Developments in Normal Mode Initialization. Part I: A Simple Interpretation for Normal Mode Initialization

PHILIP J. RASCH

National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307*

(Manuscript received 18 May 1984, in final form 6 May 1985)

ABSTRACT

An interpretation of the iterative schemes of nonlinear normal mode initialization (Machenhauer, Kitade, and Tribbia) is introduced, where the schemes are regarded as sequential applications of filters. The response functions of these filters provide a means of evaluating the convergence of the iterative methods. The filters annihilate that component of the signal corresponding to the theoretical frequency determined from the normal mode analysis. The actual frequencies are, of course, determined by the linear terms in the differential equations (which are accounted for by the normal mode analysis) and the nonlinear terms which will change the actual frequencies. The interpretation is extremely simple, but has not appeared previously in the literature. It is primarily qualitative. It explains in a qualitative sense the convergence problems encountered when attempting to initialize modes with small equivalent depths, or models which include diabatic physical processes. It complements the analyses of Phillips and Errico. The analysis of Tribbia's higher order initialization suggests that higher order steps can be more sensitive to convergence problems than the earlier methods.

1. Introduction

Initialization is the process of adjusting the data prior to a model integration so that the forecast shows a minimum of noise (the high-frequency transient behavior). Nonlinear normal mode initialization (nonlinear NMI) has, over the last few years, become the method of choice for the initialization of data for many research and operational forecast models of large scale atmospheric motion. A good discussion can be found in Daley (1981) and Machenhauer (1983). The essence of many studies (cf. Bengtsson, 1978, 1981; Lorenz, 1982) suggests that the specification of the initial state is as much to blame for errors in forecasts as the mathematical formulation of the model or the physical parameterizations. The initialization can also improve the analysis–forecast cycle by reducing the amount of data rejected during the analysis.

NMI was introduced in various forms by Dickinson and Williamson (1972), Williamson (1976), Machenhauer (1976), and Baer (1976), and developed by numerous authors since then. It is computationally very efficient, and has also contributed to the theoretical understanding of nonlinear atmospheric dynamics and its relationship with quasi-geostrophic theory (Leith, 1980; Errico 1982).

Despite its success, NMI is not without problems (reviewed below). In this series of papers some new developments in NMI which address these problems are described.

- In Part I is provided a presentation of NMI which we hope is accessible to readers completely unfamiliar with NMI, as well as the NMI practitioner. The paper 1) reviews the problems with NMI; 2) reviews the initialization schemes and establishes a notation; and 3) introduces a new interpretation, which we believe aids in the understanding of NMI.
- The interpretation leads to the hypothesis that the relative success or failure of the current schemes depends on the nature of the nonlinear terms in the system of equations. In Part II (Rasch, 1985) we offer a new scheme which is less sensitive to the nonlinear terms' characteristics, and verify that the nonlinear terms do indeed possess the characteristics which lead to problems in current schemes for NMI.

As the use of NMI has increased, at least two problems have become apparent (Williamson and Temperton, 1981; Temperton and Williamson, 1981; Puri and Bourke, 1982). These problems lead to an increase in the amplitude of the high-frequency transients generated at model start-up, rather than the intended decrease. The first and more important problem arises when one attempts to perform the nonlinear NMI on a model that incorporates parameterizations of physical processes (e.g., convective parameterizations) which we loosely call *physics* throughout the rest of the paper. The inclusion of these diabatic processes leads to acute problems in initialization in tropical regions where they play a particularly important role. When models are

* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

initialized in the absence of such parameterizations (an "adiabatic" nonlinear NMI) those disturbances which are forced by the physics are absent and this may detrimentally affect the forecast. Thus the Hadley circulation initially present in data is removed if the initialization does not include physics. When the highly nonlinear processes are included in the current approaches, (a "diabatic" nonlinear NMI), an increase in high-frequency oscillations is seen. Puri and Bourke (1982), and Puri (1983) have suggested that to retain the Hadley circulation in the initial data, those modes which are responsible for the Hadley circulation should be excluded from an adiabatic normal mode initialization. This technique provides a method for retaining the Hadley circulations but is somewhat unsatisfactory (at least to this author), since there is, in principle, no reason to restrict the initialization to the adiabatic case, or to exclude some fast modes while including others with equivalent time scales. Kitade (1983) has outlined an alternate solution, which is discussed in some detail later in this paper.

The other problem observed is the difficulty in adjusting the normal modes that correspond to internal gravity waves of small equivalent depth. These modes describe fine vertical structure in the lower atmosphere. Although slower than the external gravity modes they are often fast enough that it is desirable to initialize them. When they are included in the initialization there is again an increase in high frequency transients. This problem is elaborated upon in Section 3.

In this paper an attempt to provide insight into the failures of nonlinear NMI will be made. It begins by introducing a concise algebraic notation for the equations of motion; then the normal mode decomposition, a fundamental part of the nonlinear NMI method, is described. Following are sections describing the NMI and an interpretation of the current nonlinear NMI methods as filters. This interpretation is extremely simple, but has not previously been described in the literature. It is useful in providing intuition into the convergence problems of the iterative initialization schemes. It complements the work of Errico (1983) and Phillips (1981). It also extends the analysis to Tribbia's (1984) higher order initialization scheme.

2. The normal mode decomposition

Let us consider a vector $\mathbf{z}$ which contains the relevant dependent variables for the model after discretization (by a finite difference or spectral method) in space but not time; e.g.

$$\mathbf{z} = (\zeta_1 \zeta_2 \cdots \zeta_N)^{\mathrm{T}}$$

and (1)

$$\zeta_p = (u_p, v_p, \phi_p)^{\mathrm{T}}$$

where u_p, v_p, and ϕ_p represent the values of velocity and geopotential at the pth grid point, or amplitudes of the pth basis function, in a grid point or spectral model, respectively.

We use boldfaced lowercase characters to denote vectors, uppercase boldface to denote matrices, and normal lowercase to denote scalers. The superscript T indicates a matrix (vector) transpose. The conjugate transpose is implied where appropriate; i denotes the square root of -1. The general system of equations describing any model can be written, using this notation as an autonomous set of ordinary differential equations:

$$\dot{\mathbf{z}} = i\mathbf{L}\mathbf{z} + \mathbf{n}(\mathbf{z}), \tag{2}$$

where $\dot{\mathbf{z}} = \partial \mathbf{z}/\partial t$, t is time, $\mathbf{L}$ a matrix describing the linear portion of the equations and $\mathbf{n}$ a nonlinear term, which generally includes such processes as advection and friction, and ideally the physical parameterizations like cumulus convection and the radiative transfer processes as well.

We regard $\mathbf{z}$ as a *state vector* characterizing the model, or a finite dimensional representation of the atmosphere at any time. The linear operator (matrix) $\mathbf{L}$ (assumed Hermitian here) has associated with it an eigenvalue problem

$$\mathbf{L}\mathbf{e}_j = \omega_j \mathbf{e}_j. \tag{3}$$

The normal mode decomposition of the state vector $\mathbf{z}$ is attained by letting the eigenvectors form a new basis (coordinate system) for the dynamical system (2). We let

$$\mathbf{z} = \sum_{j=1}^{3N} a_j \mathbf{e}_j \tag{5}$$

and transform (2) to the space described by the eigenvectors in the form

$$\dot{a}_j = i\omega_j a_j + r_j(a_1, a_2, \ldots, a_N), \quad j = 1, 2, \ldots, 3N, \tag{6}$$

where

$$r_j = \mathbf{e}_j^{\mathrm{T}}\mathbf{n}, \quad j = 1, 2, \ldots, 3N. \tag{7}$$

If $\mathbf{n}$ is identically zero (i.e., the model is linear), we can view the model variables as a linear combination of the eigenvectors whose coefficients oscillate at frequencies corresponding to their eigenvalues.

If $\mathbf{n}$ is nonzero, we may still view the state vector $\mathbf{z}$ as a linear combination of the eigenvectors, but the coefficients are coupled to each other and are governed by (6).

If $\mathbf{L}$ is separable in horizontal and vertical directions, the index j can be separated into a double index k, ν; where ν specifies the vertical character of a mode, and k the horizontal character. Modes with the same index ν share a common vertical structure and the horizontal structure functions associated with a given vertical structure function form a complete set of eigenvectors for an equivalent barotropic model with a particular mean depth. We can thus characterize groups of eigenvectors interchangeably by a vertical mode index ν, its corresponding vertical structure function, or the *equivalent depth* for the corresponding barotropic model. The modes of a particular equivalent depth are generally divided into groups by their eigenvalues. For each equivalent depth, the eigenvalues separate into three regions; small eigenvalues whose eigenvectors we identify with westward propagating Rossby modes, and larger eigenvalues whose eigenvectors we identify with eastward and westward propagating inertia-gravity waves. The fastest gravity modes are associated with the largest equivalent depth, the slowest gravity modes with the smallest equivalent depth.

The coefficients of the modes are partitioned into somewhat arbitrary subsets, to condense the notation. One set of modes is, for convenience, labeled as the fast modes. These modes are generally identified in either (or a combination) of two ways: the traditional way is through the separation in eigenvalues which occurs for a particular vertical structure function or equivalent depth, i.e. into Rossby (slow) and gravity (fast) waves for a particular equivalent depth; the second way is via an absolute cutoff frequency. The modes with frequencies whose modulus exceeds some critical amplitude ω_{crit} belong to the fast class. We denote the complementary set of modes as the slow modes.

The normal modes are sensitive to details of the model and the linearization used (viz., the temperature and winds of the basic state). Luckily the initialization results seem to be insensitive to the details of the normal mode structure. Temperton and Williamson (1981) found the structure functions to be relatively insensitive to changes in the temperature profiles. Machenhauer (1977) found the initialization to be relatively insensitive to whether the initialization used modes derived from a basic state at rest, or from one with a geostrophic zonal wind.

3. Nonlinear normal mode initialization

Nonlinear NMI takes into account the nonlinear interaction between modes. Two different, but related, methods were developed simultaneously by Machenhauer (1976, 1977), and Baer (1976, 1977). We will deal here with Machenhauer's version, and Tribbia's (1984) technique. The latter may be considered in some sense an extension of that of Baer, which depends on a two-time scale asymptotic expansion in a small parameter.

Historically, NMI calculations were first performed with barotropic models, using a mean depth of 8–10 km, which corresponds approximately to the external modes of a baroclinic model. The successful methods were then tested using smaller mean depths (near 100 m), to simulate the internal baroclinic modes. Finally the methods were used in a full baroclinic model, where interactions between the modes of various equivalent depths can take place.

Early tests with adiabatic baroclinic models revealed that attempts to initialize the modes with small, equivalent depth using a Machenhauer scheme inevitably lead to an increase in the imbalance of the modes, rather than the intended decrease. Surprisingly, attempts to initialize barotropic models with similar equivalent depths usually do converge. These tests are difficult to carry out in practice, as the perturbation height field can easily exceed the mean depth of the fluid—an unrealizable physical scenario—which requires working with very small winds and height perturbations. It can be shown (Rasch, 1984) for the very simple barotropic model of Tribbia (1982), that with a set of mild restrictions, when the perturbation height field is constrained to being less than the mean depth, the Machenhauer scheme will always converge to a physically valid balanced initial condition. Although we cannot make rigorous statements about the convergence properties of the Machenhauer schemes for more complex barotropic primitive equation models—we conjecture that a similar convergence characteristic extends to this case. Try as we might, we have not been able to find initial conditions designed so the perturbation height field will remain less than the mean depth (as small as 1 m) when integrated (but was otherwise chosen arbitrarily) and so would cause the Machenhauer scheme to diverge. This perturbation height constraint is manifested differently in baroclinic models, where the equivalent depth for a baroclinic mode enters only as an artifact of the vertical decomposition. Indeed, for forced modes, the equivalent depth can even be viewed as negative (cf. Chapman and Lindzen, 1970).

Because of the problem with initializing the internal modes, the fast mode category is often restricted to modes with large equivalent depth, even when there are faster modes with small equivalent depth which remain uninitialized. In practice this has not caused any great difficulty because most of the kinetic and available potential energy of the atmosphere, even for the fast modes, is represented by modes of large equivalent depths. Nonetheless, it would be desirable to initialize all modes with short periods, rather than restrict the initialization to a subset of those modes, or at least to understand the reason why some modes cannot be initialized.

a. *Machenhauer's scheme*

Machenhauer (1977) observed that when the model is in balance, the left-hand term in (6) is small for the 'fast' gravity modes. That is,

$$\dot{a}_j \approx 0.$$

Therefore he defined the balanced state to be

$$\dot{a}_j \equiv 0 = i\omega a_j + r_j \tag{8}$$

or

$$a_j = -r_j/i\omega_j. \tag{9}$$

He suggested the iterative scheme

$$a_j^{k+1} = -r_j(a_1^k, a_2^k, \ldots, a_N^k)/(i\omega_j) \qquad (10)$$

where k is the iteration number.

Such a scheme is often call 'Picard,' or 'direct iteration' in the iterative solution of equations, and has been used as far back as Cauchy (cf. Wouk, 1964; Ortega and Rheinbolt, 1970).

b. Kitade's scheme

Kitade (1983) has suggested a modification to NMI which amounts to an underrelaxation of the Machenhauer initialization. We identify a Machenhauer correction as

$$a_j^M = -r_j(a_1^k, a_2^k, \ldots, a_N^k)/(i\omega_j) \qquad (12)$$

and make the new iteration a linear combination of the Machenhauer correction and the previous iteration:

$$a_j^{k+1} = (1 - \alpha)a_j^k + \alpha a_j^M. \qquad (13)$$

Letting $\alpha = 1/2$, and combining (12) and (13) gives

$$a_j^{k+1} = \frac{a_j^k - r_j^k/(i\omega_j)}{2}. \qquad (14)$$

c. Tribbia's higher order initialization

Tribbia (1984) suggested a method for higher order initializations in which constraints on a higher time derivative of the normal mode coefficients are applied.

His technique derives from the general solution to (6) (obtained from a series expansion to r)

$$a(t) = \left\{a(0) + \sum_{n=0}^{\infty} \frac{1}{(i\omega)^{n+1}} \frac{\partial^n r(0)}{\partial t^n}\right\} e^{i\omega t} + \sum_{n=0}^{\infty} \frac{1}{(i\omega)^{n+1}} \frac{\partial^n r(t)}{\partial t^n}. \qquad (15)$$

Here, we have dropped the subscript to reduce clutter. The condition that no oscillations occur with frequency ω becomes

$$a(0) + \sum_{n=0}^{\infty} \frac{1}{(i\omega)^{n+1}} \frac{\partial^n r(0)}{\partial t^n} = 0. \qquad (16)$$

This equation is implicit, since r depends on a. Tribbia suggested a sequence of approximations of (16) constructed to be asymptotically equivalent to the Baer–Tribbia process for the first few steps. His scheme can be written

$$a^k = \sum_{n=0}^{k-1} \frac{-1}{(i\omega)^{n+1}} \frac{\partial^n r(a^{k-1})}{\partial t^n}, \quad a^0 = 0. \qquad (17)$$

The equation is complex enough so it is useful to write the first few steps explicitly.

$$a^0 = 0, \qquad (18)$$

$$a^1 = -r(a^0)/i\omega, \qquad (19)$$

$$a^2 = -r(a^1)/i\omega - \dot{r}(a^1)/(i\omega)^2. \qquad (20)$$

The first two steps (a^0 and a^1) correspond to the original Baer–Tribbia (1977) formulation, and Machenhauer's, if the fast modes are zeroed first. Step 3 (a^2) introduces some small approximations to the Baer–Tribbia scheme. In practice, Tribbia has carried this iteration out to a computation of a^3 in a barotropic model with a mean height of 1 km. The method shows considerable success at removing the transients for this case. Machenhauer (1983) discusses the rationalization behind a similar scheme which differs in nonessential details.

4. The response function interpretation of NMI

Here, we outline a context in which a qualitative analysis of NMI methods can be made, suggest why Kitade's variant (underrelaxation) is more robust than Machenhauer's NMI, and why Tribbia's higher order method may be less robust. The framework is essentially that used in the simplest of filtering theory.

We will view the schemes outlined as sequential applications of filters applied to a sequence of time series. The time series are just the normal mode coefficients at two or three time steps. The filter coefficients vary with the mode being filtered. The sequence of time series are related to each other through the prognostic equations.

Making the last paragraph explicit, let us drop the subscript for the moment and consider any fast coefficient of (6):

$$\dot{a} = -i\omega a + r. \qquad (21)$$

The Machenhauer iteration is

$$a^{k+1} = -r(a^k)/i\omega. \qquad (22)$$

We can combine these equations to eliminate r and form

$$a^{k+1} = a^k - \dot{a}^k/i\omega. \qquad (23)$$

This is the form most often used in operational NMI, with a discrete approximation made to $\dot{a}$ (most often a forward difference). This form of the iterative scheme eliminates the need to compute **n**, and then r, explicitly. In version (23) of the iteration, the model is used like a subroutine to the initialization; it provides an estimate of $\dot{a}$. Equation (22) can also be used, but entails a careful reproduction of the model in continuous form. In principle, for consistent finite difference schemes, as the time step length is reduced to zero the two implementation schemes are equivalent. In practice, the equations may change as the time step changes. The difficulty in translating the discrete implementation of certain physical processes (for example moist and dry convective adjustment) used within the model to the continuous form provides an additional incentive for the use of (23). Some of these problems are addressed in Part II of this sequence of papers.

If we presume that $a(t)$ is of the form

$$a = \chi e^{i\omega_n t}, \tag{24}$$

where ω_n is the frequency of oscillation at which the mode propagates when the nonlinearities (the forcing) are included, then (23) can be written

$$\chi^{k+1} = \chi^k\left(1 - \frac{\omega_n}{\omega}\right). \tag{25}$$

Here we have assumed that the frequencies ω and ω_n are independent of time. We comment further on this assumption toward the end of the paper. In terms of a response function we rewrite (25) as

$$R_M(\omega_n, \omega) = \frac{\chi^{k+1}}{\chi^k} = 1 - \frac{\omega_n}{\omega} \tag{26}$$

where the subscript M identifies the Machenhauer response function.

A similar equation can be constructed when discrete approximations are made to the time derivative $\dot{a}$. For the forward time step used most often we find

$$R_M{}^{f.d.} = 1 - \frac{e^{i\omega_n\delta t} - 1}{i\omega\delta t}. \tag{27}$$

Some immediate conclusions can be made in the light of (26); the conclusions are the same when made with (27).

- The filter annihilates that component of the time series corresponding to ω, the theoretical frequency determined from the normal mode analysis.
- When $|\omega| \gg |\omega_n|$, the response is near unity, suggesting that low frequencies are well preserved by the filter.
- The filter damps only when $0 < (\omega_n/\omega) < 2$.
- A singularity exists at $\omega \equiv 0$; hence, NMI becomes more sensitive to perturbations of ω_n from ω as $|\omega|$ decreases. For small ω the filter will amplify any high frequency component of the observed spectrum ω_n. For example, for a mode with frequency corresponding to a period of 12 hours, only those components of the time series with periods longer than 6 hours are damped; those components with shorter periods are amplified.
- If the iterative scheme converges, the first derivative with respect to time ($\dot{a}$) is minimized.

The response function interpretation is probably useful only in a qualitative sense, since the behavior of a particular mode is governed by a superposition of many frequencies ω_n, whose spectrum changes with time and iteration. The fact that the iterative scheme works so well with unforced modes of large equivalent depth implies that the high frequencies associated with a particular mode are (not surprisingly) predominantly those of the associated normal mode eigenvalue. As $|\omega|$ gets smaller, application of NMI becomes inappropriate, since it damps only very near ω, and amplifies higher frequencies. This scenario is similar to the case where internal gravity modes are initialized and the nonlinearities become very important. Since the phase speed of these modes is small, the advective terms become important and the Doppler shifting of frequencies can perturb significantly, ω_n from ω. The frequency cutoff approach to defining the fast modes is justified by this argument.

The problems encountered when physics are included in the initialization even with modes having a large ω are more subtle. One can speculate that the forcing generates some high frequency components in the model which are consequently amplified. This possibility is confirmed in Part II, in a complex baroclinic model where we show certain parameterizations often present in sophisticated forecast and general circulation models (notably *moist and dry convective adjustment,* and *large scale condensation*) do impose a high frequency forcing (whose time scale is proportional to the time step) and that, in their absence, the convergence is much improved.

Kitade's modification generates a response function

$$R_K = 1 - \frac{\omega_n}{2\omega}. \tag{28}$$

The modification has effectively doubled the range of frequencies over which the iterative scheme is convergent, extending the interval of convergence in the right half line. The filter is not as sharp as the Machenhauer version, so more iterations are required to reduce the amplitude of the high frequency components of the time series. Some of these results have also been found by Errico (1983), who investigated the results of the convergence of Kitade's scheme by using a more sophisticated matrix formulation. For example, both analyses suggest modification has not improved the convergence of the iteration, when ω and ω_n have opposing signs.

After some algebraic manipulation of (18) through (21) one can derive an alternate form of Tribbia's scheme.

$$\left.\begin{aligned} a^0 &= 0 \\ a^1 &= a^0 - \dot{a}^0/\mathrm{i}\omega = \dot{a}^0/\mathrm{i}\omega \\ a^2 &= a^1 - \ddot{a}^1/(i\omega)^2 \end{aligned}\right\}, \tag{29}$$

or generally,

$$a^k = a^{k-1} - \frac{1}{(i\omega)^k}\frac{\partial^k a^{k-1}}{\partial t^k}. \tag{30}$$

The transfer functions which correspond to each step of the Tribbia higher order initialization are:

$$\left.\begin{aligned} R_{T,0} &= 0 \\ R_{T,1} &= 1 - \omega_n/\omega \\ R_{T,2} &= 1 - \omega_n^2/\omega^2 \end{aligned}\right\}, \qquad (31)$$

or, generally,

$$R_{T,k} = 1 - (\omega_n/\omega)^k. \qquad (32)$$

The following comments about his scheme can be made:

- Instead of applying one filter to a sequence of time series, this version of NMI applies a sequence of filters to a sequence of time series, each with a different transfer function.
- Step 1 sets the zeroth derivative of a to zero.
- Step 2 is identical with the M filter. The correction attempts to minimize $\dot{a}$.
- Step 3 is a sharper filter than that used in Machenhauer NMI, with a different frequency interval over which damping takes place. The transfer function is nearer unity within this interval so the longer period components are better preserved. Note that it attempts to zero the second derivative with respect to time rather than the first. On the other hand when $|\omega_n/\omega|$ is outside the interval $[0, \sqrt{2}]$, the filter amplifies much more strongly.
- The higher order filters amplify more and more strongly outside the interval of convergence as k increases, and the interval of convergence as k increases. This interval of convergence for the kth is just

$$0 < \left(\frac{\omega_n}{\omega}\right) < 2^{1/k} \qquad (33)$$

which reaches a limiting value for large k of $[0, 1]$, just half the interval of convergence for Machenhauer's scheme.

The estimation of derivatives from a time series is notoriously difficult, and the difficulty increases with higher derivatives. We have seen from the filter interpretation that the higher order initializations are even more sensitive to deviations than the Machenhauer or Kitade versions of the initialization, increasing the possibility of problems. In practice, Tribbia has suggested that the derivatives be evaluated by a least squares fit to the discrete data points. This will result in a smoothing in time of the data and (potentially) a smaller value for the effective frequency ω_n. The filter interpretation implies this is beneficial to the initialization.

5. Discussion

This analysis is strictly valid only when the frequencies ω and ω_n are independent of time. The analysis is probably qualitatively correct whenever the variation in ω and ω_n is on a time scale which is large with respect to the time scales defined by ω or ω_n. Nevertheless, the analysis provides the simplest possible framework in which any scheme can be tested. If a scheme shows problems in this context, one may anticipate potential problems for more complex scenarios.

The analysis also leads to speculation about extensions/variations to the schemes. The method suggested by Kitade can be interpreted as a relaxation technique. The relaxation coefficient α in (13) and (28) could presumably be made a function of iteration. Starting with a very small α would remove the fastest components of each mode first; a gradual increase in the amplitude of α would gradually remove slower and slower components. Since the relaxation technique for the Machenhauer scheme increases the interval of convergence only where ω_n has the same sign as ω, this cannot alleviate all the problems.

Tribbia's scheme is attractive because it attempts to balance the higher derivatives. The $k = 2$ filter is independent of the relative signs of ω_n and ω. The very large amplification by the response function encountered when $|\omega_n/\omega|$ are large could be reduced with a similar use of an underrelaxation technique.

Finally, one might consolidate the two techniques by using a linear combination of the two schemes with more desirable response function characteristics.

We outline a substantially different approach toward NMI in Part II of this paper.

6. Summary

We have reviewed most current techniques of NMI and offered a new interpretation of the iterative techniques as sequential applications of filters, which explains in part the problems with convergence of the iteration often seen with these techniques.

The interpretation suggests that the success of the schemes is sensitive to the ratio of the time scales (alternatively, the frequencies) defined by the normal mode decomposition and those time scales inherent in the projection of the forcing terms on that mode. There exists a critical ratio for each iterative scheme which defines the regime where the scheme will converge. This explains why the modes corresponding to small equivalent depths can diverge, and why the forcing by such processes as cumulus convection can cause divergence. In the first case the inherent time scales are very large. In the second case the forcing time scale is very small. In Part II of this sequence of papers we offer evidence that the short time scale forcing does indeed exist in current models which include diabatic processes. We also introduce a new version of NMI which can adapt to the impact of this forcing. We test this scheme in a reasonably complex model, and make comparisons with the currently used versions of NMI.

Acknowledgments. The work comprises a small portion of my Ph.D. dissertation. I want to thank Drs. C. E. Leith, T. N. Krishnamurti and D. L. Williamson for their help as thesis advisors during my appointment as a graduate assistant at NCAR, and the NCAR Advanced Study Program for financial support. I also enjoyed and benefited from many discussions with Drs. J. Tribbia and R. Errico.

REFERENCES

Baer, F., 1976: Suppression of gravity oscillations by initialization in nonlinear flows. *Ann. Meteor.*, **11,** 101–104.

——, 1977: Adjustment of initial conditions required to suppress gravity oscillations in nonlinear flows. *Beitr. Phys. Atmos.* **50,** 350–356.

Bengtsson, L., 1978: Growth rate and vertical propagation of the initial error in baroclinic models. *Tellus,* **23,** 323–334.

——, 1981: Numerical prediction of atmospheric blocking—A case study. *Tellus,* **33,** 19–42.

Chapman, S., and R. S. Lindzen, 1970: *Atmospheric Tides.* Reidel, 200 pp.

Daley, R., 1981: Normal Mode Initialization. *Rev. Geophys. Space Phys.*, **19,** 450–468.

Dickinson, R., and D. Williamson, 1972: Free oscillations of a discrete stratified fluid with application to numerical weather prediction. *J. Atmos. Sci.*, **29,** 623–640.

Errico, R. M., 1982: The generation of gravity waves by quasi-geostrophic forcing. *J. Atmos. Sci.*, **39,** 573–386.

——, 1983: Convergence properties of Machenhauer's initialization scheme. *Mon. Wea. Rev.*, **111,** 2214–2223.

Kitade, T., 1983: Nonlinear Normal Mode Initialization with Physics. *Mon. Wea. Rev.*, **111,** 2194–2213.

Leith, C., 1980: Nonlinear normal mode initialization and quasi-geostrophic theory. *J. Atmos. Sci.*, **37,** 958–968.

Lorenz, E. N., 1982: Atmospheric predictability experiments with a large numerical model. *Tellus,* **34,** 505–513.

Machenhauer, B., 1976: An initialization procedure based on the elimination of gravity oscillations in a spectral barotropic primitive equation model. *Ann. Meteor.*, **11,** 135–138.

——, 1977: On the dynamics of gravity oscillations in a shallow water model, with applications to normal mode initialization. *Contrib. Atmos. Phys.*, **50,** 253–271.

——, 1983: Fundamentals of nonlinear normal mode initialization. *The Interaction between Objective Analysis and Initialization,* R. Daley, J. Derome and D. Williamson, Eds, NCAR Tech. Note NCAR/TN-204+PROC, 193 pp. [Available from NCAR, PO Box 3000, Boulder Co. 80303.]

Ortega, J. M., and W. Rheinbolt, 1970: *Iterative Solution of Nonlinear Equations of Several Variables.* Academic Press, New York.

Philips, N., 1981: Treatment of normal and abnormal modes. *Mon. Wea. Rev.*, **109,** 1117–1119.

Puri, K., 1983: The relationship between convective adjustment, Hadley circulation and normal modes of the ANMRC spectral model. *Mon. Wea. Rev.*, **111,** 23–33.

——, and Bourke, W., 1982: A scheme to retain the Hadley circulation during nonlinear normal mode initialization. *Mon. Wea. Rev.*, **110,** 327–335.

Rasch, P. J., 1984: Extending normal mode initialization: A new method. Ph.D. Dissertation. [Available as NCAR Cooperative Thesis No. 81 from the author.]

——, 1985: Developments in normal mode initialization. Part II: A new method and its comparison with currently used schemes. *Mon. Wea. Rev.*, **113,** 1753–1770.

Temperton, C., and D. Williamson, 1981: Normal mode initialization for a multi-level grid point model. Part I: Linear aspects. *Mon. Wea. Rev.*, **109,** 729–743.

Tribbia, J. J., 1982: Nonlinear normal mode initialization and the ellipticity condition. *Mon. Wea. Rev.*, **109,** 1751–1761.

——, 1984: A simple scheme for higher order nonlinear normal mode initialization. *Mon. Wea. Rev.*, **112,** 278–284.

Wouk, A., 1964: Direct iteration, existence and uniqueness. *Nonlinear Integral Equations,* P. Anselone, Ed., University of Wisconsin Press 3–34.

Williamson, D., 1976: Normal mode initialization procedure applied to forecasts with the global shallow water equations. *Mon. Wea. Rev.*, **104,** 195–206.

——, and C. Temperton, 1981: Normal mode initialization for a multi-level grid point model. Part II: Nonlinear aspects. *Mon. Wea. Rev.*, **109,** 729–743.