Vertical Interpolation of Meteorological Variables

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ABSTRACT

The initialization of numerical prediction models usually requires the transformation of variables observed in a p-coordinate system into some other coordinate frame of reference (e.g., σ-coordinates or Θ-coordinates). Such transformations require the application of interpolation or curve-fitting techniques. The present study demonstrates that the choice of an appropriate interpolation scheme can become a critical issue for the skill of a low-resolution prediction model. First we show that the interpolation scheme, when applied to more than one meteorological variable, should satisfy the balance requirements that exist between these variables. Not all of the currently used schemes meet this condition. Next we provide evidence indicating that interpolation schemes used to convert p- into σ-coordinates, and then back into p-coordinates, do not necessarily replicate the original, observed field distributions of these meteorological variables. Such double transformations usually are required, because the numerical output in model coordinates has to be translated back to p-coordinates for verification of model results. Because of the limitations of certain interpolation procedures, even a correct model prediction may exhibit low predictive skill because of errors introduced in this final coordinate transformation process.

1. Introduction

For the integration of a set of prognostic equations in a numerical prediction model it is necessary to prepare initial values of all time-dependent variables. Not all of these variables can be acquired in a form suitable for direct input into the model. The first task in numerical simulation, therefore, is to prepare the necessary input data. Many models require the interpolation of p-coordinate data, e.g., variable fields observed on standard isobaric surfaces, to nonpressure coordinates, such as sigma or isentropic coordinate systems (e.g., see Bleck, 1984). To provide convenient model output, it may be necessary to interpolate the prediction results again to p-coordinates. For low-resolution models, more so than for high resolution ones, the adequate vertical interpolation of meteorological variables becomes a critical issue. [Low-resolution models carry the advantage of greater economy in terms of required computer time, often without undue penalties in terms of prediction skill (Anthes and Warner, 1978).] If "correct" values of the meteorological fields cannot be derived from their representation in p-coordinates by suitable interpolation, a model running on nonpressure coordinates obviously will take off from wrong initial conditions. On the other hand, if model physics and dynamics produced good results, little would be gained if they could not be transferred adequately to p-coordinates.

An adequate model initialization procedure should prepare grid point data that describe the meteorological fields as strictly as possible in conformance with the fields in the "real" atmosphere, maintaining the characteristics of these variables, such as their vertical profiles and the balances that must exist between them.

Data interpolation for model initialization remains a complex problem because one does not know, *a priori*, which interpolation scheme will result in that profile of a variable which best approximates the real atmosphere. This problem has not yet been resolved adequately. For instance, one usually assumes that the thermodynamic variables, T, Θ, Θ_e, etc., follow a linear dependence on the logarithm of pressure, whereas other variables, such as the geopotential Φ, the three wind components u, v, w, relative vorticity ζ, specific humidity (q) and relative humidity (RH), etc., are supposed to depend linearly on pressure. As we shall see in the subsequent discussion, certain of these assumptions are unrealistic and possibly destroy the basic equilibrium relationships between these variables, which do exist in the real atmosphere. High-resolution models should be less sensitive to such interpolative misrepresentations of atmospheric structure than are the more economic low-resolution models.

Problems with vertical interpolation, especially in mountainous regions, are well recognized (Sundqvist, 1976, 1982; Phillips, 1979). To better address this problem Shen *et al.* (1985) designed a limited domain primitive equation model in sigma coordinates, which uses observed values of both ϕ and T as well as of u, v, and q. In order to maintain the atmosphere's static equilibrium, appropriate vertical interpolation methods were used. This paper describes the considerations that went into the vertical interpolation techniques used in the Shen *et al.* model.

In the subsequent sections we shall examine realistic vertical distributions of some meteorological parameters and the relationships between them. The impact of several interpolation schemes on problems related to model initialization will be considered.

2. Vertical distribution of key meteorological parameters in the real atmosphere

Plots of the annual mean profiles of T, Φ, u, q, and ω at 55, 35 and 15°N as a function of the logarithm of pressure [obtained from Oort and Rasmusson (1971) data not given here] show that each of these variables has a different vertical profile that differs with geographic location and with altitude. Temperature tends to increase linearly with ln$p$ in the troposphere, but has a small lapse rate in the stratosphere, with the average tropopause location being a function of latitude on an annual-average and dependent on the synoptic condition on short time scales. A linear relationship between T and ln$p$ may be satisfactory within certain layers of the atmosphere, but not in the transition zones between these layers.

The change of curvature in the vertical profiles of the geopotential is masked by the large mean value of this parameter, but is, nevertheless, present. A linear decrease with ln$p$ is an adequate approximation in the stratosphere, but more complex conditions prevail in the troposphere. Variations also exist with geographic location.

Vertical profiles of the zonal wind offer an example of strong nonlinearity. The presence of a jet stream near tropopause level is a prime example. Easterlies at low and high altitudes are characteristic of low latitude regions.

Moisture decreases quickly with height above the lowest layers of the atmosphere and its vertical profiles show a strong dependence on geographic location.

In addition to the dependence of vertical profiles of meteorological variables on geographic location there is a dependence on season. The average tropopause height tends to be lower in winter than in summer, and the tropopause jet streams are stronger in winter than in summer, whereas the moisture content of the lower troposphere is higher in summer than in winter. Unless one uses a high-resolution numerical model, it is quite obvious that a set of simple linear proportionalities with ln$p$ cannot provide suitable approximations of the vertical profiles of most meteorological variables.

Plots of variables as functions of pressure, p, instead of ln$p$ clearly show that linear vertical interpolation of temperature, geopotential, zonal wind, vertical velocity or moisture with p is not adequate.

Daily profiles, exemplified by those of u, v, Φ, and T obtained at Urumqi (43°47'N, 83°37'E) at 1200 GMT on 8 June 1979 (Fig. 1) are even more complex than the space and time averaged ones. More examples are provided in Figs. 4 and 5. Lapse rates in the surface layers often can exceed the dry adiabatic one and can vary greatly between significant points along the tropospheric as well as stratospheric portions of the individual soundings. Often, more than one strong inversion near the tropopause level appears in the sounding. Linear interpolation with ln$p$ within the framework of a low-resolution model will not do justice to the description of the initial conditions.

Other parameters, such as u and v shown in Fig. 1, reveal even more complex vertical structures that could not be accommodated by linear interpolation with ln$p$ over the relatively large layer increments of a low-resolution model. Of course, different interpolation functions or factors could be used for different variables, in order to achieve optimum approximation to reality. However, if errors exist in the interpolated values of different variables, the basic balances between these variables, such as the quasi-geostrophic and quasi-static states, may be destroyed, leading to incorrect predictions and even to computational instabilities, one of the reasons why most of the numerical prediction models currently in use have to resort to elaborate initialization procedures. Furthermore, the use of different interpolation procedures for different variables will make the computer coding of a numerical model more complicated.

3. Impact of equilibrium requirements between variables in the real atmosphere on the interpolation accuracy

Observational evidence suggests that in the real atmosphere the wind and pressure fields are in quasi-

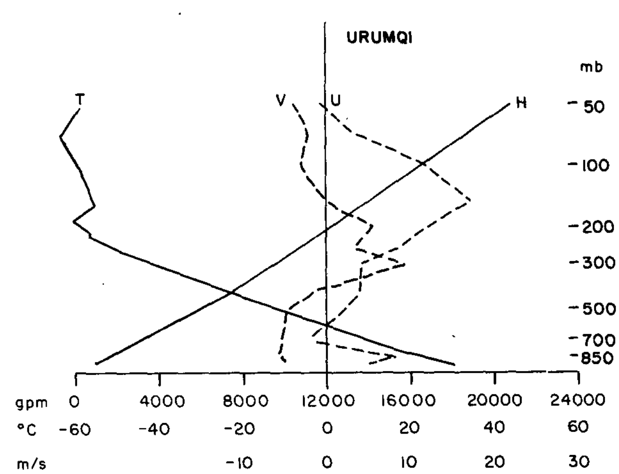

FIG. 1. The profiles of geopotential, temperature and wind at Urumqi (43°47'N, 83°37'E) for 1200 GMT 8 June 1979.

geostrophic balance, while the geopotential and temperature fields are in a quasi-hydrostatic equilibrium. Thus, the key variables in the atmosphere are interdependent. Interpolation in a low-order model, therefore, is not simply a matter to be considered separately for each variable. The problem will be illustrated for a polytropic atmosphere, i.e., an atmosphere in hydrostatic equilibrium and having a constant lapse rate

$$T = T_0 - \gamma Z \tag{1}$$

where $\gamma = -\partial T/\partial Z$. The hydrostatic approximation

$$\partial\Phi/\partial(\ln p) = -RT \tag{2}$$

usually is justified, except for motions with a horizontal scale of less than about 20 km.

In the real atmosphere changes of lapse rate with height occur on such an irregular and frequent basis that $T(Z)$ cannot be described by an elementary function of altitude or pressure. However, the atmosphere can always be divided into layers, each of which can be approximated by a constant lapse rate. Under such conditions, substitution of Eq. (1) into (2) and integration yields

$$T/T_0 = (p/p_0)^{R\gamma/g} \tag{3}$$

which gives the relation between temperature and pressure, similar to that prescribed by a polytropic process, i.e., by

$$T/T_0 = (p/p_0)^k \tag{4}$$

where the coefficient of polytropy is given by

$$k = (C_p - C_\pi)/(C_v - C_\pi) \tag{5}$$

and singly determines the state of the polytropic atmosphere. The specific heats, C_i, are for isobaric, isosteric and polytropic processes.

We shall now examine specific cases and their impact on vertical interpolation.

a. The isothermal atmosphere

With $\gamma = 0$ and T = const, the geopotential

$$\Phi = -RT(\ln p) + C \tag{6}$$

becomes linearly dependent on the logarithm of pressure, a condition that usually approximates the stratosphere reasonably well (see Fig. 1).

b. The standard atmosphere

With $\gamma = 0.0065°\mathrm{C\ m^{-1}}$ for $Z \leqslant 10769$ m we obtain from Eq. (3)

$$T = T_0(p/p_0)^{0.1904}. \tag{7}$$

Expansion into a Taylor series for small increments of p yields

$$T = T_0\left[1 + 0.1904\left(\ln\frac{p}{p_0}\right) + \frac{0.1904^2}{2!}\left(\ln\frac{p}{p_0}\right)^2 + \frac{0.1904^3}{3!}\left(\ln\frac{p}{p_0}\right)^3 + \cdots\right] \tag{8}$$

and in first approximation

$$T = T_0\left[1 + 0.1904\ln\left(\frac{p}{p_0}\right)\right] = a\ln p + b \tag{9}$$

where $a = 0.1904T_0$, and $b = (1 - 0.1904\ln p_0)T_0$. Thus, temperature is a linear function of altitude, as prescribed by Eq. (1), also proportional to pressure with the power of 0.1904, as indicated by Eq. (7), and linearly dependent on $\ln p$ in the standard atmosphere as shown by Eq. (9).

Under hydrostatic conditions [Eq. (2)] the geopotential should have the vertical distribution given by

$$\Phi = a(\ln p)^2 + b(\ln p) + c. \tag{10}$$

Therefore, the geopotential should depend on the squared logarithm of pressure in the troposphere of the standard atmosphere, and not simply on the logarithm of pressure as was suggested for the temperature according to Eq. (9). The two variables, Φ and T, thus cannot rely on the same interpolation approach.

c. The dry adiabatic atmosphere

The lapse rate condition $\gamma = \gamma_d = g/C_p = 0.0098°\mathrm{C\ m^{-1}}$ yields

$$T = T_0(p/p_0)^{\gamma_d R/g} = T_0(p/p_0)^{0.286}, \tag{11}$$

i.e., the temperature is proportional to $p^{0.286}$. Equation (11) also can be expanded into a Taylor series, but for better accuracy the squared term will have to be retained in the approximation

$$T = a(\ln p)^2 + b(\ln p) + c \tag{12}$$

$$\Phi = a_1(\ln p)^3 + b_1(\ln p)^2 + c(\ln p) + d \tag{13}$$

where a, b, a_1, b_1, c and d are dependent on T_0 and p_0 at a given point. The geopotential now is related to the cubic power of the logarithm of pressure. In general, the larger the lapse rate, the more complex the vertical distributions of temperature and geopotential will be. These distributions are determined by the basic equilibrium (i.e., the hydrostatic approximation) between the two variables, which should be taken into consideration if vertical interpolations are to be performed in a low-resolution model.

We shall now examine vertical distributions of the wind components and of other kinetic parameters. For the sake of brevity we assume the wind and pressure fields to be in geostrophic equilibrium, hence

$$\left.\begin{aligned} fv_g &= \partial\Phi/\partial x \\ -fu_g &= \partial\Phi/\partial y \end{aligned}\right\}. \tag{14}$$

Introduction of Eq. (10) yields

$$\left.\begin{aligned} fv_g &= (\partial a/\partial x)(\ln p)^2 \\ &\quad + (\partial b/\partial x)\ln p + \frac{\partial c}{\partial x} + (2a+b)\frac{\partial \ln p}{\partial x} \\ -fu_g &= (\partial a/\partial y)(\ln p)^2 \\ &\quad + (\partial b/\partial y)\ln p + \frac{\partial c}{\partial y} + (2a+b)\frac{\partial \ln p}{\partial y} \end{aligned}\right\} . \quad (15)$$

According to this equation the vertical interpolation of the geostrophic wind not only depends on $(\ln p)^2$, but also on the horizontal gradients of temperature and pressure through the differentials of the coefficients a, b and c. Because strong winds always are associated with strong horizontal pressure and temperature gradients, linear interpolation of winds in low-resolution models must be of dubious quality. Ageostrophic components in the wind fields of the real atmosphere add an additional degree of complexity to the interpolation problem.

In a similar manner, the vorticity, divergence and vertical velocity fields have to be related to $(\ln p)^2$ and to the horizontal distributions of T and p. Furthermore, the divergence fields have to be restricted by the nondivergent state of the total air column.

The foregoing discussion has provided sufficient demonstration of the fact that meteorological parameter interpolation has to meet certain balance requirements, which are usually nonlinear. Consequently, models assuming linear relationships of u, v, ω, and ζ, div, etc., with pressure are in error and thus provide inconsistent initial conditions, more so if they are of low rather than high vertical resolution.

We also have demonstrated that one simple interpolation procedure will not do justice to the requirements of all variables. This complexity is one of the reasons why the interpolation problem has not yet been solved satisfactorily, and why numerical prediction models have resorted to elaborate initialization procedures instead of using the observed fields of the key variables. We should keep in mind that an interpolation of the thermodynamic variables with the (linear) logarithm of pressure, and of the kinematic variables simply with pressure, will not satisfy the balance requirements of the real atmosphere, and thus will produce erroneous initial fields and cause errors in the forecasts.

There are several interpolation schemes widely used at present, such as linear interpolation, Lagrange's polynomial interpolation and spline interpolation. Each has its own advantages and limitations and may work better under certain conditions, but not under others. In the following discussion we shall examine linear interpolation of variables with respect to a function of pressure, such as p, $\ln p$, p^μ (where μ is a positive constant), and Lagrange's polynomial interpolation, which describes a variable in terms of the square or cubic power of the logarithm of pressure.

4. Choice of interpolation schemes

a. The linear interpolation scheme

A variable α in the new coordinate system is defined at level L of that system by

$$\alpha_L = \alpha_{k+1} - \frac{\alpha_{k+1} - \alpha_k}{\Delta F} \cdot \Delta F' \quad (16)$$

where k and $k + 1$ are two levels in the old coordinate system, usually located on either side of the new level L, and ΔF and $\Delta F'$ represent the differences in the pressure functions p, $\ln p$ or p^μ given by

$$\Delta F = \begin{cases} p_{k+1} - p_k \\ \ln(p_{k+1}/p_k) \\ p_{k+1}{}^\mu - p_k{}^\mu \end{cases} \quad (17)$$

$$\Delta F' = \begin{cases} p_{k+1} - p_L \\ \ln(p_{k+1}/p_L) \\ p_{k+1}{}^\mu - p_L{}^\mu . \end{cases} \quad (18)$$

Because of the discussion in the preceding chapter we will assume a proportionality of the variable to the logarithm of pressure.

b. Lagrange's polynomial interpolation

Assuming that the variable α is proportional to the squared logarithm of pressure (see preceding section) its value at the new coordinate level L can be described by

$$\left.\begin{aligned} \alpha_L &= C_1 \ln(p_L/p_k)\ln(p_L/p_{k+1})\alpha_{k-1} \\ &\quad + C_2 \ln(p_L/p_{k-1})\ln(p_L/p_{k+1})\alpha_k \\ &\quad + C_3 \ln(p_L/p_{k-1})\ln(p_L/p_k)\alpha_{k+1} \\ C_1 &= [\ln(p_{k-1}/p_k)\ln(p_{k-1}/p_{k+1})]^{-1} \\ C_2 &= [\ln(p_k/p_{k-1})\ln(p_k/p_{k+1})]^{-1} \\ C_3 &= [\ln(p_{k+1}/p_{k-1})\ln(p_{k+1}/p_k)]^{-1} \end{aligned}\right\} . \quad (19)$$

Usually, the level L will lie between levels $k - 1$ and $k + 1$ of the old coordinate system. There is little difference in the solutions if L lies above or below k. However, if L lies outside of level $k - 1$ or $k + 1$ then Eq. (19) is used for extrapolation outside the defined reach of the old coordinate system. If such an extrapolation is carried over too large an interval, serious errors may arise in comparison to the structure of the real atmosphere.

c. The top of the model atmosphere

In a conversion from *p*-coordinates to nonpressure coordinates (such as σ-coordinates) and back to *p*-coordinates extrapolation usually cannot be avoided altogether. In one coordinate system the upper- and lowermost levels usually lie outside the defined range of the other coordinate system. Experience shows that it is better to use interpolation (rather than extrapolation) when converting from the *p*- to a nonpressure coordinate system if the model calculations are defined in the latter system. In this way the errors introduced into the calculations will be minimized. Extrapolation, then, is used to translate the model output (i.e., the forecast) to the lowest or highest *p*-coordinate surfaces for display purposes, because in this final step the introduction of extrapolation errors is less harmful since they will not propagate into further iterative calculations.

As a test example we chose a six-level model in sigma-coordinates, with $\sigma = (p - p_t)/(p_s - p_t)$, where p_t is the pressure at the top, and p_s at the surface of the model atmosphere. The original data were taken from the mandatory levels 100, 200, 300, 500, 700 and 850 mb. Four experiments were conducted, involving linear and Lagrangian interpolation, as well as atmospheric tops in the model at 0 and at 60 mb:

Scheme 1: Linear interpolation using Eqs. (16–18), atmospheric top at $p_t = 60$ mb.

Scheme 2: Lagrange's interpolation, Eq. (19), atmospheric top at 60 mb.

Scheme 3: Linear interpolation, atmospheric top at 0 mb.

Scheme 4: Lagrange's interpolation, atmospheric top at 0 mb.

For the sake of brevity, the discussion of results will be restricted to the parameters Φ, T and u. Data fields were initialized for 1200 GMT 8 June 1979, to cover a domain including most of China, with an emphasis on the Qinghai-Xizang (Tibet) Plateau. The stations Urumqi (43°37′N, 87°37′E), Germu (36°25′N, 94°54′E), Lhasa (29°40′N, 91°08′E), Chengdu (30°40′N, 104°01′E) and Haikou (20°02′N, 110°21′E) provided the observed data. The high plateau elevations within the model domain lend an additional degree of importance to the testing of appropriate interpolation procedures.

5. Results

a. Scheme 1

With $p_t = 60$ mb the highest level in sigma-coordinates will lie below the 100 mb level if the surface pressure is larger than 542 mb, thus allowing interpolation (rather than extrapolation) of the σ-coordinates from the *p*-coordinates at the top of the model. Over high plateau regions this consideration is of importance.

Figure 2 depicts the observed 500 mb geopotential height field at 1200 GMT 8 June 1979. As evident

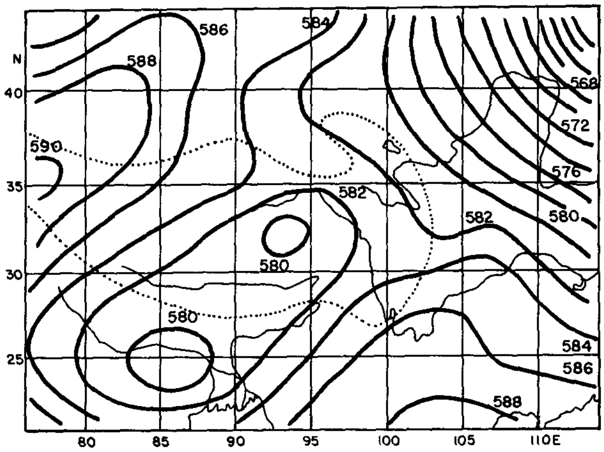

FIG. 2. Observed geopotential height field, 1200 GMT 8 June 1979.

from the previously discussed diagrams (Fig. 1), the curvature of the profiles of geopotential height Φ against ln*p* as vertical coordinate is rather small. It is tempting, therefore, to use ln*p* for linear interpolation of Φ from the pressure to the sigma-coordinate system. Interpolation from the σ-system back to the *p*-system, required by the model output procedures, revealed, however, that the geopotential field at 500 mb became rather erratic (Fig. 3a) and only vaguely resembled the original field shown in Fig. 2. Furthermore, the differences between these two fields (Fig. 3b) were not distributed randomly, as one would expect with observational errors, but were, to a large extent, controlled by the topography within the model domain. Over the plateau the differences were almost nil, while over the surrounding lowlands the reinterpolated 500 mb heights were consistently lower than the observed heights. The vortex located near 32.5°N, 93.0°E (Fig. 2) suffered greatly under the interpolation procedure and became nearly extinct (Fig. 3a). The high-pressure ridge over northwestern China became much weaker because of the general reduction of geopotential heights there, and a false anticyclone appeared to the southeast of the plateau because of the lesser reductions in the interpolated geopotential height values there. Other levels in the model revealed similar discrepancies, the largest appearing near 200 mb in the transition region between troposphere and stratosphere, to be discussed later.

The discrepancies demonstrated here were due only to the use of a certain interpolation procedure widely used in coordinate transformation (linear with ln*p*). Even though the model may have forecast the geopotential field well, the results would have been displayed poorly. The model skill, therefore, would have received poor marks. For a low-resolution model the use of inadequate interpolation procedures thus might constitute a fatal flaw.

Figure 4 shows the vertical temperature profiles at the five test stations within the model domain. The dots mark the temperatures derived by interpolation

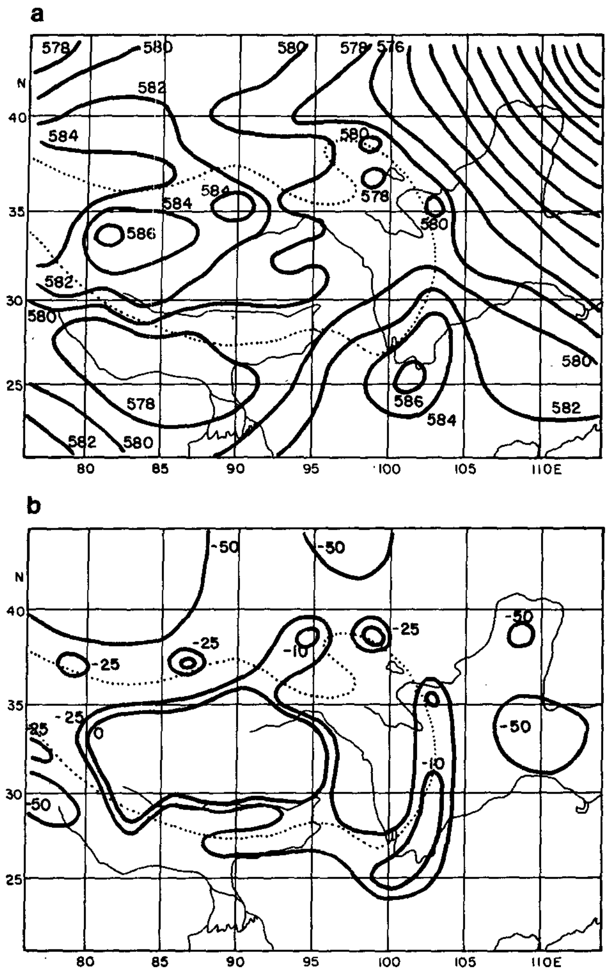

FIG. 3. 500 mb geopotential heights, 1200 GMT 8 June 1979: (a) obtained after reinterpolation by scheme 1 (dam); (b) differences (gpm) between observed and reinterpolated values.

from p- to σ-coordinates, and the dashed lines represent the reinterpolated temperature profiles, converting from a σ-system back to a p-system. Since in the troposphere vertical temperature profiles follow rather well a linear dependence on $\ln p$; as demonstrated in section 2, the two interpolation steps (p to σ, and back again to p) yielded quite acceptable results in the lower levels of the model atmosphere. Serious errors appear, however, where the tropopause lies below 100 mb. The reinterpolated temperatures at Urumqi, for instance, were much colder than the observed ones at 100 mb, and much warmer at 200 mb.

The observed profiles of the zonal wind component, u, at the five stations mentioned earlier are shown in Fig. 5, together with the interpolated and the reinterpolated profiles. Linear interpolation with $\ln p$ does extremely poorly in describing wind profiles. The jet stream, generally, is weakened by the interpolation procedure, and the wind maximum often appears at the wrong level in the low-resolution model.

b. *Scheme 2*

Interpolation for p- to σ-coordinates, using the Lagrangian scheme, yielded profiles of the geopotential similar to those obtained by scheme 1. However, interpolation back to the p-coordinate system revealed the superiority of Lagrange's technique. Figure 6a shows the reinterpolated 500-mb map. The originally observed pattern (see Fig. 2) was retained well, including the vortex over the central plateau. The differences between reinterpolations and observations (Fig. 6b) were smaller and generally more evenly distributed. The ridge over northwestern China and the vortex over southern Asia (near 25°N) were strengthened somewhat by the interpolation procedure. The differences at 300 mb and at 700 mb were even less than those at 500 mb. Largest errors were encountered at 100 mb, because extrapolation had to be used over some regions to arrive at the field in the σ-coordinate systems, amplifying the errors in the reinterpolation to the p-system.

The Lagrangian scheme also did considerably better in the interpolation and reinterpolation of the vertical temperature profiles (Fig. 7). The transition between

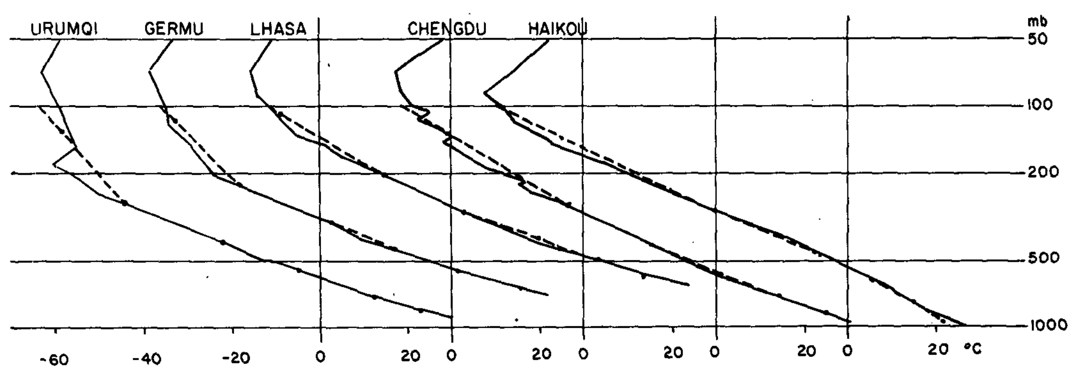

FIG. 4. Temperature profiles from five stations. The solid lines are the observed profiles, the dashed–dotted represent the interpolated temperatures in σ-coordinates, and the dashed lines are the reinterpolated profiles, using scheme 1.

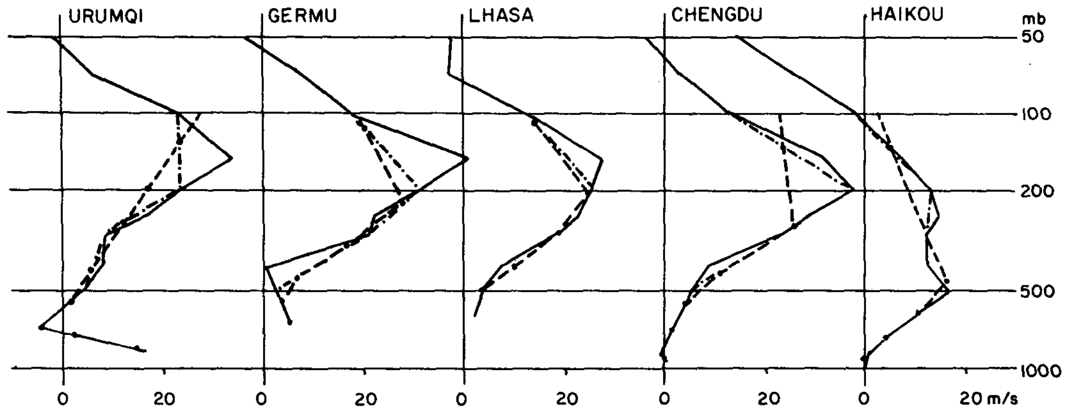

FIG. 5. U-component profiles from five stations. The solid lines are the observed profiles, the dashed–dotted lines are the interpolated profiles and the dashed lines are the reinterpolated profiles, using scheme 1.

troposphere and stratosphere was handled much better than by scheme 1, as were the changes in lapse rate within this transition zone. Equation (19) actually constitutes a second-order polynomial in p_L, so that the temperature at level L can be written as

$$T_L = a(\ln p_L)^2 + b(\ln p_L) + c. \tag{20}$$

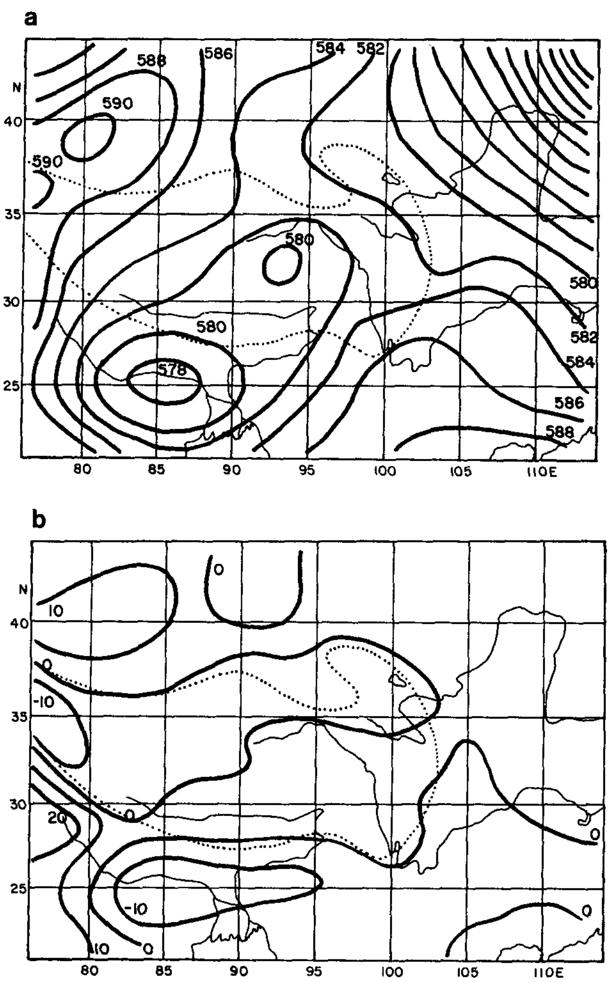

FIG. 6. Similar to Fig. 3 except scheme 2 was used for interpolation.

The coefficients C_1, C_2, C_3 and the quantities p_k, p_{k+1}, p_{k-1} and α_k, α_{k+1}, α_{k-1} (i.e., T_k, T_{k+1}, T_{k-1}) are given for each point in the profile. Equation (20), therefore, is a more accurate approximation of Eq. (8) than a linear relationship with $\ln p$ would be.

Although Lagrangian interpolation also tends to reduce the wind maximum in sharply defined jet streams (Fig. 8), the wind profiles represented by the interpolation as well as the reinterpolation procedures are considerably more realistic than those obtained by scheme 1.

From this discussion it becomes quite obvious that Lagrangian interpolation approximates conditions in the real atmosphere better when conversion from a p- to a σ-coordinate system is called for, but also when the σ-system has to be converted back to a p-system. The errors in reinterpolated fields at 100 mb were larger than at other surfaces, mainly because the uppermost level in this model had to rely, at least over some regions, on extrapolation rather than interpolation.

The better performance of the Lagrangian scheme has to be ascribed to the fact that it meets some of the basic equilibrium requirements between atmospheric variables which were discussed earlier. Simulation of the jet stream intensity suffers under certain conditions because, as we have shown earlier, wind speed there depends also on horizontal gradients of pressure and temperature not accounted for by this interpolation procedure.

c. Schemes 3 and 4

In these schemes $p_t = 0$ mb, therefore the values of all variables at the topmost σ-coordinate level will have to be obtained by extrapolation rather than by inter-

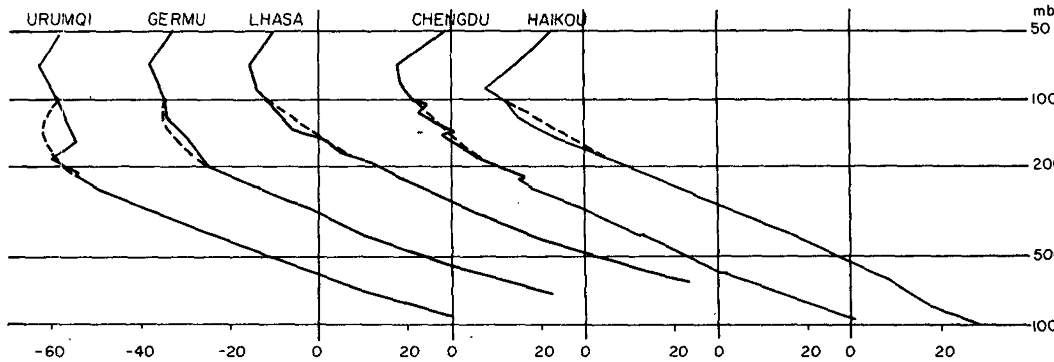

FIG. 7. Temperature profiles from five stations. The solid lines are the observed profiles and the dashed–dotted lines are the interpolated profiles, using scheme 2.

polation. Since the uppermost pressure level with observed data is 100 mb in the present model, all other σ-coordinate levels, with the exception of the top level, are obtained by interpolation, hence are similar to the values obtained by schemes 1 and 2. (In some places with low elevations the values at the lowest σ-level also may have to be obtained by extrapolation.)

Extrapolated values at the uppermost σ-level had to rely on observations from the 100 and 200 mb levels for linear interpolation, and from the 100, 200 and 300 mb levels for Lagrangian interpolation. Serious errors will have to be expected because the distributions of parameters above 100 mb usually differ from those below 100 mb in the real atmosphere.

In Fig. 9 we show the temperatures calculated for the uppermost σ-level by linear interpolation (scheme 3). Temperatures over the Tibetan plateau at this level were generally below $-90°C$ and went as low as $-95.5°C$, much lower that the observed temperatures. Where the tropopause was located below the 200 mb level, linear extrapolation of the lapse rate between 200 and 100 mb would give relatively small errors, as for instance over the northern regions of the model domain. However, the reinterpolated values in the upper atmosphere of the p-coordinate system would not fare so well. Over the plateau and over the southern regions of the model domain the tropopause was located above the 200 mb level, in places even above the 100 mb level. Extrapolation to the uppermost σ-level, therefore, yields ludicrous results. The horizontal gradients in the errors introduced by extrapolation would establish unrealistic pressure gradient forces, which would adversely affect the model results.

The Lagrangian scheme (scheme 4) is not immune to these problems. As an example we show the u-component field at the uppermost σ-level, based upon extrapolation from the 300, 200 and 100 mb values. The wind maximum of the jet stream usually is located near one of these levels, causing strong vertical wind shears. These extrapolated shears cause a fictitious easterly jet over the plateau between 30 and 35°N with maximum zonal components of -72 m s^{-1} above the observed position of the westerly jet (Fig. 10). If such a strong, and erroneous, easterly wind field were imposed upon the model's initial conditions, large errors in the model predictions should be expected.

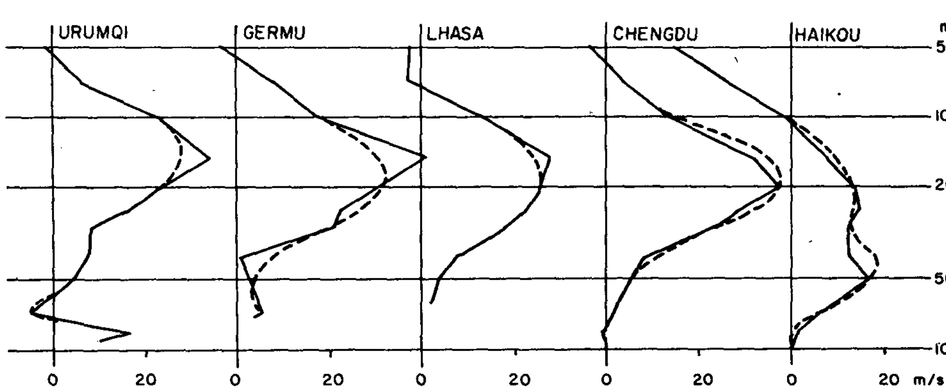

FIG. 8. Similar to Fig. 7 but for u-component profiles.

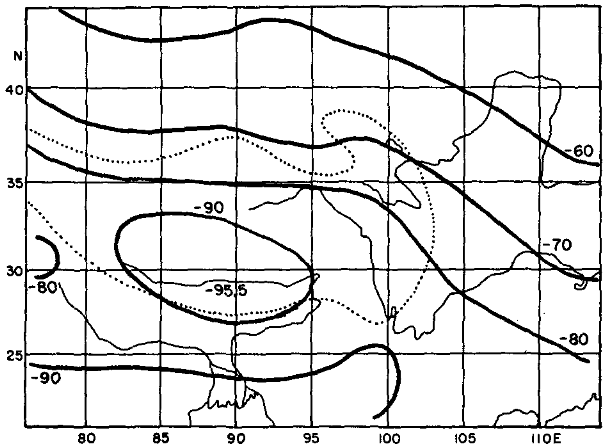

FIG. 9. The interpolated temperature field at the highest level in σ-coordinates obtained by scheme 3.

Lagrange's interpolation with a third-order polynomial was also tested under conditions of $p_t = 0$, but yielded similarly unreasonable results at the uppermost σ-level. We therefore have to conclude that no low-resolution model should attempt to choose $p_t = 0$ as the upper boundary of its atmosphere.

Perhaps the obvious should be pointed out: If σ-levels in a model lie close to mandatory pressure levels in a p-coordinate system the interpolated and reinterpolated profiles of any variable will resemble rather closely the observed profiles. Such fortuitous choices, however, are usually not available for model applications over terrain with large elevation differences.

d. Summary of reinterpolation errors

In all the schemes tested above the reinterpolated values differed from the originally observed ones. Because of the unacceptable extrapolation errors inherent in schemes 3 and 4 we will summarize error evaluations only for schemes 1 and 2, with sigma defined as $\sigma = (p - p_t)/(p_s - p_t)$. Twelve evenly spaced grid points from the model domain were chosen for statistical evaluation. Tables 1 and 2 list the differences between reinterpolated values and observed values for five important meteorological parameters at mandatory pressure levels. At two of these points the 700 mb level was located below ground, and at three points the 850 mb level did the same. These points were not included in the statistics of scheme 1 (linear interpolation). In Table 2, depicting the results of scheme 2, all twelve points were considered in the 700 mb statistics, but only nine points in the 850 mb statistics.

Because $p_t = 60$ mb in these two schemes, the values at the highest σ-level of this model version were obtained by interpolation from the p-coordinate system, but the reinterpolated values in the p-system had to be obtained by extrapolation from the σ-system. The errors of reinterpolation at 100 mb were larger with scheme 2 than with scheme 1, especially in the temperature and u-component fields. Two points out of twelve had errors in excess of 5°C. The maximum error in scheme 2 reached −10.1°C, whereas it was only −4.7°C with scheme 1. Final extrapolation to the p-coordinate system accounted for most of these errors.

At 200 mb, where the tropopause was located over most of the region, scheme 2 proved to be superior to scheme 1. The geopotential only had one point with errors greater than 50 gpm (i.e., 53 gpm) in scheme 2, while four points exceeded 100 gpm with scheme 1, and the maximum error with that scheme reached −165 gpm. The maximum error in temperature with scheme 2 was 4.3°C, while with scheme 1 the errors at three points exceeded 5.0°C. The poor performance of scheme 1 in handling the wind field is quite apparent. In the u- and v-components nine and eight points out of twelve exceeded error limits of 2 m s^{-1}, with maximum errors of −14.6 and 8.1 m s^{-1}, respectively. Scheme 2 cut these errors in half.

At 300 mb the Lagrangian scheme also proved to be of superior quality. In the lower troposphere both schemes revealed similar error performance, however, the reinterpolated wind fields, using Scheme 1, sometimes appeared to reverse directions. Such errors would result in the mistaken displacement of weather systems.

In the statistics of reinterpolation errors the Lagrangian interpolation scheme delivered superior performance, with the exception of the 100 mb level. Weather systems in the middle and lower troposphere were well represented by this scheme. Linear interpolation, on the other hand, had serious problems in adequately displaying output from a low-resolution model.

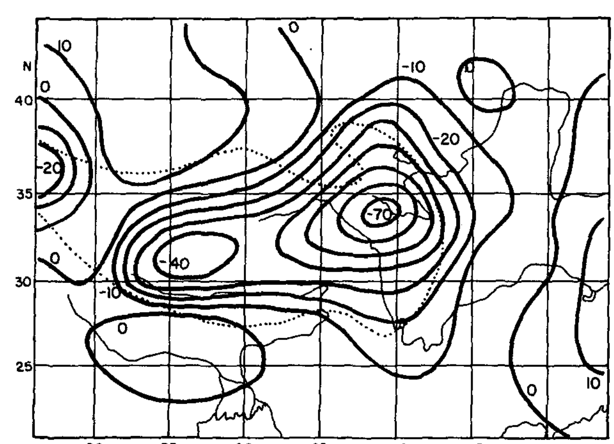

FIG. 10. The interpolated U-component field at the highest level in σ-coordinates obtained by scheme 4.

TABLE 1. The reinterpolation differences from the original values for five meteorological parameters in scheme 1.

Level (mb)	$\Delta\phi$ (gpm)						ΔT (°K)					Δq (g kg^{-1})				Δu (m s^{-1})				Δv (m s^{-1})			
	≤10	11–25	26–50	51–100	>100	Max	≤1.0	1.1–2.0	2.1–5.0	>5.0	Max	0.0–0.2	0.3–0.4	≥0.5	Max	≤1.0	1.1–2.0	>2.0	Max	≤1.0	1.1–2.0	>2.0	Max
100	3	2	2	2	3	140	3	4	5	0	−4.7					2	4	6	12.1	5	2	5	−6.7
200	1	1	3	3	4	−165	2	1	6	3	7.4					1	2	9	−14.6	2	2	8	8.1
300	1	4	4	3	0	−60	9	2	1	0	2.5	11	0	1	0.6	10	1	1	2.1	9	1	2	−2.3
500	2	0	8	2	0	−60	10	2	0	0	1.2	7	3	2	1.1	10	1	1	2.7	10	2	0	2.0
700	6	2	2	0	0	−32	9	1	0	0	1.8	7	0	3	1.1	10	0	0	0.8	9	1	0	1.3
850	9	0	0	0	0	3	9	0	0	0	−0.2	9	0	0	−0.2	9	0	0	−0.2	9	0	0	0.1

TABLE 2. The reinterpolation differences from the original values for five meteorological parameters in scheme 2.

	$\Delta\phi$ (gpm)						ΔT (K)					Δq (g kg^{-1})				Δu (m s^{-1})				Δv (m s^{-1})			
	≤10	11–25	26–50	51–100	>100	Max	≤1.0	1.1–2.0	2.1–5.0	>5.0	Max	0.0–0.2	0.3–0.4	≥0.5	Max	≤1.0	1.1–2.0	>2.0	Max	≤1.0	1.1–2.0	>2.0	Max
100	5	1	2	3	1	−112	3	1	6	2	−10.1					4	2	6	20.2	6	0	6	−8.5
200	6	2	3	1	0	53	5	3	4	0	4.3					6	1	5	−8.6	6	2	4	3.6
300	9	3	0	0	0	−14	12	0	0	0	−0.7	12	0	0	0.1	10	2	0	1.8	12	0	0	0.8
500	11	1	0	0	0	15	9	1	2	0	2.6	8	3	1	−1.1	8	4	0	1.9	8	2	2	−3.3
700	11	1	0	0	0	14	11	0	1	0	−2.7	9	1	2	−0.8	12	0	0	−0.6	9	3	0	2.0
850	8	1	0	0	0	13	9	0	0	0	0.6	8	0	1	−1.0	9	0	0	−0.8	8	1	0	1.1

6. Summary and conclusions

Mesoscale numerical modeling is usually applied to weather situations that develop rapidly, such as squall lines, mesoscale convective complexes and synoptic and subsynoptic scale vortices under the influence of complex terrain features. Even though it is possible to describe with reasonable completeness, either explicitly or by suitable parameterizations, the dynamic processes in the atmosphere that govern these rapid developments, such models have not always been successful in predicting the time and place of occurrence within acceptable error limits. Errors in the boundary and initial conditions have an important impact on the quality of model predictions. We have noticed, for instance, that objective analyses produced by the National Meteorological Center (NMC) over the United States are well suited under most circumstances for model initialization, whereas NMC analyses over China and Tibet are of much poorer quality. In our own work we had to resort to hand-analyzed maps, containing a lot more observational data than available to NMC, in order to achieve satisfactory model runs (Shen *et al.*, 1985). The necessity of model initialization with improved data sets over China well illustrates the sensitivity of our, and presumably other, models to errors in the input data.

In the foregoing discussion we demonstrated that it will not suffice to acquire reasonably well-analyzed datasets for initial input into the model. The necessary interpolations in space, especially along the vertical coordinate, that have to be carried out during coordinate conversion procedures, can bring havoc to the vertical profiles of meteorological parameters, which are interrelated through the set of dynamic and thermodynamic equations.

In order to avoid the problems arising from the necessity that the meteorological parameter fields must be internally consistent, some models resort to elaborate initialization procedures, relying on only one observed variable, such as the temperature fields, and computing the other variables (geopotential height, wind) from hydrostatic and geostrophic balance requirements forcing the vertical integral of divergence in the air column over each grid point to zero. Even though such a procedure stabilizes the model mathematically, it may, at least in part, obliterate some of the details in the input fields that may be instrumental in the observed weather systems developments on relatively short time scales. Furthermore, these initialization procedures are rather time consuming.

The problems arising from interpolations and extrapolations of vertical profiles of meteorological variables may have been minimized in some models (see e.g., Anthes and Warner, 1979; Kaplan *et al.*, 1982) due to their high resolution, such as 30 layers, even though the essential dynamic processes might be resolved by a much lower resolution, which would make the model faster, more economic in its use, and therefore more attractive for routine applications.

In the foregoing discussion we have demonstrated that in low-resolution models the problems arising from errors introduced by interpolation (and extrapolation) of vertical profiles are amplified. Such errors are introduced when variables given in a p-coordinate system are translated into a σ-coordinate system (or Θ-coordinate system, see Bleck, 1984). These errors will then propagate through the time integration steps, leading to erroneous forecasts, perhaps even to mathematical instability of the model. When the model output needs to be translated back into a p-coordinate system to provide comparisons with the real atmosphere, an additional source of interpolation (or extrapolation) errors is being tapped, which can cause erroneous displays for variables that actually might have been forecast rather well.

In order to make low-resolution models perform well, it is suggested that extrapolation not be used with the coordinate transformations necessary to initialize the model, but only in the transformation of the model results to coordinates suitable for display. Furthermore, the interpolation scheme should satisfy the basic equilibrium requirements between the meteorological variables. Application of different curve-fitting schemes to different variables should be viewed with caution (e.g., approximation of temperature profiles as linear functions of the logarithm of pressure, while geopotential and/or wind are interpolated as linear functions of pressure), because these schemes might not meet the balance requirements between the variables. The quest for one universally applicable, simple interpolation scheme will be frustrated by these balance requirements. To resolve the discontinuities between troposphere and stratosphere and in the vicinity of sharply defined jet streams, data from the 150 and 250 mb levels will help to improve interpolation during the initialization procedure.

Ideally, an interpolation scheme should not only meet the balance requirements mentioned above, but should also return the originally observed meteorological parameter fields, at least with respect to their main characteristics, when two coordinate transformations are performed in opposite directions. This criterion should be considered the "acid test" for any interpolation procedure. Adoption of an inadequate procedure will be faced with the same dilemma as the computer translation of "Out of sight—out of mind" from English to Chinese and back to English again, which returned "Blind and crazy" as final output.

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