Reply

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Two iterative methods for solving the balance equation, the Shuman (1957) or Miyakoda (1956) method and the 'Arnason (1958) method, were extended by Paegle and Tomlinson (1975). In the following it is assumed that the successive approximate solutions of the balance equation satisfy the ellipticity condition.

'Arnason showed that his method might be divergent if the relative vorticity calculated with approximate streamfunctions takes large positive values. Based on 'Arnason's work, Paegle and Tomlinson considered a modification of this method, by the introduction of a relaxation parameter, which is convergent in the case of large positive vorticity. Applying the same modification to the method of Shuman they found that the modified method also converged where the original method diverged. No convergence analysis was attempted.

From a convergence analysis of the modified Shuman method, however, it appears impossible to prove, due to the nonlinearity of the error equation, that the introduction of a relaxation parameter has an effect comparable to that in the modified 'Arnason method, where the relaxation parameter changes a divergent method into a convergent one.

The only thing one can prove, at least in our opinion, is that the relaxation parameter may accelerate convergence in the case of a very slowly converging, near-perfect oscillating solution.

To be more specific, let us consider the error equation of the modified Shuman method, which is described by Eq. (2.13) of our paper (Bijlsma and Hoogendoorn, 1983) or Eq. (13) of Iversen and Nordeng (1982):

$$(1+\gamma)\nabla^2\epsilon^{n+1} = \gamma\nabla^2\epsilon^n + \frac{(\bar{\psi}_{xx}-\bar{\psi}_{yy})(\epsilon^n_{xx}-\epsilon^n_{yy}) + 4\bar{\psi}_{xy}\epsilon^n_{xy}}{2f+\nabla^2\bar{\psi}+(1+\gamma)(\nabla^2\epsilon^{n+1}+\nabla^2\epsilon^n)},$$

where the relaxation parameter is denoted by γ. Further, $\epsilon^n = \psi^n - \psi^{n-1}$, and $\bar{\psi} = \psi^n + \psi^{n-1}$. This equation is identical to equation (15) of Iversen and Nordeng, if we write $\nabla^2\epsilon^n = \eta^n - \eta^{n-1}$ with $\eta^n = f + \nabla^2\psi^n$. For $\gamma = 0$ it is the error equation of the Shuman method given by Eq. (2.4) of our paper. If we consider near-perfect oscillating solutions of this equation (including $\gamma = 0$) such that we may neglect $\eta^{n+1} - \eta^{n-1}$ compared to $\eta^n + \eta^{n-1}$ we may drop the term $(1+\gamma)(\nabla^2\epsilon^{n+1} + \nabla^2\epsilon^n)$ in the denominator, so that we tacitly assume that the Shuman method converges, no matter how slowly, as follows from the analysis of p. 998 of our paper. From that analysis it is also evident that the introduction of γ has a beneficial effect on the convergence of the method since it reduces the oscillating character of the solutions.

However, the procedure described above is not a nonlinear convergence analysis, nor is the analysis of Iversen and Nordeng. A nonlinear analysis requires that either we have to consider the complete error equation without neglecting the term $(1+\gamma)(\nabla^2\epsilon^{n+1} + \nabla^2\epsilon^n)$, or that Iversen and Nordeng should use

$$\begin{aligned} 2\bar{\eta} &= \eta^{n+1} + \eta^n + \gamma(\eta^{n+1} - \eta^{n-1}) \\ &= \eta^n + \eta^{n-1} + (1+\gamma)(\eta^{n+1} - \eta^{n-1}) \end{aligned}$$

in Eq. (16) of their paper, so that their results are no longer valid. Without the assumption of a near-perfect oscillation the quantities $\bar{\eta}$ are not comparable for different values of γ. The nonlinear case has to be treated heuristically. Possibly the "worst" solutions of the Shuman method do not really diverge, but tend to the near-perfect oscillation we considered above.

REFERENCES

'Arnason, G., 1958: A convergent method for solving the balance equation. *J. Meteor.*, **15**, 220–225.

Bijlsma, S. J., and R. J. Hoogendoorn, 1983: A convergence analysis of a numerical method for solving the balance equation. *Mon. Wea. Rev.*, **111**, 997–1001.

Iversen, T., and T. E. Nordeng, 1982: A convergent method for solving the balance equation. *Mon. Wea. Rev.*, **110**, 1347–1353.

Miyakoda, K., 1956: On a method of solving the balance equation. *J. Meteor. Soc. Japan*, **34**, 364–367.

Paegle, J., and E. M. Tomlinson, 1975: Solution of the balance equation by Fourier transform and Gauss elimination. *Mon. Wea. Rev.*, **103**, 528–535.

Shuman, F. G., 1957: Numerical methods in weather prediction. Part I: The balance equation. *Mon. Wea. Rev.*, **85**, 329–332.

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