Comments on "A Generalized Class of Time-Dependent Solutions of the Vorticity Equation for Nondivergent Barotropic Flow"

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In the author's knowledge, the most general solution of the vorticity equation for a nondivergent barotropic flow has been given by Ertel [see e.g., Kochin *et al.* (1964)]. It can be written

$$\psi(\lambda, \mu, t) = -r^2\omega\mu + Y_n(\lambda - \Omega_n t, \mu), \quad (1)$$

where ψ denotes streamfunction, λ longitude, μ the sine of latitude, t time, r the radius of the Earth, ω a constant,

$$\Omega_n = \omega - \frac{2(\omega + \Omega)}{n(n+1)},$$

Ω the angular velocity of the Earth, n a nonzero positive integer, and Y_n an arbitrary harmonic function of degree n, which satisfies

$$\Delta Y_n = -\frac{n(n+1)}{r^2} Y_n. \quad (2)$$

It is possible to rewrite (1) in the form

$$\psi(\lambda, \mu, t) = -r^2\omega\mu + \sum_{m=-n}^{n} \psi_n{}^m P_n{}^m(\mu) e^{im(\lambda - \Omega_n t)}, \quad (3)$$

which shows that Ertel's solutions consist of a zonal current and perturbations of different zonal wavenumbers m, but with the same degree n. The vorticity patterns rotate without deformation with the angular velocity Ω_n.

These solutions are more general than the classical Haurwitz' wave solutions used after Phillips (1959) to test numerical models.

Thompson's (1982) solutions can be written in exactly the same form as (3), since one can relate a surface spherical harmonic in two different systems of coordinates using

$$Y_n^{m'}(\lambda', \mu') = \sum_{m=-n}^{+n} C_n^{m,m'} Y_n{}^m(\lambda, \mu), \quad (4)$$

[see Machenhauer (1979)]. This is the reason why Thompson's solutions are less general than Ertel's. In Thompson's solutions, the coefficients $\psi_n{}^m$ in (3) are not independent and are related by

$$\psi_n{}^m = AC_n^{m,m'},$$

where A denotes the amplitude of the perturbation ψ', defined by Eq. (8) of Thompson's paper.

The formulae (4) are consequences of the property (2), which does not depend on the system of coordinate. Another interesting consequence of the property (2), relevant to spectral modeling, is that triangular truncation is invariant in a coordinate change.

Finally, it must be pointed out that: 1) Ertel's solutions have never been used in numerical modeling and 2) they are more general than Haurwitz' wave solutions, allowing for nonlinear wave interactions (and not only nonlinear interactions between a wave and the mean flow).

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