Further Observational Characteristics of Bimodal Planetary Waves: Mean Structure and Transitions

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(Manuscript received 20 October 1986, in final form 3 August 1987)

ABSTRACT

Further results concerning the mean states of the bimodal wavenumber 2 to 4 amplitude probability density distribution are presented followed by composites of transitions from one side of this distribution to the other. The data used are ECMWF analyses from the four winters from 1980/81 to 1983/84. Cross sections of the mean states associated with the two modes reveal that both modes exhibit a baroclinic vertical structure, but that the difference between the two is more nearly equivalent barotropic. The composite transitions between the low-amplitude and high-amplitude states indicate that the transition time for the onset or decay of the large amplitude waves is about 4 days. The kinetic energy and available potential energy of wavenumbers 2 to 4 increases (or decreases) by 50 percent in this same time interval during the onset (or decay) of the large amplitude state. Nonlinear interaction with intermediate-scale waves is the only apparent source for the observed kinetic energy tendency during the transition from the low amplitude to the high amplitude mode. Thus, the growth of the large amplitude events does not strictly resemble that of a classical baroclinic instability. During the decay of the large amplitude waves, nonlinear interaction between the wavenumber 2 to 4 ensemble and wavenumber 1 accounts for the decline in kinetic energy, while nonlinear interaction between wavenumbers 2 to 4 and smaller-scale waves accounts for the decline in available potential energy. Examples of individual cases are presented to corroborate the composite results.

Finally, a case study of the synoptic evolution of a large-amplitude event is presented to illustrate the event's life cycle.

1. Introduction

Search for the underlying causes of the observed low frequency variability in the midlatitude circulation has been a topic of great interest in the past few years (e.g., Wallace and Blackmon, 1983). A significant problem in dealing with these relatively large-scale features (compared to a synoptic-scale cyclone) from an observational viewpoint has been objectively identifying events with similar characteristics. The term “blocking” has been used, often with very little discretion, to describe a broad array of seemingly disparate events. Certainly, different blocking events may arise from quite different physical mechanisms (Wallace and Blackmon, 1983; Hansen and Sutera, 1984; Hansen, 1986, Schilling, 1986).

Recent observational studies by Sutera (1986) and Hansen and Sutera (1986) have revealed the presence of a bimodal probability density distribution for the amplitude of the planetary-scale wave packet formed by zonal harmonic wavenumbers 2 to 4. In these studies, essentially similar results were obtained from two independent datasets that span a total of 20 winters. Figure 1 illustrates a probability density estimate of the amplitude of the wavenumber 2 to 4 geopotential height, \[Z_{2-4}\], obtained from the four winters of European Centre for Medium Range Weather Forecasts (ECMWF) data to be considered in the present study. The two modes are separated by more than one standard deviation of the total wavenumber 2 to 4 variability. Thus, this separation represents a major part of the total variance occurring at these length scales.

Grid-point differences between the mean 500 mb heights of the low amplitude mode (Mode 1) and the high amplitude mode (Mode 2) exceed 100 m in certain locations (Sutera, 1986; Hansen, 1986; Hansen and Sutera, 1986). The statistical significance of the bimodality and the techniques used to obtain it are discussed in detail in Sutera (1986) and Hansen and Sutera (1986). Statistical sampling was employed in the latter study to establish the very low probability that the bimodal probability density distributions were obtained by random chance from a normal or even a skewed unimodal population. In contrast, the speed, horizontal shear, and vertical shear of the zonal mean wind exhibit unimodal distributions that cannot be distinguished statistically with the techniques employed from normal distributions (Hansen and Sutera, 1987).

Filtered time series of \[Z_{2-4}\] for each of the four winters (1 December through 28 February) are pre-
sent in Fig. 2. The large amplitude events can persist anywhere from a few days to 3 weeks and in general, transitions from Mode 1 to Mode 2 and vice versa are quite rapid. Note that the transitions occur randomly as there is no evidence of periodicity in the duration of the high or low amplitude events (Hansen and Sutera, 1986).

The bimodality provides a framework for unifying the study of a large class of persistent, large-scale, large-amplitude circulation features. Synoptically, the 500 mb patterns associated with the large amplitude mode of the bimodal distribution are characterized by a pattern of large-scale troughs and ridges with broad meridional extent. Rex (1950) made a distinction between this type of “amplified wave” pattern and the dipole, split-flow pattern now referred to by synopticians as “Rex blocking.” Localized events like Rex blocks leave little systematic signature in the wavenumber 2 to 4 amplitude. Energetically, the time–mean large amplitude wave mode is characterized by strong, steady baroclinic energy conversions, primarily at wavenumbers 2 and 3, and a diminution of cyclone-scale activity compared to the low amplitude mode (Hansen, 1986).

A minimum in the probability density distribution strongly suggests the existence of an instability at low wavenumbers. Recent theoretical studies have explored the possible link between the existence of the bimodality and a nonlinear resonance phenomenon (Benzi et al., 1986a,b). The work of Benzi et al. (1986a,b) has its roots in the multiple equilibrium theory of Charney and DeVore (1979) in which the instability was orographic in nature. Alternative explanations of the bimodality include a mechanism associated with the observed teleconnection patterns (Wallace and Gutzler, 1981), specifically the “Pacific–North American” (PNA) pattern, and a wave interference mechanism between free traveling Rossby waves and forced stationary waves (e.g., Lindzen et al., 1984). Madden (1983) has illustrated the characteristic energy cycle that is indicative of the interference of stationary and traveling wavenumber 1. It is also possible that some other wave instability unrelated to the nonlinear resonance noted above could explain the bimodality (e.g., baroclinic instability). Each of these potential explanations have aspects that require further study before conclusions concerning the origin of the bimodality can be made.

The purpose of the present study is to investigate the mean vertical structure and composite transitions between the modes in order to gain information that may be useful in interpreting the physical origins of

FIG. 1. Probability density distribution of the amplitude of the ensemble of the wavenumbers 2 to 4, [Z_{2-4}], in the 22.5°–78.75°N latitude zone for the four winters from 1980 through 1984 (for details, see Sutera, 1986).

FIG. 2. Time series of the wave amplitude indicator, [Z_{2-4}], in the 22.5°–78.75°N zone for the 90 days from 1 December through 28 February of (a) 1980/81, (b) 1981/82, (c) 1982/83 and (d) 1983/84.
the wave amplitude bimodality. The study concentrates on the same four winter ECMWF dataset used by Sutera (1986) and Hansen (1986). After briefly describing the dataset, geopotential height and temperature cross sections are presented to clarify the mean-wave structure of the two modes. The composite of 13 large amplitude wave events taken from the four winters from 1980/81 through 1983/84 are then examined to illustrate systematically important processes occurring during the transition periods in wave amplitude from one mode of the probability density distribution to the other. Finally, a case study of the physical space developments during the transitions is presented to exemplify the energetics results. The question of the physical interpretation of the bimodality is revisited in section 7.

2. Data

The data used in the present study are the initialized fields for the operational model of the ECMWF. Data for the horizontal wind, vertical velocity, geopotential height and temperature were obtained at 10 standard levels (1000, 850, 700, 500, 400, 300, 250, 200, 150 and 100 mb) interpolated to a $2.5^\circ$ by $2.5^\circ$ latitude-longitude grid. Once daily data for 1 December through 28 February of the years 1980/81, 1981/82, 1982/83 and 1983/84 were used. As the balanced initial fields of an operational model, they are necessarily model dependent, but the fact that they include assimilated data from numerous sources in addition to conventional radiosondes (e.g., satellite soundings and aircraft reports over the oceans) largely compensates the disadvantage of model dependency.

3. Mean structure

The vertical structure of the wavenumber 2–4 height and temperature fields for the 2 modes is revealed by longitude–height cross sections along $50^\circ$N ($50^\circ$N is the latitude of largest $Z_{2-4}$ amplitude). Although the increase in the wavenumber 2–4 height and temperature for Mode 2 compared to Mode 1 is quite marked (Figs. 3 and 4), both modes exhibit a baroclinic vertical structure, especially at levels below 500 mb. Notice the westward tilt with height of the height wave and the eastward tilt of the temperature wave. The phase difference between $Z_{2-4}$ and $T_{2-4}$ is quite marked below 400 mb for both modes. However, examination of the difference between the Mode 2 and Mode 1 cross sections (Fig. 5) indicates that the difference field is more characteristic of an equivalent barotropic structure. Note the lack of any appreciable tilt in the height difference and the very slight eastward tilt in the temperature difference field. The equivalent barotropic structure is confirmed by the nearly in phase relationship of the $\Delta Z_{2-4}$ and $\Delta T_{2-4}$ fields, particularly compared to either the Mode 1 or Mode 2 $Z_{2-4}$ and $T_{2-4}$ cross sections. Analogous results are evident from examination of a larger NMC dataset (Hansen and Sutera, 1986).

The large increase diagnosed in the baroclinic energy conversion terms for Mode 2 compared to Mode 1 in the present dataset (Hansen, 1986) results from the increased amplitude of the temperature field and strengthened height gradients for Mode 2, rather than from a “more baroclinic” structure of the waves present during the large amplitude events. However, it is not clear from illustrations of this kind whether the growth of the large amplitude waves can or cannot be attributed to a baroclinic instability mechanism. For example, observations of an occluded cyclone reveal an equivalent barotropic structure, but this does not imply that the growth of the cyclone was not associated with baroclinic processes.

Latitude–pressure cross sections of the zonal mean wind, $u_z$, and zonal mean temperature, $T_z$, reveal essentially no difference on average between the modes in the speed, horizontal shear and vertical shear of $u_z$ and $T_z$ (not shown). No simple relationship between the wave dynamics and the zonal mean wind and temperature fields is suggested by this result. Further examination of these potential interrelationships using a larger dataset has reached a similar conclusion (Hansen and Sutera, 1987).
4. Composite transitions

In order to determine what systematic behavior accompanies the transitions between the two modes, composite time series of various quantities were formed from all the transitions from Mode 1 to Mode 2 as well as those from Mode 2 to Mode 1 occurring in the dataset. By compositing cases, we hope to average out signals not pertinent to the Mode 2 events, and to empirically identify those processes of systematic importance to the development and decay of the large amplitude waves.

In the figures that follow, the time axis runs from 4 days before the first day in which \( [Z_{2-4}] \) switched from one side of the minimum in the probability density distribution (denoted Day -4) to 4 days after the transition (denoted Day +4) for a total of 8 days. The transition in each case occurs within the 24 h period between Day -1 and Day +1. If the onset of a given case occurs within 8 days of its decay or if the decay of one case occurs within 8 days of the onset of another case, observations occurring in the overlapping period of the two transitions are not used in forming the composites. Mode 1 and Mode 2 events of less than 4 days duration are not included. There were a total of 13 transitions in each case during the four winters considered.

a. Composite wave amplitude transitions

Consider the composite time series of \( [Z_{2-4}] \) for the Mode 1 to Mode 2 transition (Fig. 6a) and for the Mode 2 to Mode 1 transition (Fig. 6b). Henceforth, these transitions will be denoted 1-2 and 2-1 respectively. Note in each case that once the wave amplitude begins to increase or decrease, the total time for the transition is roughly 4 days. For comparison, the average duration of both Mode 1 and Mode 2 events is 10 to 11 days. Inspection of the actual time series of \( [Z_{2-4}] \) for each winter (Fig. 2) shows that the transitions can occur in as little as 2 days. Thus, the onset and decay of the Mode 2 events occur on a time scale comparable to that of synoptic-scale waves. Similarly rapid transitions have been noted in the growth and decay of persistent grid point height anomalies (Dole, 1986). The relative speed of the transitions in the present case is perhaps more remarkable given the hemispheric-scale nature of our wave amplitude indicator.

b. Composite energetics

Now consider the composite energetics of the wave number 2 to 4 ensemble during the transition from the low amplitude state, Mode 1, to the large amplitude state, Mode 2. (The notation and symbolic formulation of the energetics equations are given in the Appendix.)
The wavenumber 2 to 4 kinetic energy, $K_{2-4}$, and available potential energy, $A_{2-4}$ (Fig. 7) have nearly identical magnitudes and time developments during this composite transition. Both increase by roughly 50 percent in magnitude ($\sim 1.7 \times 10^5 \ J \ m^{-2}$) in a period of 4 days. The entire change in $K_{2-4}$ and $A_{2-4}$ observed between the mean states of the two wave amplitude modes is accomplished during this 4-day period. These energy levels are the average for the entire midlatitude troposphere ($20^\circ-80^\circ$N; 1000 mb–100 mb) and their close correspondence to the amplitude indicator $[Z_{2-4}]$, which is determined from 500 mb height data alone, emphasizes the utility of the indicator in identifying energetic large-scale phenomena.

Examination of the composite time evolution of the baroclinic conversions $C(A_z, A_{2-4})$ and $C(A_{2-4}, K_{2-4})$ (Fig. 8) reveals that both of these conversions tend to vary in phase with $K_{2-4}$ and $A_{2-4}$ during the onset of the Mode 2 events. The surplus of $C(A_z, A_{2-4})$ over $C(A_{2-4}, K_{2-4})$ appears to account for the increase in $A_{2-4}$ with a small contribution from the wave–wave interaction, $C_{4}(2-4)$ (not shown).

The composite rate of change of $K_{2-4}$, $\partial K_{2-4}/\partial t$, and the rate of change of $K_{2-4}$ due to wave–wave interaction with all other waves, $C_K(2-4)$, are illustrated in Fig. 9a. Large positive values of $\partial K_{2-4}/\partial t$ span the 4-day period from Day $-2$ to Day $+2$ in the composite. The composite rate of change of cyclone scale (wavenumbers 5 to 10) kinetic energy due to wave–wave interactions, $C_K(5-10)$, indicates that the loss of energy from this band is more than sufficient to account for the positive values of $C_K(2-4)$ (Fig. 9b). The intermediate-scale baroclinic conversion, $C(A_z, K_{2-4})$, is the apparent source of this kinetic energy (Fig. 9c). Wavenumber 1 and smaller scale waves account for the balance of the kinetic energy export from the cyclone waves. No other sources of $K_{2-4}$ are evident. The wave–mean flow interaction, $C(K_z, K_{2-4})$, represents a sink of wave energy during the composite $1-2$ transition and all the other
budget terms are either relatively small or show little variation during the transition. From the present results, it appears possible that baroclinic instability of intermediate-scale waves may lead to the initiation of the Mode 2 events through nonlinear coupling.

On average, for the composite 4 day Mode 1 to Mode 2 transition period, significant positive contributions to $\partial K_{2-4}/\partial t$ are made by the baroclinic term and the wave–wave interaction term with negative contribu-

Fig. 9. Composite time series for the Mode 1 to Mode 2 transition of (a) the observed kinetic energy tendency, $\partial K_{2-4}/\partial t$ (dashed line), the contribution to $K_{2-4}$ from wave–wave interactions with all other wavenumbers, $C_K(2-4)$ (solid line), (b) the change in intermediate-scale kinetic energy due to wave–wave interactions with all other waves $C_K(5-10)$; and (c) the baroclinic conversion, $C_{A_{5-10}, K_{5-10}}$, for intermediate-scale wavenumbers. Note that the time axis begins 6 days before the transition in (c).

tions from wave–mean flow interaction and dissipation (estimated either as a residual or based on a crude parameterization, Table 1). If the kinetic energy balance for this transition is compared to the mean kinetic energy balance for Mode 1, the initial state from which this transition begins, it appears that the wave–wave interaction term accounts for most of the observed kinetic energy tendency (Table 1), since the other contributions change relatively little from their Mode 1 mean value. In the composite mean for this transition, the change in the wave–mean flow interaction term makes a small contribution opposing the increase in $K_{2-4}$.

During the composite transition from Mode 2 to Mode 1, $K_{2-4}$ and $A_{2-4}$ also decline more or less in phase with the decline of $[Z_{2-4}]$ with a 50 percent reduction in each over the 4-day transition period (Fig. 10). $C(A_{2-4}, A_{2-4})$ also declines quite sharply from Day $-2$ to Day $+1$ (Fig. 11). By the later stage of the composite Mode 2 event’s life cycle, interaction between the waves and the mean flow provides a source of wave kinetic energy at Days $-4$ and $-3$ before dropping to near zero (not shown). The baroclinic generation of kinetic energy actually increases on average right at the time of the transition. Considerable variability is present in both of these results, however.

The strikingly systematic aspect of the Mode 2 to Mode 1 transition is that the rates of change of $K_{2-4}$ and $A_{2-4}$ can be accounted for entirely by wave–wave interactions. For this transition $\partial K_{2-4}/\partial t$ is illustrated in Fig. 12a. The $C_K(2-4)$ (not only accounts for $\partial K_{2-4}/\partial t$ but also offsets the noted increase in $C(A_{2-4}, A_{2-4})$ at Day $-1$ and Day $+1$. The recipient of this energy is wavenumber 1. The gain of kinetic energy by wavenumber 1 due to wave–wave interaction, $C_K(1)$, is also illustrated in Fig. 12a. The loss of $K_{2-4}$ to wavenumber 1 occurs systematically in the individual cases and implies that the wave 2–4 pattern decays due to barotropic damping to the larger scale wave upon which it is superimposed. $C_K(2-4)$ and $\partial A_{2-4}/\partial t$ are illustrated in Fig. 12b. The wave–wave interaction also accounts for the majority of the decline in $A_{2-4}$. Unlike the kinetic energy, however, smaller scale waves are the recipients of the wavenumber 2 to 4 available potential energy.

For the composite average of the 4-day Mode 2 to Mode 1 transition period, $\partial K_{2-4}/\partial t$ is negative with significant negative contributions coming from the wave–wave interaction and (estimated) dissipation terms with a large positive contribution coming from the baroclinic term. In general, the combined effect of the wave–wave interaction and dissipation overwhelms the baroclinic source leading to the negative kinetic energy tendency. Comparing the kinetic energy balance of the 2–1 transition to the Mode 2 mean state, it again appears that the kinetic energy tendency is accounted for primarily by the wave–wave interaction term. Based on these results and the fact that the baroclinic conversion is always positive with a magnitude of 1.0 to 1.5 W
TABLE 1. Composite mean kinetic energy budget of wavenumbers 2-4 during the 4-day transition periods for Mode 1 to Mode 2 (1-2) and Mode 2 to Mode 1 (2-1) transitions. Units on \( K_{2-4} \) are J m\(^{-2} \) and all conversions are in W m\(^{-2} \).

<table>
<thead>
<tr>
<th></th>
<th>( K_{2-4} (\times 10^3) )</th>
<th>( \frac{\partial K_{2-4}}{\partial t} )</th>
<th>( C(K_{2-4}, K_{2-4}) )</th>
<th>( C(A_{2-4}, K_{2-4}) )</th>
<th>( C_d(2-4) )</th>
<th>( F(K_{2-4}) + F(\Phi_{2-4}) )</th>
<th>Residual*</th>
<th>Est.* ( D(K_{2-4}) )</th>
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<tr>
<td>1-2 Difference Compared to Mode 1</td>
<td>4.3</td>
<td>0.42</td>
<td>-0.16</td>
<td>1.31</td>
<td>0.29</td>
<td>-0.05</td>
<td>-0.97</td>
<td>-1.19</td>
</tr>
<tr>
<td>2-1 Difference Compared to Mode 2</td>
<td>0.3</td>
<td>0.42</td>
<td>-0.15</td>
<td>0.02</td>
<td>0.40</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>4.9</td>
<td>-0.46</td>
<td>0.07</td>
<td>1.44</td>
<td>-0.63</td>
<td>-0.08</td>
<td>-1.26</td>
<td>-1.37</td>
</tr>
<tr>
<td></td>
<td>-0.3</td>
<td>-0.46</td>
<td>0.04</td>
<td>-0.03</td>
<td>-0.63</td>
<td>-0.05</td>
<td>0.21</td>
<td>0.10</td>
</tr>
</tbody>
</table>

* The residual equals the tendency minus all budget terms.

The estimated dissipation is determined from \( D(K_{2-4}) = -\kappa K_{2-4} \) where \( \kappa = 2.8 \times 10^{-8} \) s\(^{-1} \) (Hansen, 1986). Note the close agreement between the residual and the estimated dissipation.

m\(^{-2} \) for the wavenumber 2 to 4 band, we can suggest that the baroclinic conversion's role in the dynamics of the wave 2 to 4 ensemble is to balance the dissipation, whereas the wave-wave interaction appears to drive the major, rapid changes in the kinetic energy level. This impression is reinforced by the individual examples presented in the next section. Similar conclusions about the influence of wave-wave interactions on the kinetic energy of planetary waves have been reached by earlier investigations (e.g., Kao and Chi, 1978; Tsay and Kao, 1978; Hansen and Chen, 1982; Hayashi and Golder, 1983; Itoh, 1983; Fischer, 1984).

Turn now to the \( A_{2-4} \) budget during the transitions (Table 2). For the average, composite 1 to 2 transition, the net baroclinic contribution \( (C(A_{2-4}, A_{2-4}) - C(A_{2-4}, K_{2-4})) \) is the only significant contributor to the \( A_{2-4} \) tendency and accounts for nearly all of the observed tendency. For the 2 to 1 transition, the wave-wave interaction \( C_d(2-4) \) is the only contributor toward the negative \( A_{2-4} \) tendency as the net baroclinic contribution drops near zero. These results are expected based on the composite time series.

Standard deviations of the daily energy levels \( (K_{2-4} \) and \( A_{2-4} \)) are typically \( \pm 4 \times 10^4 \) to \( \pm 7 \times 10^4 \) J m\(^{-2} \) during the transitions. Standard deviations of the conversions tend to be relatively larger [for example, 0.3 to 0.5 W m\(^{-2} \) for \( C(A_{2-4}, K_{2-4}) \)]. This is a partially a function of having only 13 cases in the sample and partly due to not having sufficient data to remove the seasonal trends and interannual variability from the energetics results. In section 5, examples of individual Mode 2 events are shown that lend confidence to the composite results.

The onset of the Mode 2 events is not analogous to what is expected from a finite-amplitude baroclinically unstable wave. To illustrate this point, consider the evolution of a cyclone-scale baroclinic wave ensemble during late December 1980. The evolution of the kinetic energy for wavenumbers 5 through 10 \( (K_{5-10}) \), as

Fig. 10. Composite time-series of the wavenumber 2 to 4 kinetic energy, \( K_{2-4} \) (solid line) and available potential energy, \( A_{2-4} \) (dashed line) during the Mode 2 to Mode 1 transition.

Fig. 11. Composite time series of \( C(A_2, A_{2-4}) \) (dashed line) and \( C(A_{2-4}, K_{2-4}) \) (solid line) for the Mode 2 to Mode 1 transition.
well as the energy tendency \( \partial K_{5-10}/\partial t \), and baroclinic kinetic energy conversion \( C(A_{5-10}, K_{5-10}) \) are illustrated in Fig. 13a, b. Note the close correspondence between the baroclinic conversion and the \( K_{5-10} \) tendency during the waves' growth phase. Similar results are evident in Simmons and Hoskins (1980) model simulations. Barotropic effects modulate the baroclinic growth rate and ultimately determine the maximum amplitude and decay rate of the baroclinic waves. The baroclinic conversion drops off sharply after the maximum kinetic energy is achieved. During the cyclone waves' decay, the nonlinear transfer of energy to other wavenumbers overcomes the remaining baroclinic conversion to determine the cyclone wave kinetic energy decay rate. For Mode 2 events, the baroclinic conversion of kinetic energy has comparable values during both the onset and decay and appears to only balance the dissipation as suggested earlier, while the wave-wave interaction apparently accounts for the kinetic energy growth and decay.

5. Individual examples

As an example of how the temporal evolution of individual cases of Mode 2 events is reflected in the wavenumber 2 to 4 ensemble energetics, consider the 1982/83 winter. This winter was chosen because it includes four complete Mode 2 events and the onset of a fifth. The 1982/83 winter was characterized by a very intense El Niño event which may have resulted in the fact that the mean zonal wind was stronger during this winter than in the other three in the dataset and that the Mode 2 events were generally of shorter duration and their amplitudes lower than those in the other three winters. Nonetheless, their temporal behavior was similar to the composites presented earlier. (Analogous results from the other years were generally similar.)

The time series of \( K_{2-4} \) and \( A_{2-4} \) for 1982/83 winter are shown in Fig. 14a. The time periods of the Mode 2 events as determined from the analysis of the wave amplitude indicator are given by the heavy line along the bottom of the figure. Notice that both \( K_{2-4} \) and \( A_{2-4} \) are relatively large for the Mode 2 events as expected from both the mean energetics of the modes (Hansen, 1986) and from the composite transitions presented in the previous section. The total energy of this wave ensemble defined as \( \text{TE}_{2-4} = K_{2-4} + A_{2-4} \) shows the sharp changes in energy level accompanying

**Table 2.** Composite mean available potential energy budget of wavenumbers 2–4 for Mode 1 to Mode 2 transitions (1–2) and Mode 2 to Mode 1 transitions (2–1). Units on \( A_{2-4} \) are J m\(^{-2}\) and all conversions are in W m\(^{-2}\).

<table>
<thead>
<tr>
<th>( A_{2-4} \times 10^9 )</th>
<th>( \partial A_{2-4}/\partial t )</th>
<th>( C(A_2, A_{2-4}) )</th>
<th>( -C(A_{2-4}, K_{2-4}) )</th>
<th>Net baroclinic effect</th>
<th>( C_\alpha(2-4) )</th>
<th>( F(A_{2-4}) )</th>
<th>Residual</th>
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<tbody>
<tr>
<td>1–2</td>
<td>4.1</td>
<td>0.42</td>
<td>1.72</td>
<td>-1.31</td>
<td>0.42</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>2–1</td>
<td>5.5</td>
<td>-0.36</td>
<td>1.45</td>
<td>-1.44</td>
<td>0.01</td>
<td>-0.35</td>
<td>-0.03</td>
</tr>
</tbody>
</table>
the growth and decay of the Mode 2 events with the possible exception of the fourth event (Fig. 14b). However, not all Mode 2 events achieve the same energy level. Similar curves from the other three winters show equally good correspondence between the wave amplitude, $[Z_{2.4}]$, and TE$_{2.4}$. Of the 13 Mode 2 events, only 3 are not clearly represented in the TE$_{2.4}$ time series: the first case in 1981/82, the fourth case in 1982/83 and the third case in 1983/84. In addition, one event in 1983/84 appears to have been missed.

Next, consider the time series of the energy conversions from the 1982/83 winter. Relatively large values of the baroclinic conversions (Fig. 15a) tend to accompany the Mode 2 cases but there is little indication that $C(A_{2.4}, K_{2.4})$ systematically leads to growth in wave kinetic energy. In contrast, the $C_K(2-4)$ time series
shows consistent positive contributions to $K_{2-4}$ during the growth and consistent negative contributions during the decay of the Mode 2 events (Fig. 15b). Note that not all of the marked nonlinear contributions to wave energy increase or decrease accompany the growth or decay of a Mode 2 event. However, periods identified by the growth or decay of the hemispheric wave amplitude indicator $Z_{2-4}$ are nearly always accompanied by the characteristic nonlinear wave–wave interactions identified in the composites.

6. A case study

Finally, let us briefly consider the synoptic evolution of a case study Mode 2 event. The event occurring from 15 through 28 December 1983 was chosen as an “average” case. This event persisted for two weeks and would generally be referred to as a period of Pacific blocking. The transitions in this case began at wave amplitude values less than (more than) the most probable Mode 1 (Mode 2) value and end at amplitude values larger (smaller) than the most probable Mode 2 (Mode 1) value. As a result, the transitions show very sharp contrasts in the height field. The temporal evolution of the energetics during this event was similar for the most part to that of the composite both during onset and decay (not shown). Strong amplification occurred from 13 through 17 December. Decay was most rapid between 27 and 29 December. Average kinetic energy gain via wave–wave interaction for the onset was 0.87 W m$^{-2}$ (14–17 December) and $-0.85$ W m$^{-2}$ for the decay (27–30 December). Positive peaks during onset occurred on 14 and 17 December. Baroclinic conversions at wavenumbers 2–4 were actually larger during the decay than during the onset ( 0.93 W m$^{-2}$ average from 13 to 17 December during onset versus 1.25 W m$^{-2}$ average for 27–30 December during decay).

Prior to the onset of the event (10–14 December), the 5-day mean 500 mb height exhibited a fairly uniform zonal flow (Fig. 16a), and the synthesized wavenumber 2 to 4 height field (Fig. 16b) exhibited weaker features than the Mode 1 mean state (Hansen, 1986). After the onset of the event, the mean heights from 17 to 26 December illustrate the strong planetary-wave amplification in the total field and in the wavenumber 2 to 4 field (Fig. 16c, d). This time period may be considered the “mature” phase of this particular case. Inspection of daily maps reveals the presence of the fairly stationary Pacific blocking ridge. Notice also the amplification of the Hudson Bay low and the western Pacific–eastern Asian low, and the less noteworthy amplification of a ridge over eastern Europe and northwestern Asia.

The 5-day mean for the period following the return to Mode 1 (29 December 1983–2 January 1984) shows the reestablishment of a fairly zonal flow pattern to the hemisphere with a strong wave 1 component supplemented by the very strong flow over the Atlantic and Europe (Fig. 16e, f). The west Pacific trough and east Pacific ridge have flattened and the North American low has broadened in the eastward direction. The strong wave 1 influence present in Fig. 16e is consistent with the prominent amplification of wave 1 during the decay of this event. This is a rather well-marked realization of the strong energy transfer from waves 2 through 4 to wave 1 that accompanies the decline of the composite Mode 2 events. In this particular case, the rate of energy loss by waves 2 to 4 reached a negative peak of $-2.2$ W m$^{-2}$ on 28 December.

In general, the mechanism for wave–wave interaction is momentum transport by waves of certain wavenumbers up or down the velocity gradients of another wavenumber which then accelerates or decelerates that flow. This implies tilting of trough and ridge axes of intermediate-scale waves (for example) relative to the gradient of the larger-scale flow field. Localizing this interaction can be difficult if not problematic, although some attempts have been made in this direction. A particularly clear example of wave–wave interaction forcing the development of a Rex-type block was discussed in detail by Hansen and Chen (1982). Interpretation in their case was straightforward because only one major cyclone system was present in the Northern Hemisphere during the block’s development so masking of this signal by independent processes occurring elsewhere in the hemisphere was minimized.

For the onset of Mode 2 events, however, a more typical scenario from the present dataset appears to be that of more than one major cyclone in differing geographic locations acting to enhance an underlying planetary wave pattern. Prior to the onset of the present event, cyclone-scale waves (wavenumbers 5 through 10) reached a hemispheric maximum in their baroclinic energy conversion on 12 December followed by a kinetic energy maximum on the 13th. The cyclone waves present prior to the onset of this event could be viewed as 2 wave trains, one over the central Pacific, another over eastern North America and the North Atlantic. By 14 December, the Pacific train had developed a slight northwest–southeast tilt over the eastern Pacific which may be crudely viewed as transporting easterly momentum into the region that eventually contained the easterlies to the south of the blocking high in Fig. 16c, d. Further cyclone–wave development over North America and the western Atlantic on 15–17 December resulted in westerly momentum flux convergence into the region of westerlies south of the Hudson Bay trough in Fig. 16c, d by southwest–northeast tilted intermediate-scale wave features (Fig. 17). Other interactions might be inferred elsewhere in the hemisphere.

It is tempting to speculate that the cumulative effect of these cyclone-scale features and their attendant nonlinear interactions is to drive the larger-scale flow pattern toward the Mode 2 configuration which, once achieved, remains relatively stable. Unfortunately, es-
Fig. 16. Average 500 mb heights for (a) 10–14 December 1983, (b) 10–14 December but only wavenumbers 2 to 4, (c) 17–26 December, (d) 17–26 December but only wavenumbers 2 to 4.

7. Discussion

As mentioned in the introduction, several mechanisms may be put forward to explain the bimodality. At this point, a few remarks on the issue of physical interpretation are in order.

Consider first the interference mechanism between free, traveling Rossby waves and forced stationary waves. Several problems exist with this approach including: 1) free Rossby waves are not thought to routinely have sufficient amplitude in the troposphere to explain the height differences between the modes; 2) it is difficult to explain the persistence of events which can be as long as 2 to 3 weeks (e.g., Fig. 2) with traveling waves possessing shorter periods than this; 3) there is no reason to expect the traveling waves to have preferred amplitudes as would be required in order for
their superposition with steady forced waves to lead to a bimodal amplitude probability density; and 4) the systematic wave–wave interactions occurring during the transitions would either have to be purely coincidental or their presence would require a modification of the conventional linear framework of Rossby wave analysis. Wave interference effects may be more important in the stratosphere and mesosphere where the free waves and stationary waves both have large amplitudes (e.g., Salby, 1984; Hirooka, 1986).

A major drawback of the PNA teleconnection scenario is the time scale of the transitions. For example, Wallace and Gutzler (1981) used monthly mean data to reveal the PNA pattern. Subsequent investigation by Blackmon et al. (1984) reinforced the impression that the PNA teleconnection pattern exhibits a predominant time-scale longer than one month. Conversely, the composite transitions between the modes take only a few days and the mean duration of the Mode 2 events is roughly 10 days. Also, if the teleconnections are the result of wave propagation as has been suggested (e.g., Hoskins, 1983), these waves would also require a preferred amplitude in order to achieve the bimodality, and the diagnosed wave–wave interactions would be difficult to understand.

As noted earlier, circumstantial evidence from the present energetics analysis appears incompatible with the notion that a pure baroclinic instability can explain the bimodality.

The energetics of the transitions are not inconsistent with the nonlinear resonance theory. Extension of the ideas of Benzi et al. (1986b) to accommodate the diagnosed wave–wave interactions reported here will be reported in a forthcoming paper. However, the question of midlatitude confinement of resonant waves (Held, 1983) remains open. Study of the potential for nonlinear wave self-confinement is in progress and preliminary results indicate this may be a viable mechanism to allow midlatitude resonance (Speranza, 1987; personal communication).

Any possible explanation of the bimodality, including possible instability theories, must explain the evident importance of barotropic effects (e.g., wave–wave nonlinearity) and the equivalent barotropic structure of the difference field. Conceivably, baroclinic instability could enter the problem at higher wavenumbers which in turn force development of the longer waves through nonlinear coupling as is at least suggested by the composite energetics results. In any case, the presence of a bimodal probability density distribution in an observed large-scale variable strongly argues in favor of the application of nonlinear dynamical systems theory (e.g., Lorenz, 1963).

8. Summary

Further results concerning the mean states of the bimodal wavenumber 2 to 4 amplitude probability density distribution were presented followed by composites of transitions from one side of this distribution to the other. The data used were ECMWF analyses from the 4 winters from 1980/81 to 1983/84. The following observations can be made:

1) The largest increases in the height and temperature wave amplitudes for Mode 2 compared to Mode 1 occur in midlatitudes in the middle and upper troposphere for $Z_{2-4}$ and the middle to lower troposphere for $T_{2-4}$. 
2) Both modes exhibit a baroclinic vertical wave structure, but the difference between the two is more nearly equivalent barotropic, in agreement with results from another dataset (Hansen and Sutera, 1986).

3) The total transition time between one mode of the amplitude probability density distribution to the other is 4 days for both the onset and decay of the large amplitude mode.

4) The tropospheric averaged wave kinetic energy and available potential energy vary in phase with the 500-mb wave amplitude during both transitions.

5) During the transition from the low amplitude to high amplitude mode, wave–wave interaction with intermediate-scale waves provides the major source for the observed increase in wavenumber 2–4 kinetic energy. However, the increase in $A_{2-4}$ is accomplished by the surplus of $C(A_2, A_{2-4})$ over $C(A_{2-4}, K_{2-4})$. It is suggested that the baroclinic source of kinetic energy $C(A_{2-4}, K_{2-4})$ serves only to balance the effects of dissipation during this transition.

6) During the decay of the composite large amplitude events, substantial barotropic transfer of energy from the wavenumber 2–4 ensemble to wavenumber 1 accounts for the rapid decline of $K_{2-4}$. The decline of $A_{2-4}$ appears to be largely accomplished by nonlinear transfer of $A_{2-4}$ to smaller scale waves.

The upscale cascade of energy from cyclone-scale eddies to planetary-scale eddies is an expected result of two-dimensional turbulence (e.g., Fjortoft, 1953). Synoptic manifestations of these intermittent bursts of upscale energy cascade may take several forms, but it is apparent that one common synoptic occurrence is the growth of the bimodal waves. Further upscale cascade to the scale of wavenumber 1 occurs during their
decay. The apparent importance of this sort of nonlinear dynamics provides additional evidence that theoretical understanding of the bimodality should be sought within a nonlinear framework.

Acknowledgments. I am grateful to Dr. Alfonso Sutera for many useful and stimulating discussions of the topic under consideration. The helpful comments of the reviewers are appreciated. This study was supported by the Climate Dynamics Program of the National Science Foundation under Grants ATM-8403372 and ATM-8518507.

APPENDIX I

Energetics Formulation

The formulation of the spectral energetics equations employed in this study is given by Saltzman (1970). In symbolic form, the rates of change of wavenumber $m$ kinetic energy ($K_m$) and available potential energy ($A_m$) are given by:

$$\frac{\partial}{\partial t} K_m = C(K_z, K_m) + C(A_m, K_m) + C_k(m|n, l)$$

$$+ F(K_m) + F(\Phi_m) + D(K_m)$$

$$\frac{\partial}{\partial t} A_m = C(A_z, A_m) - C(A_m, K_m) + C_A(m|n, l)$$

$$+ G(A_m) + F(A_m).$$

Here, a $z$ subscript denotes a zonal mean quantity:

$$(-)z = \frac{1}{2\pi} \int_{0}^{2\pi} (-) d\lambda$$

where $\lambda =$ longitude.

and an $m$ subscript denotes a departure from this mean for wavenumber $m$. The notation $C(A, B)$ represents a conversion of energy from reservoir $A$ to reservoir $B$. The $C_k(m|n, l)$ denotes rate of increase of the quantity $x$ (where $x = K$ or $A$) at wavenumber $m$ due to nonlinear triad interactions with all possible combinations of wavenumbers $n$ and $l$.

The $F(K_m)$ and $F(A_m)$ denote the combined horizontal and vertical boundary fluxes of $K_m$ and $A_m$ and $F(\Phi_m)$ is the boundary flux of geopotential energy. The kinetic energy dissipation, $D(K_m)$, and the generation of available potential energy, $G(A_m)$, cannot be computed explicitly.

In the present study, these equations were integrated from $20^\circ$ to $80^\circ$N and from 1000 to 100 mb. All of the budget terms were evaluated for each day of the four-winter dataset. Values of a given variable for a particular ensemble of wavenumbers will be denoted with subscripts. For example, the kinetic energy of wavenumbers 2 through 4 will be represented as

$$K_{2-4} = \sum_{m=2}^{4} K_m.$$ 

For the wave–wave interaction terms, the rate of change of $K_{2-4}$, for example, due to this mechanism is denoted

$$C_k(2-4) = \sum_{m=2}^{4} C_k(m|n, n \pm m).$$

REFERENCES


