

2. Formulation

The existence of the dynamic center may be demonstrated through consideration of the kinematics of a closed vortex. By Helmholtz's theorem, the horizontal wind field can be represented as the sum of an irrotational and a nondivergent part. Only the latter has nonzero circulation or vorticity. Let the nondivergent motions be described by a streamfunction ψ, such that the horizontal velocity $\mathbf{V}$ is given by $\mathbf{V} = \mathbf{k} \times \nabla\psi$, where $\mathbf{k}$ is the vertically pointing unit vector. Further, let the horizontal variations of density ρ be neglected.

In Fig. 1 the outermost closed streamline of the vortex is S. Along S, $\psi = \psi_s$, a constant. Since the flow along S is nonzero and cyclonic, ψ must everywhere decrease toward the inward side of S. Therefore, by Rolle's theorem of the calculus, ψ must have at least one local minimum (where $\mathbf{V} = \mathbf{k} \times \nabla\psi = 0$) within the region enclosed by S. Since the vortex is more or less circular, normals to the wind converge toward the minimum. Thus, if the speed of the divergent part of the wind can be neglected, the minimum is the dynamic center that an aircraft would locate—apart from possible ambiguity due to the presence of multiple local minima, such as described in Typhoon Abby by Huntley and Diercks (1981).

We now demonstrate an additional property of the dynamic center. For any point P within S, the horizontal flux of air across L, a curve of arbitrary geometry joining P with S is:

$$\Phi = \rho \int_{\mathrm{P}}^{\mathrm{S}} \mathbf{k} \times \nabla\psi \cdot (\mathbf{n} dL) = (\psi_{\mathrm{S}} - \psi_{\mathrm{P}}), \qquad (1)$$

where ψ_{P} is the value of the streamfunction at P and $\mathbf{n}$ is a unit vector pointing in the cyclonic sense normal to L. Thus, Φ, the "circulating flux" of air across a curve joining any given interior point to the streamline, is a constant, regardless of the geometry of the curve or the location of its endpoint on the streamline. If P is now made to correspond to C, the point where minimum ψ resides, the circulating flux is larger than for any other choice of P. The foregoing discussion shows that a dynamic center must exist in any closed vortex and that it is, in a fundamental sense, "the center" because it is the point within the vortex about which the circulating flux is largest.

Although a skillful navigator or flight director can locate the dynamic center by guiding the aircraft very nearly through it, we are interested in accurate center determination when the aircraft traverses the vortex core, but does not pass exactly through the center. The center coordinates can then be determined from wind and pressure data in a time window centered on the CPA. In this version of the algorithm, the window comprises observations taken 50 s before and after CPA to the center, so that all the observations used for center location are taken within the eye. The CPA itself can be approximated as the time when the minimum of either $V = |\mathbf{V}|$ or D, the departure of a selected isobaric height from its value in the standard atmosphere, is observed. In practice, however, the two minima seldom coincide exactly so that we identify the CPA as the minimum of H, a single arbitrarily designed function that incorporates both quantities:

$$H = V^2 + gD, \qquad (2)$$

where g is the gravitational acceleration. Both terms in (2) have units of velocity squared. The first term is positive–definite, and the second is generally negative with its magnitude increasing toward the center. H is usually positive in the outer vortex. Within the radius of maximum wind (RMW) it changes sign and attains large negative values near the dynamic center.

Fig. 2 shows the geometry of center location with the present algorithm. In an idealized circular vortex with gradient balance, a line drawn normal to the aircraft track at minimum H would pass through C and would constitute a line of position (LOP) for the center. Similarly, lines drawn normal to the wind each second along the track would be LOP's. Real hurricanes are somewhat asymmetric, and real data contain errors so that the LOP's do not intersect at a point, but converge in a region perhaps a kilometer or two in extent. A rational choice for the center coordinates $(X_{\mathrm{C}}, Y_{\mathrm{C}})$ would, in some sense, minimize the separation between the center and the closest points on the set of LOP's. We define this separation for the nth LOP to be s_n. An elementary geometric argument allows us to express these components in terms of (X_n, Y_n), the aircraft coordinates at the time of the nth observation, the (unknown) center coordinates and θ_n, the wind or track direction (measured clockwise from north), as appropriate:

$$s_n = (X_n - X_{\mathrm{C}}) \sin\theta_n + (Y_n - Y_{\mathrm{C}}) \cos\theta_n . \qquad (3)$$

With (3) it is possible to express S_L, the weighted rms value of s_n,

$$S_L^2 = (\sum W_n)^{-1} \sum W_n s_n^2, \tag{4}$$

where W_n is a weighting factor set to 1 when θ_n is a wind direction, or set to 10 when it is a track angle at CPA. The W_n's were chosen by trial and error. Generally the CPA LOP's contribute little to determination of the center unless the aircraft passes exactly through the center. In that case the wind LOP's are all nearly parallel to the track so that the CPA LOP is required to keep the center's location from being indeterminate. S_L^2 is minimized if X_C and Y_C are chosen such that:

$$\partial S_L^2/\partial X_C = 0, \quad \partial S_L^2/\partial Y_C = 0. \tag{5}$$

Solution of (5) for X_C and Y_C involves solution of a 2×2 linear system with coefficients that depend upon summed functions of X_n, Y_n and θ_n. This least-squares algorithm can compensate for asymmetries, cross-isobar flow and noisy data. Nevertheless, it is possible that some vortex geometries may result in the apparent center's displacement from the actual dynamic center. This may or may not be important in research applications, but in an operational setting it is sufficient that the centers represent trackable features of the circulation.

Once N successive centers have been located, the track is constructed by fitting with least squares a series of cubic splines to the center coordinates using time as a parameter. The acceleration is constrained to be zero at the ends of the track, so that the number of degrees of freedom (DF) is the number of splines minus 2. If DF = N, the track goes exactly through all the centers. By reducing the DF, increasing amounts of smoothing can be obtained. At the outset the DF = N track is unsatisfactory because it tends to meander and to have a center at the apex of each bend. The amplitude of the oscillations is about a kilometer so that they may arise from the inherent statistical uncertainty in location of the centers, navigational inaccuracy, or real but unresolved, high-frequency motions of the center. After some experimentation, two different amounts of smoothing seemed to have merit. In the first, DF = $N/2$ with upward rounding if N is odd. This is termed the detailed track because it is capable of following oscillations with periods of several hours, but effectively smooths most of the influence of occasional erroneous positions or of motion whose periods are comparable to the interval between center fixes ($\sim$1 h). In the second amount of smoothing, DF = 2. This is termed the mean track and represents roughly the same time resolution (6 h) as the "best tracks" that NHC prepares in post analysis. It is very nearly the same as a simple linear fit to the centers. As we shall see below, the mean track may have operational applications.

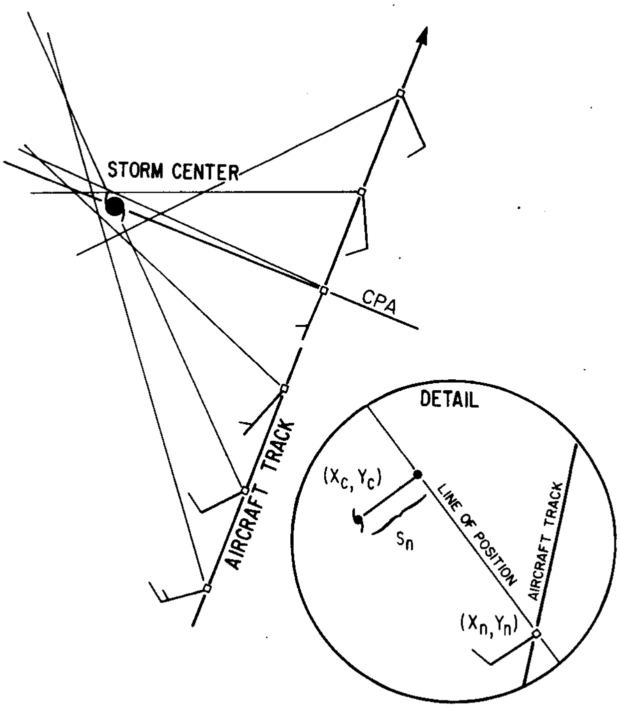

FIG. 2. The geometric relations among the aircraft track, lines of position (LOP's) and the dynamic center.

In research, when one wishes to composite data with respect to the moving storm center, it is desirable that the dynamic centering use winds expressed in a coordinate system moving with the storm rather than in one fixed to the Earth, that is, upon relative rather than absolute winds. This contrasts with operational applications which can use absolute winds. To obtain relative wind centers with the present algorithm, all the absolute centers are first determined, and the absolute track is approximated as described above. The coordinates of the center thus become available as continuous functions of time. The absolute track is then differentiated to obtain the center's velocity and the winds are transformed into relative coordinates. The transformed winds are then used iteratively to determine the relative centers and track. Generally, a single iteration is sufficient to provide convergence to within a few hundred meters. In practice, the track based upon realative winds is parallel to that based upon absolute winds and lies a few kilometers to its right facing in the direction of motion. This arises because conversion to relative winds makes the wind back ahead of the center and veer behind it. Thus the intersections of the LOP's move to the right of the track when the wind field is transformed.

3. Case studies

In this section, we present examples of centers and tracks determined with the present algorithm using data obtained by NOAA's Research Facilities Center

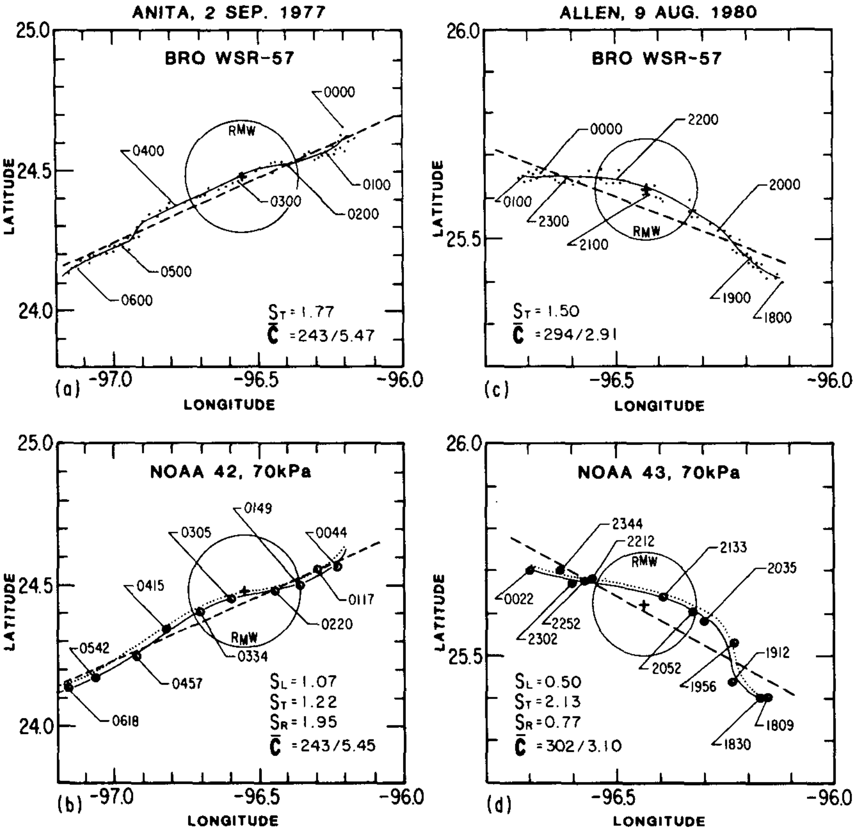

FIG. 3. Comparison between storm tracks determined by (a) radar and (b) aircraft in Hurricane Anita between 0000 and 0600 GMT on 2 September 1977. (c, d): A similar comparison for Hurricane Allen between 1800 and 0100 GMT on 9–10 August 1980. The solid and dashed curves represent, respectively, the detailed and mean tracks of the absolute-wind or radar centers, and the dotted curves in the lower panels are the relative-wind detailed tracks. The individual centers are marked by dots in the radar data and by circles in the aircraft data. The numbers connected to the centers indicate the time of observation (GMT). The large solid circles represent the radius of maximum wind (RMW) and are centered on the radar tracks. They are plotted in the same geographical position in both radar and aircraft panels to facilitate comparison.

(RFC) aircraft in Hurricanes Anita of 1977 and Allen of 1980. [See Lawrence (1978) and Lawrence and Pelissier (1981) for summaries of these storms.] These particular cases are selected because they allow comparison between independent storm tracks observed either by aircraft and land-based radar or by multiple aircraft.

Fig. 3 illustrates two sets of comparisons between tracks determined simultaneously by the Brownsville, TX WSR-57 radar and a single aircraft. The data describe Hurricane Anita on 2 September 1977 and Hurricane Allen on 9 August 1980. In Figs. 3 and 5 the following statistical measures of the centering algorithm's performance are displayed:

S_L: The rms separation in kilometers between the absolute-wind centers and closest points on the LOP's. This is the quantity that is minimized in locating the centers.

S_T: The rms separation between the absolute-wind (or radar) centers and the detailed track.

S_R: The rms separation between the absolute and relative centers.

$\bar{C}$: The velocity of the storm center computed from the mean track and expressed as direction (degrees)/speed (m s^{-1}).

The values of S_L and S_T are virtually identical to the corresponding quantities for the relative-wind centers, so that the latter quantities are not displayed. In Fig. 3 the magnitudes of S_L indicate that the LOP's define regions a kilometer or less in extent within which the centers lie. Since S_T is larger than S_L, the smoothing inherent in construction of the detailed track must compensate for other sources of error besides the statistical uncertainty represented by S_L. The detailed track thus appears to approximate the actual path of the absolute wind center to an accuracy approximately comparable to S_T rather than S_L. The values of S_R and the dotted tracks in Fig. 3 show that, as anticipated in the previous section, the relative-wind track parallels the absolute-wind track and lies somewhat to the right of it.

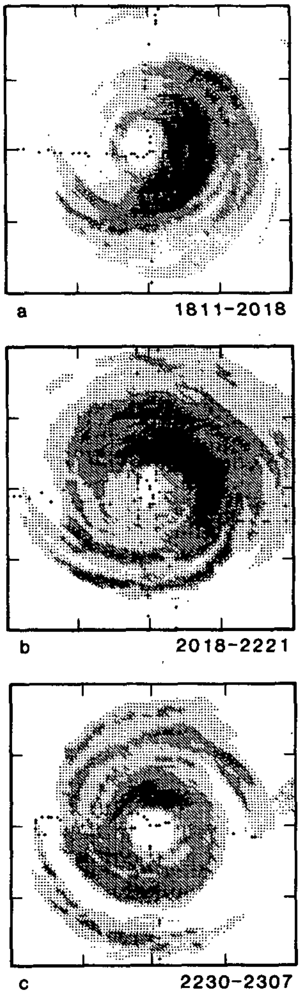

FIG. 4. Aircraft radar composites of Hurricane Allen's inner core for three time periods on 9 August 1980 showing the intense convective event that influenced the detailed track. The darkest shade indicates radar reflectivity > 40 dB(Z), and successively lighter shades indicate >35, >30 and >25 dB(Z). The tick marks along the edges are at 24 km intervals, and the dots mark the aircraft's track.

For both of these cases, the radar data are exceptionally suitable for center tracking. The hurricanes had small, well-defined eyes and were close (within 200 km) to the radar. Consequently, the agreement between the radar and dynamic centers is better than we would expect under less favorable conditions. For Anita, the two independent tracks are virtually identical. In Allen, however, the oscillation of the dynamic center is larger, and its mean path is directed more toward the north in comparison with the radar track. This appears to be related to the influence of a region of intense convection and 40+ dB(Z) radar reflectivity shown in Fig. 4. This feature was located in the eye wall southeast of the center at 1810 GMT and moved 90° cyclonically around the eye during the observing period. By 2300 GMT the convection had moved north of the center and had largely dissipated. Aboard the aircraft it was evident in real time that the dynamic center was displaced from the geometric center of the eye toward the convective cell. Research aircraft have observed dynamic centers located adjacent to the most convectively active portion of the eye wall, rather than at the center of the eye in several other hurricanes, notably David and Frederic of 1979 and Floyd, Gert, and Irene of 1981. Apparently this displacement, in combination with the cell's orbital motion, accounts for the difference between the two tracks. Trochoidal track oscillations have been observed before by both radar (e.g., Jordan, 1966) and satellites (Lawrence and Mayfield, 1977). The mean track was introduced in order to remove the effect of these motions and so obtain representative velocities for initialization of forecast models. Nevertheless, it is significant that the radar and aircraft tracks have so many details in common and that the differences between them can, at least in this case, be related to the presence of convection in the eye wall.

Fig. 5 shows two more sets of storm tracks; this time, determined by three aircraft operating at different altitudes in Hurricane Allen on 5 and 8 August 1980. The aircraft are identified by their radio call letters: NOAA-43, NOAA-41 and NOAA-42. They flew at the 85 kPa, 64 or 70 kPa, and 50 kPa pressure levels, respectively.

S_L and S_T indicate that the statistical uncertainty of the centering is larger for 5 August than in the previous cases—possibly because the vortex core became disorganized as the outer concentric eye supplanted the inner (Willoughby *et al.*, 1982). On the other hand, the statistics for 8 August are consistent with Anita and with Allen on 9 August. As before, the relative wind tracks parallel and lie to the right of the absolute wind tracks. The displacement between the relative and absolute wind tracks increases with the storm's speed of translation, but is also related to the radial profile of the tangential wind within the eye. On both days the mean-track velocities of the centers agree to within 3 or 4° of direction and 0.5 m s^{-1} of speed. For 5 August the zonal components of the detailed-track velocities for all three aircraft remain nearly constant at slightly less than 10 m s^{-1}, as the meridional component accelerates from 2 to 4 m s^{-1}. This is manifested as a subtle northward bend of the tracks. For 8 August both components of the detailed-track velocity decelerated by somewhat more than 1 m s^{-1}; thus, except in NOAA-43's case, the tracks are nearly straight. On both days the upper- and middle-level aircraft agreed with each

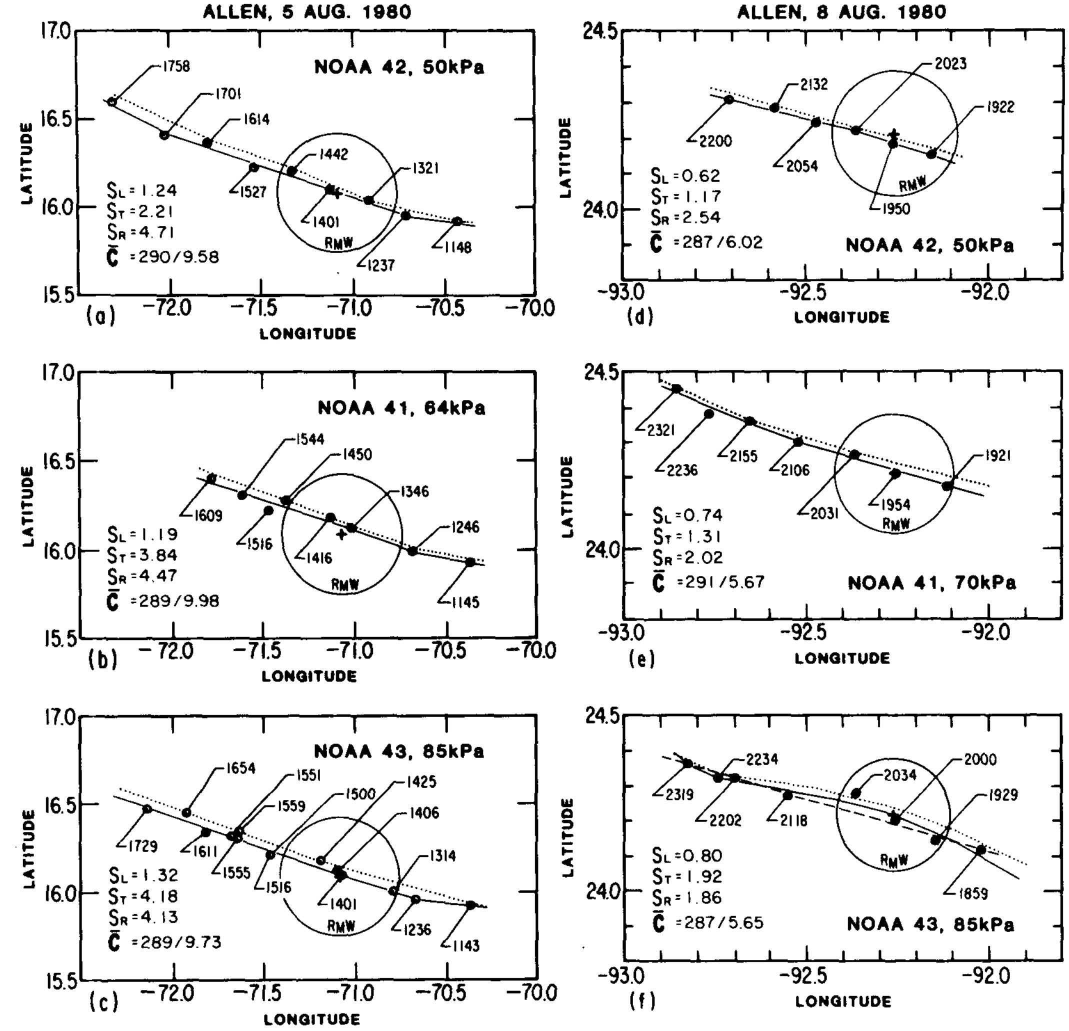

FIG. 5. Independent storm tracks from three aircraft operating in Hurricane Allen on 5 August (a, b, c) and 8 August (d, e, f) 1980. The data are represented in the same fashion as in the lower panels of Fig. 3, except that the mean track is shown only for NOAA-43 on 8 August and the RMW circles are centered on the low-level aircraft's detailed track.

other more closely than with the low-level aircraft. In general the instantaneous velocities computed from the detailed tracks differ by no more than 10° of direction and 1.5 m s^{-1} of speed.

Although the standard of navigation aboard the RFC aircraft is excellent, the position data do contain significant errors due to navigational limitations. The navigational errors apparently have both random and bias components. The effect of the random error is largely eliminated by the smoothing inherent in construction of the track from the center positions, but the bias errors remain. Errors of this kind did not appear in the previous cases because proximity to land made it possible to update the INE with radar and shore-based navigational aides. Even though each aircraft carries two INE's, as well as Omega electronic navigation equipment, the best real-time estimates of position will drift without such updates. Fig. 6 shows the terminal errors (the difference between the indicated INE position and the known position of the airfield upon landing) in comparison with the mean bias errors among the absolute tracks. The bias errors explain ~75% of the variance among the tracks. On 5 August the bias and terminal errors are comparable in magnitude, but on 8 August the terminal errors are considerably larger, apparently because most of the drift occurred after the aircraft left the storm. Relative to the means of the three tracks on both days, the low-level aircraft was biased to the east, the middle-level to the north and the high-level to the west. On 5 and 8 August, NOAA-41's northward bias agrees with the sense of that aircraft's terminal errors. Similar agreement is evident for the other aircraft on 5 August, but not on 8 August. Tilt of the vortex axis,

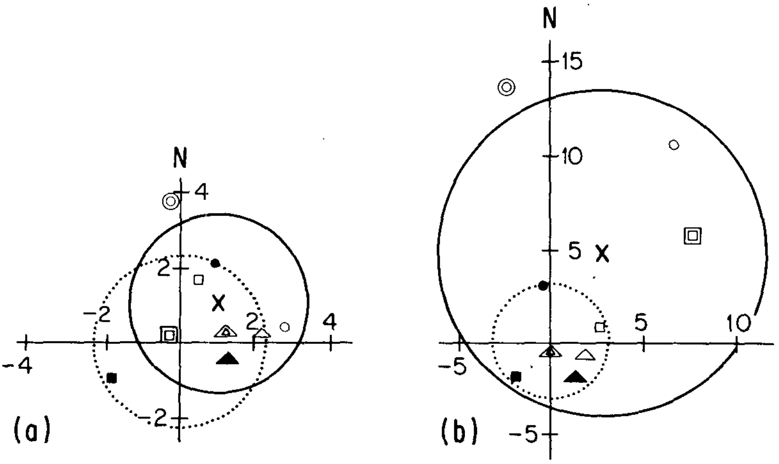

FIG. 6. The vector-mean errors among the storm tracks compared to the terminal navigation errors: (a) on 5 August 1980 and (b) on 8 August 1980. The aircraft are indicated as follows: circles NOAA-41, squares NOAA-42, and triangles NOAA-43. The solid symbols are vector average differences between the individual aircraft center positions and the mean of all three aircraft centers located at the origin. The dotted circle indicates the RMS separation of the aircraft centers from their mean. The open symbols are the terminal errors with the double symbols representing the INE used for track determination. The cross locates the mean of all six terminal errors, and the solid circle is the RMS terminal error. The axis coordinates are given in kilometers.

such as described by Huntley and Diercks (1981), may contribute to the bias, but the bias does not show a consistent pattern of variation with height so that INE drift seems to be a more probable source—particularly for a storm of this intensity.

Whatever its origin, the rate of change of the bias is slow in comparison with the speed of the storm center, so that repeatable estimation of the storm's mean motion is possible if one does not mix data from different aircraft with different accumulated biases. With modern satellite navigational equipment (Smith and Criss, 1976) or improved INE, this limitation could probably be overcome.

4. Conclusions

The analysis and computations above support the following conclusions:

a) The dynamic center located by aircraft is a fundamental feature that must exist in any closed vortex. It is the point about which circulating flux, as defined by (1), is largest.

b) Detailed tracks, determined by land-based radar and by aircraft, agree well when the radar center is well-defined and located close to the radar.

c) Displacements of the dynamic center from the geometric center of the radar or visual eye occur and may be related to convection in the eye wall in some cases.

d) Taking navigational limitations into account, the present technique allows center locations to be determined with an accuracy of 3 km and storm motion vectors averaged over a period of four to six hours with an accuracy of 4° of direction and 0.5 m s^{-1} of speed.

The cases presented in Section 3 are consistent with a much larger body of experience with the centering algorithm. At the National Hurricane Research Laboratory, post-flight composites of dynamic and radar data are prepared routinely using detailed, relative-wind tracks generated by this technique. The principal advantages of the algorithm in a research mode lie in its ability to construct objective, accurate, relative wind tracks with a minimum of human intervention. This can be accomplished even when the aircraft does not attain a small CPA separation from the center.

The algorithm does not, however, represent a dramatic increase in accuracy over the best real-time penetration fixes of absolute wind centers. Some navigators and flight directors consistently attain CPA distances < 1 km from the objectively determined centers. The present results do emphasize the great accuracy of the aircraft as a reconnaissance platform and its inherent ability to give accurate motions as well as positions. The algorithm has not been implemented in real time, but it does seem to have promise in operational situations. If computing resources were available aboard the aircraft, center location could be made virtually automatic and would not require special maneuvering within the eye. The fix coordinates could be transmitted over the existing satellite data link so that the forecaster would receive 6–8 automatic fixes per aircraft sortie, compared with the 2 or 3 manually-transmitted fixes presently available. Even without improvements in navigational capability, this should improve real-time track evaluation and provide better initialization of objective forecast models.

Acknowledgments. This study could not have been accomplished without the skill and professionalism of the RFC flight crews. Without diminishing the contributions of the others we would like to thank individually: the aircraft commanders Jim Gunnoe, Howard Ticknor, Dave Turner and Fred Werley; flight directors Dick Darby, Harlan Davis, Jim McFadden, John Michie and Stig Rossby; and navigators Bill Adams, Tom Bergner, Jerry Schissel and Cris Nelson. At NHRL Dave Jorgensen provided invaluable advice and assistance in handling the data. He and Frank Marks prepared the radar composites. Jeanne Clos and Jackie Piotrowicz did much of the actual data processing. Frank Marks, Jim Brown and Lloyd Shapiro read and commented on earlier versions of the manuscript. Angel Tillman and Dale Martin typed the manuscript and drafted the figures respectively. We are most grateful for their able assistance.

REFERENCES

Huntley, J. E., and J. W. Diercks, 1981: The occurrence of vertical tilt in tropical cyclones. *Mon. Wea. Rev.*, **109**, 1689–1700.

Jordan, C. L., 1966: Surface pressure variations at coastal stations during the period of irregular motion of Hurricane Carla of 1961. *Mon. Wea. Rev.*, **94**, 454–458.

Kutzbach, G., 1979: *The Thermal Theory of Cyclones,* Amer. Meteor. Soc., 255 pp.

Lawrence, M. B., and B. M. Mayfield, 1977: Satellite observations of trochoidal motion during Hurricane Belle 1976. *Mon. Wea. Rev.*, **105**, 1458–1461.

——, 1978: Atlantic hurricane season of 1977. *Mon. Wea. Rev.*, **106**, 534–545.

——, and J. E. Pelissier, 1981: Atlantic hurricane season of 1980. *Mon. Wea. Rev.*, **109**, 1567–1582.

Neumann, C. J., and J. M. Pelissier, 1981: Models for the prediction of tropical cyclone motion over the North Atlantic: An operational experiment. *Mon. Wea. Rev.*, **109**, 522–538.

Smith, D. L., and G. W. Criss, 1976: GPS, Navstar global positioning system. *Astronaut. Aeronaut.*, **14**, 26–32.

Willoughby, H. E., J. M. Clos and M. G. Shoreibah, 1982: Concentric eye walls, secondary wind maxima and the evolution of the hurricane vortex. *J. Atmos. Sci.*, **39**, 395–411.