Comments on "A Numerical Study of the Interactions between Two Tropical Cyclones"

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1. Introduction

Chang (1983) has presented some interesting results from numerical simulations of the interaction between two tropical cyclones. In his study, Chang simulates the interaction between two cyclonic vortices using a nondivergent barotropic model and a three-dimensional tropical cyclone model. The results from the three-dimensional model showed that on an f-plane, with no mean wind, the motion of two tropical cyclones consists of a mutual cyclonic rotation combined with a decreasing separation distance, resulting in an eventual merging of the two systems. This interaction between vortex pairs was first described by Fujiwhara (1923) and is often referred to as the Fujiwhara effect. The results from Chang's three-dimensional model simulations are in good agreement with Fujiwhara's description as well as with more recent observations of the interaction between pairs of tropical cyclones in regions of weak horizontal wind shear (Dong and Neumann, 1983).

Chang also simulated the interaction between two cyclonic vortices of equal strength using a nondivergent barotropic model. For this case, the two vortices rotated around each other, but the separation distance increased in all of the simulations presented. From this result, Chang suggested that the divergent component of the wind rather than vorticity advection must be responsible for the mutual attraction of vortex pairs. In this comment it will be shown that the mutual attraction can be explained by vorticity advection alone, and that the attraction (or lack of) depends strongly on the initial wind profiles of the vortices.

2. Conceptual description

In order to illustrate the mutual attraction of a pair of cyclonic vortices of equal strength as a result of vorticity advection, a conceptual argument is presented. First, consider a cyclonic vortex with a tangential wind profile given by

$$V(r) = V_m\left(\frac{r}{r_m}\right)\exp\left\{\frac{1}{b}\left[1 - \left(\frac{r}{r_m}\right)^b\right]\right\}, \tag{1}$$

where r is radius, V_m the maximum tangential wind, r_m the radius of maximum wind, and b is a factor which determines the rate at which the tangential wind decays with radius. In this example, we consider $V_m = 30$ m s^{-1}, $r_m = 100$ km, and $b = 1.0$ or 0.5. The vorticity profile which corresponds to (1) is given by

$$\zeta_{(r)} = \frac{2V_m}{r_m}\left[1 - \frac{1}{2}\left(\frac{r}{r_m}\right)^b\right]\exp\left\{\frac{1}{b}\left[1 - \left(\frac{r}{r_m}\right)^b\right]\right\}. \tag{2}$$

Radial profiles of V and ζ outside the radius of maximum wind for $b = 1.0$ and 0.5 are shown in Figs. 1 and 2, respectively.

First, consider the case when $b = 1.0$ (the solid lines in Figs. 1 and 2). Now suppose there is a second cyclonic vortex (referred to as vortex B) located 500 km from the $b = 1.0$ vortex shown in Fig. 1 (referred to as vortex A). For the purposes of this discussion it is assumed that vortex B is to the east of vortex A. From Fig. 1 it can be seen that the tangential wind of vortex A has a value of about 3 m s^{-1} at the position of the center of vortex B. This wind should then result in an initial northward advection of vortex B. Similarly, the tangential wind of vortex B initially advects vortex A towards the south. Thus, if this were the only process acting, it might be expected that the two vortices would rotate cyclonically around each other at a constant separation distance.

The reason the above argument does not hold is because a second advective process is taking place. In Fig. 2 it can be seen that at the position of vortex B there is a radial gradient of the vorticity of vortex A. For this case ($b = 1.0$) the vorticity from vortex A increases to the east of vortex B. This indicates that in the area north of vortex B, its tangential circulation results in positive vorticity advection due to the presence of the vorticity field of vortex A. By the same reasoning, the tangential circulation of

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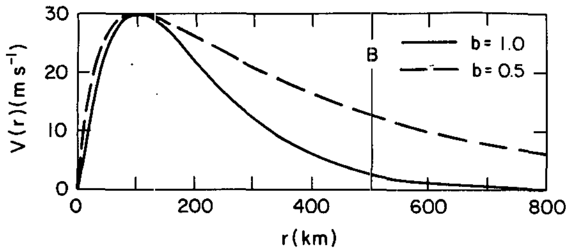

FIG. 1. Tangential wind as a function of radius for the vortex defined by (1) with V_m = 30 m s^{-1}, r_m = 100 km and b = 1.0 or 0.5. The vertical solid line B indicates a position 500 km from the vortex center.

vortex B results in negative vorticity advection in the area south of its center. This vorticity advection should then induce cyclonic motion to the north and anticyclonic motion to the south of vortex B. The induced cyclonic motion to the north and anticyclonic motion to the south then results in a net flow towards the east at center of vortex B. This flow should then advect vortex B toward the east. Assuming vortex A and vortex B have the same structure, vortex A should be advected to the west by this same process. Thus, the separation distance between the vortices should be expected to increase by the process described above, as the two vortices rotate cyclonically around each other.

Now consider the case when b = 0.5 (dashed lines in Fig. 1), and again assume there is a second cyclonic vortex (B), 500 km to the east. For this case, at the position of vortex B, the radial gradient of the vorticity from vortex A is opposite to that for the b = 1.0 case. Therefore, the tangential circulation of vortex B results in negative vorticity advection to the north and positive vorticity advection to the south of vortex B. This vorticity advection should then induce anticyclonic motion to the north and cyclonic motion to the south of vortex B, which would result in a net flow towards the west at the center of vortex B. For this case the separation distance between the two vortices should be expected to decrease as they rotate cyclonically around each other.

Thus, by vorticity advection arguments, two cyclonic vortices should be expected to rotate cyclonically around each other due to the advection of each vorticity field by the tangential wind of the opposite vortex. The interaction of the tangential wind field with the vorticity field of the opposite vortex adds a second component to the motion which can cause the separation distance to either decrease or increase, depending on the direction of the vorticity gradient.

3. Model results

In order to verify the above hypothesis, two simulations of vortex pair interactions in a nondivergent barotropic model are presented. The governing equation for this model is the conservation of absolute vorticity which can be written as

$$\frac{\partial \zeta}{\partial t} + \mathbf{V}_\psi \cdot \nabla(\zeta + f) = 0, \tag{3}$$

where ζ is the vertical component of the relative vorticity, $\mathbf{V}_\psi$ the nondivergent wind, and f the Coriolis parameter. In (3), ζ can be related to $\mathbf{V}_\psi$ by introducing a streamfunction ψ, where

$$\zeta = \nabla^2\psi, \tag{4}$$

$$\mathbf{V}_\psi = \mathbf{k} \times \nabla\psi, \tag{5}$$

for $\mathbf{k}$ a vertically oriented unit vector.

Equations (3)–(5) were solved using a spectral method with Fourier basis functions on a doubly-periodic domain. A detailed description of the numerical method used (in the context of a three-dimensional model) can be found in DeMaria and Schubert (1984). For the simulations presented here, the model was truncated at wavenumber 42 in the x and y directions on a 4000 km square domain. The shortest wave in the model then has a wavelength of about 95 km. For simplicity, the variation of the Coriolis parameter with latitude was neglected.

The model was initialized with a pair of symmetric vortices separated by 500 km. The structure of each vortex is given by (1) with V_m = 30 m s^{-1} and r_m = 100 km. The model was run for 72 hours with b = 1.0 and 0.5 to verify the discussion in Section 2.

Figure 3 shows the trajectories of the two vortices as defined by the vorticity maxima for b = 1.0 and 0.5, where the time interval between adjacent black dots is 6 h. This figure shows that for b = 1.0, the interaction between the two vortices results in a slight cyclonic mutual rotation with the separation distance between the vortices increasing with time (no mutual attraction). These trajectories are in agreement with the expected results from the previous section.

For the b = 0.5 case in Fig. 3, the interaction between the vortices results in a mutual cyclonic rotation with the separation distance between the vortices decreasing with time. In fact, the two vortices merge after about 18 h. The results for this case are

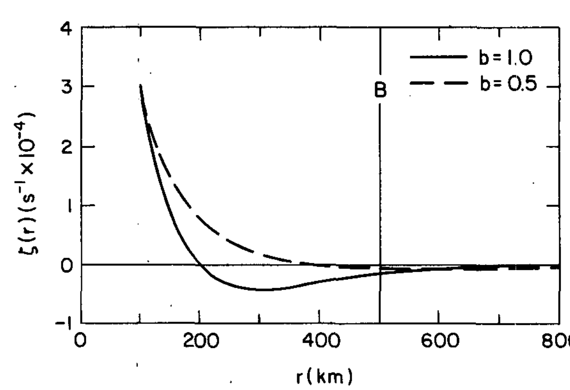

FIG. 2. Relative vorticity profiles corresponding to the vortices shown in Fig. 1.

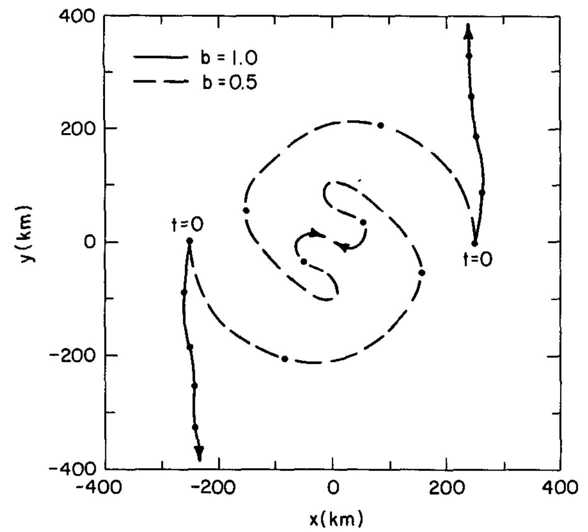

FIG. 3. Trajectories from a nondivergent barotropic model initialized with a pair of vortices defined by (1) with $V_m = 30$ m s^{-1}, $r_m = 100$ km and $b = 1.0$ or 0.5. The time interval between adjacent black dots is 6 h.

also in agreement with the expected results from the previous section.

The initial separation of 500 km used in the above example was chosen so that the difference between the vorticity gradient for the $b = 1.0$ and 0.5 cases could be easily seen in Fig. 2. The minimum value of ζ for the profile given by (2) occurs at r_c given by

$$r_c = r_m(2 + b)^{1/b}. \quad (6)$$

Since the radial gradient of the vorticity changes sign at $r = r_c$, it should be expected that when the initial separation distance is greater than r_c, the vortices should no longer exhibit mutual attraction. For $b = 1.0$ and 0.5 with $r_m = 100$ km, $r_c = 300$ km and 625 km, respectively. Thus, if the initial separation distance for the $b = 0.5$ case was greater than 625 km, the separation distance between the vortices should be expected to increase with time. This was verified using the nondivergent barotropic model initialized with the $b = 0.5$ vortices separated by 1000 km. Since r_c increases with decreasing b in (6), this implies that pairs of vortices with tangential wind profiles which decay slowly with radius will exhibit mutual attraction at larger separation distances than vortices with tangential wind profiles which decay rapidly with radius.

4. Discussion

The above results indicate that many aspects of the Fujiwhara effect (including mutual attraction) can be simulated with a nondivergent barotropic model. In Chang's nondivergent barotropic model simulations, the radial structure of the vortices used is similar to the structure of the vortex for the $b = 1.0$ case shown in Fig. 1. In all of Chang's barotropic simulations, the initial separation of the vortices was such that the vorticity field of the opposite vortex was increasing with radius at the position of each vortex (see Chang's Figs. 2 and 3). Thus, all of Chang's barotropic simulations are analogous to the $b = 1.0$ case, so that by the arguments presented here, the separation distance of the vortices should be expected to increase with time (no mutual attraction).

In Chang's primitive equation model simulations, different initial vortices were used. For this case, the vortices were generated from two stationary heat sources. These vortices (shown in Chang's Fig. 6) decay with radius at a much slower rate than the vortices used in his barotropic model. From the results in the previous section, these vortices should be expected to exhibit mutual attraction at larger separation distances than the vortices used in Chang's barotropic simulations. This implies that the difference between Chang's barotropic and three-dimensional results might partially be explained by the difference in the initial vortex structure, rather than the inclusion of diabatic effects and a divergent wind. In order to verify this, it would be necessary to determine the direction of the vorticity gradient of the opposite vortex at the position of each vortex for the initial conditions used in Chang's three-dimensional simulations.

The results presented here indicate that it is not necessary to include the effects of divergence and diabatic heating in order to simulate the mutual attraction of a pair of vortices. This does not imply, however, that these effects are unimportant for the understanding of the interactions between pairs of tropical cyclones. For example, an observational study by Chan (1984) shows the importance of including divergence in the estimate of the net vorticity change associated with tropical cyclone motion. Further studies with a three-dimensional model are needed to determine how the basic mechanism of vorticity advection described here is affected by divergence and the inclusion of physical processes such as diabatic heating and friction.

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