NOTES AND CORRESPONDENCE

Some Evidence for Inertial Oscillations in the Lower Troposphere

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ABSTRACT

Observational evidence for an inertial oscillation in a particular situation is considered at Auckland, New Zealand. Cyclogenesis occurred in the Tasman Sea during the period, a low-level jet strengthened at Auckland, and the oscillation was detectable over a wide region of the South Pacific. The possible mechanisms of Blackadar and Uccellini are discussed in relation to the evidence.

1. Introduction

Oscillations with inertial periods are well known as solutions to the horizontal equations of motion. They may be derived on the basis of a balance between the centrifugal and Coriolis forces, giving inertia circles (Holton, p. 40). If a linear change in time of a pressure field is assumed, then this can be shown to generate cycloidal oscillations with the inertial frequency (Hess 1959, p. 226). They may also be derived on the assumption of an impulsive change in the pressure field (Fleagle and Businger 1980, p. 166). Thus, isallobaric tendencies may be expected to produce them.

Inertial oscillations have been difficult to detect in the atmosphere, although they have been more commonly found in the oceans. A study of ocean current data in the Baltic showed good evidence for inertial oscillations (Gustafson and Kullenberg 1936). Starr (1945) thought conditions for the Baltic inertial observations were especially favorable, these being strong density stratification, and an enclosed sea. Kundu and Thomson (1985) found purely inertial oscillations generated in ocean currents by the forcing of an atmospheric front.

Some evidence exists for stratospheric inertial oscillations. Sawyer (1961) found a horizontal scale for the oscillations of order 100s km in a perturbation analysis, and thought them quasi-inertial. Angell (1966) found clearer evidence for them in the stratosphere than the high troposphere. Weinstein et al. (1966) also found evidence for stratospheric inertial oscillations. As a model based on their findings, they proposed a distinctly layered structure for the stratosphere. With geostrophic wind shear, overall thermal stability and the low turbulence energy there, quasi-inertial oscillations may persist.

Theoretical studies (Kelvin 1879; Rossby 1938) concerned the inertial oscillations set up in a rotating fluid, and disagreed on the period. Rossby considered the integrated momentum, while Kelvin considered what would be observed locally. Kelvin found it possible for local variations of momentum to have periods of less than 12 pendulum hours, while the integrated momentum through a region varies with exactly this time. This was pointed out by Cahn (1945). Cahn developed the solutions for the effect of rotation upon a straight parallel current of finite width suddenly generated in an unlimited homogeneous ocean of constant depth, concluding that energy dispersion occurred, with more and more of the fluid mass oscillating with a continually decreasing amplitude.

The relation between observational studies of inertial oscillations, and theoretical ideas of their origin, is generally not close. A contributory reason may be that because of dissipative and dispersive processes, they are rarely found in the atmosphere. This makes generalization from a small number of cases difficult. A further reason may be the presence of adjustment mechanisms which tend to nullify any perturbation from an equilibrium state. This note considers some statistical evidence for their existence in a particular situation, and discusses some possible mechanisms.

2. Observational evidence

a. Meteorological situations

Daily (midnight) sea level pressure maps, as routinely analyzed by the National Weather Forecasting Centre of the New Zealand Meteorological Service, were examined for the five year period, 1975–79. Slow moving frontal situations which persisted near (approaching from the north or from the west) of Auck-

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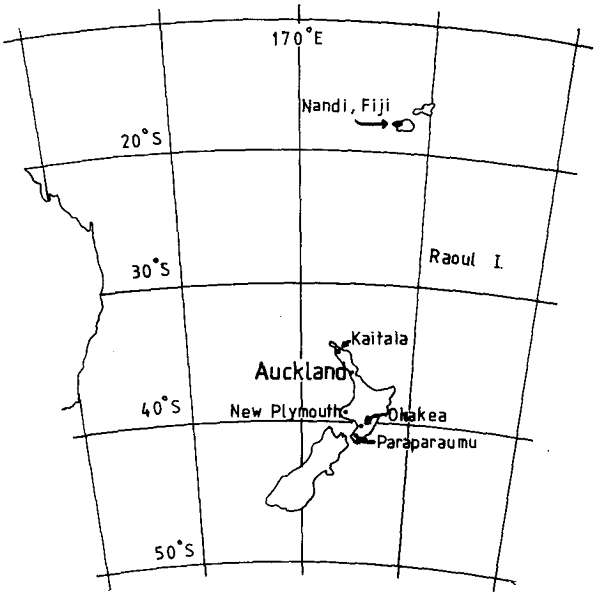


FIG. 1. Locations in South Pacific providing time series records of wind speed.

land (see Fig. 1) were examined, to help define series from an air flow without the complication of frontal changes in wind speed, or of trajectories over much of New Zealand.

This criterion was restrictive. 14 synoptic situations with at least four days satisfied this criterion. Some subjectivity is involved, as every synoptic situation is unique, so only broad classification by speed of movement and location is possible.

Hourly wind speeds from Auckland Airport were extracted for each situation. Auckland province is a generally flatter region than much of New Zealand and is surrounded by sea in the east and west. The 24 hourly record of weather there was considered suitable for such an analysis. Frontal wind changes as defined by a wind shift were then used to define the end of the series. The time series of hourly wind speeds were now plotted. Linear and quadratic detrending was carried out, as appropriate. The autocorrelation function was calculated for the residualized series. Three series gave clear evidence of oscillations different from the 24-hour cycle (by inspection of the autocorrelation functions). These situations were 16–19 February 1975, 8–14 October 1975, and 4–8 June 1978.

b. Statistical analysis

No higher order detrending than linear was judged appropriate in these cases. Histograms of the residuals from the linear detrending showed no strong departures from normality. Spectral analyses of the linearly detrended hourly wind speeds from Auckland were run. The length of the data series limits the spectral resolution, nevertheless analysis can reveal the general regions where frequency is concentrated. Table 1 presents the frequency/period estimates obtained, in each case, also shown are the latitudes of the inertial oscillations which would correspond to the analyzed frequencies.

These frequencies were then used to fit trigonometric regressions to the time series of the form

$$y = \alpha + \beta t + \delta_1 \cos\left(2\pi \frac{t}{P}\right) + \delta_2 \sin\left(2\pi \frac{t}{P}\right) \qquad (1)$$

where the period P is estimated spectrally (see Appendix for symbols). The reduction in sum of squares due to fitting the harmonic component was then tested, using the ratio of mean squares

$$F = \frac{\text{regression MS}}{\text{error MS}} \qquad (2)$$

on 2, $n - 3$ degrees of freedom, respectively, from the analysis of variance. The null hypothesis is of uncorrelated harmonic components attributable to white noise, the alternative hypothesis is that a harmonic component exists at the specified frequency. The calculated values of the F statistic are given in Table 1; all are significant at the 0.5 percent level.

A more detailed study of the situation extending over the period 4–8 June is now given, because this corresponded to an inertial frequency 'middle latitude' case, and although demonstrating significance, was the weakest oscillation detected.

In this case Fig. 2 shows the time series of hourly wind speeds, the associated autocorrelation and spectral

TABLE 1. Estimates of peak frequency and associated period from spectral analysis. Frequency (cy/h); period (h), by situation; then associated Fisher's F ratio; then inertial latitude corresponding to the analyzed frequency.

Situation	16–19 February 1975	8–14 October 1975	4–8 June 1978
Frequency/Period	$f^* = 0.0375$; $P = 26.7$ h	$f^* = 0.025$, $P = 40$ h	(1) $f^* = 0.05$; $P = 20$ h
—	—	—	(2) $f^* = 0.037$; $P = 27$ h
F	13.38	20.13	(1) 6.13
—	—	—	(2) 6.82
Latitude	26.4	17.5	(1) 36.9
—	—	—	(2) 26.4

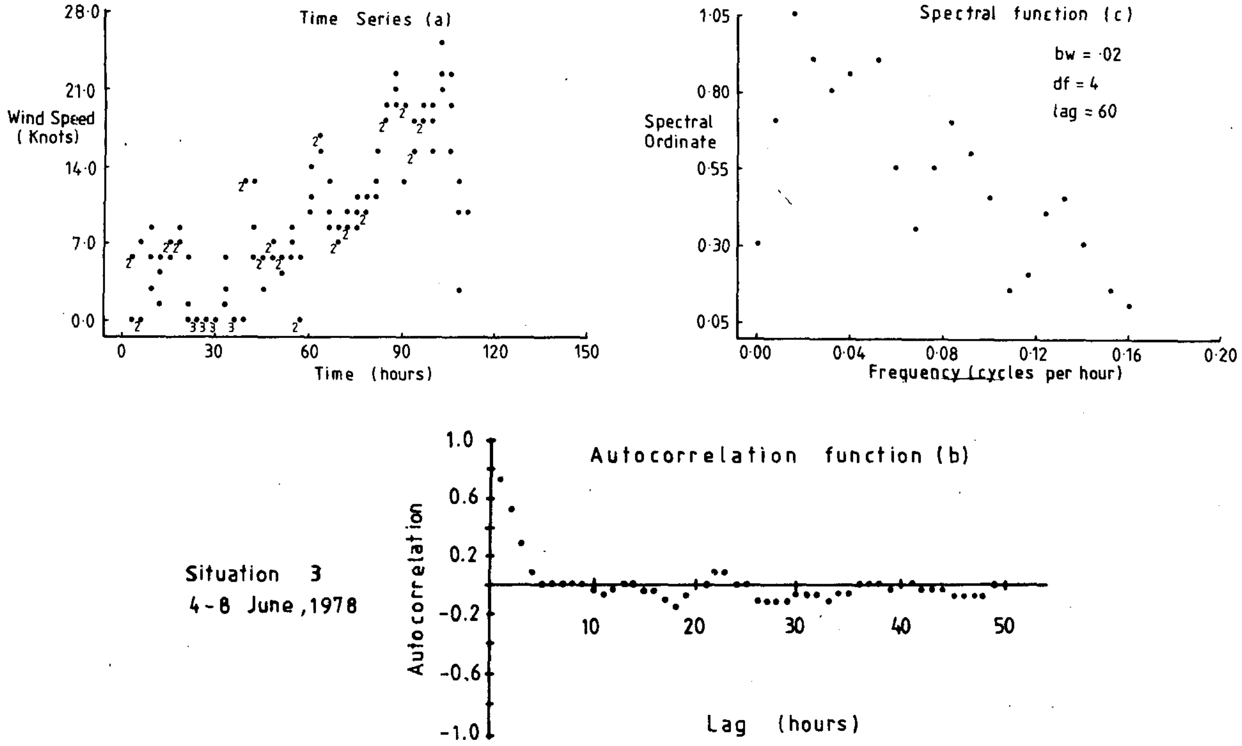


FIG. 2. (a) Time series of hourly wind speeds. (b) Autocorrelation function of the linearly residualised series. (c) Estimated spectral density function for residualised series, for situation 3, 4–8 June 1978.

density functions are plotted adjacently. (See also Table 2). A Tukey smoothing was used in the algorithm for the spectrum estimates; the associated bandwidth and degrees of freedom are indicated (Jenkins and Watts 1969, p 244). The features are weak in both time and frequency domains. Peaks appear at low frequency, and exploration with varying bandwidths shows that the location of the peaks was stable.

When a 24-hour cycle, as defined by the data, was removed, a spectral analysis was done on the residuals. Although it is not the major feature, there was still evidence for a secondary peak near 20 hours.

c. The spatial scale of the oscillation

To further test the reality of the analyzed oscillations, the hourly wind speeds from the airports at Nandi (Fiji), Raoul Island, Kaitaia, New Plymouth, Ohakea and Paraparaumu were examined. (For the locations of these sites, also see Fig. 1). The time series were linearly or quadratically detrended as appropriate, and trigonometric regressions were computed as a function of the period P. These results are shown in Table 3. No evidence for either oscillation period apears at Nandi, (and Paraparaumu) but Raoul shows evidence of a very weak peak in r^2 at 20 hours. Strong peaks at 20 hours appear at Kaitaia and Ohakea, and New Plymouth shows a comparable peak at 19 hours. Evidence for the 27-hour oscillation appears at Raoul Island, while Kaitaia shows a peak r^2 at 26 hours. The spatial separation is 11° latitude over which the 20 hour oscillation is detectable. This additional evidence supports the idea that the oscillations are not merely random features, but are definite organizations of the wind

TABLE 2. Hourly wind speed data, in knots, from Auckland Airport, 4–8 June 1978.

02	00	06	06	07	00	00	03	08	05	05	02	04	07	06
07	07	08	07	05	02	00	00	00	00	00	00	00	00	00
00	03	05	02	00	00	00	00	12	12	13	09	06	03	05
06	05	07	07	06	04	06	05	08	07	05	00	00	11	10
14	17	17	16	13	10	08	08	07	07	09	09	10	09	10
11	10	10	11	13	16	11	18	18	20	23	19	21	19	19
12	18	16	15	18	20	18	15	18	20	21	23	25	23	18
15	03	12	10	—	—	—	—	—	—	—	—	—	—	—

TABLE 3. Proportions of variance (%) explained by trigonometric regression as function of period P (hours) for time series at different locations. Nandi: N, Raoul I.: R, Kaitaia: K, Auckland: A, New Plymouth: NP, Ohakea; O.

	P													
	15	16	17	18	19	20	21	22	23	24	25	26	27	28
N	1.4	2.1	0.5	1.6	0.6	1.4	11.0	26.5	38.4	42.5	39.2	30.6	20.4	11.9
R	2.1	1.2	0.5	0.4	2.1	2.9	1.8	2.0	5.6	12.1	19.6	24.8	26.6	26.0
K	3.4	13.9	12.6	5.0	8.8	11.6	5.6	0.4	4.6	16.1	26.9	32.2	30.9	24.8
A	1.0	3.0	5.1	5.7	8.6	10.5	9.7	7.0	5.4	5.8	7.9	10.1	11.2	10.5
NP	1.1	0.8	4.0	9.5	10.3	6.4	3.0	2.7	4.7	8.9	14.4	20.5	26.7	31.9
O	2.8	1.2	2.5	4.9	9.0	11.7	10.3	6.6	2.9	0.8	0.4	0.7	1.1	1.6

flow with inertial time scales over widespread areas, comparable with the range of scales from fronts to cyclones. The analysed inertial frequency at 37°S lies within this range of latitudes.

Table 4 gives the complete analysis of variance for this situation at Auckland. Little evidence of a peak near 24 hours exists, but there are two clear peaks near 20 and 27 hours. Both correspond to significance levels above the half of one percent level. Accordingly, the further decomposition of the series into a linear trend, with harmonic components at periodicities of 20 and 27 hours, was made.

Reversing the order of the variables made no substantial change in the SS attributable to either variable. The SS attributable to the 20 hour harmonic when it was entered first, 224.7, actually increased to 315.7 when it was entered last.

The effect of autocorrelated residuals on the regression analyses is important. Here we are considering the residuals from the fitted trigonometric regressions. This question relates not only to the regression but to the closely related periodogram analysis, where the alternative hypothesis concerns an uncorrelated process $Z(t)$ superposed on a periodic term. Fitting a first-order autoregression [ar(1) process] yielded $r^2 = 0.5$. Thus if an ar(1) process describes the time series of residuals from the fitted harmonics exactly, the proportion of variance accounted for halves with each successive hour. If this is taken as the departure from independence, after 6 hours we have $(r^2)^6 = 0.0166$, which is close to the uncorrelated case. The effective degrees of freedom in the denominator of the F test is now about 18, (we have about 18 "independent" observations, spaced at every seventh observation) and even then the F is still very significant, on 2, 18 degrees of freedom. (The 1 percent point of the F distribution is 6.01). If we take 7 hours as the time to effective independence, the result is still well above significance at the 5% level and just fails at the 1% level. While this argument is only approximate, depending on the correctness of the simple ar(1) model, it is quite a severe correction and the F test is still unusual. A second-order autoregression is unlikely to make nearly so large a reduction in degrees of freedom. (A second order autoregression was fitted and this gave a 2% increase in r^2 on the first-order case.)

TABLE 4. Analysis of variance for two harmonic model. Units: kt². DF: degrees of freedom, SS: sum of squares, MS: mean squares.

Source	DF	SS	MS
Linear	1	2844.5	2844.5
Residual	107	2149.3	20.1
Total	108	4993.9	—
Harmonic (20 h)	2	224.71	112.35
Residual	105	1924.64	18.32
Total	107	2149.35	—
Harmonic (27 h)	2	332.40	166.20
Residual	103	1592.24	15.45
Total	105	1924.64	—

A treatment is also given in terms of a periodogram analysis. (Jenkins and Watts 1969; Fuller 1976). The sampling theory for the periodogram states that

$$\frac{4\pi}{\sigma^2} I_N(\lambda)$$

is distributed as chi squared, (on two degrees of freedom in this case) where the periodogram $I_N(\lambda)$ is given by

$$I_N(\lambda) = \frac{N}{2\pi} \| d_N(\lambda) \|^2$$

$$d_N(\lambda) = \frac{1}{N} \sum_{0}^{N-1} X(t) \exp(-i\lambda t).$$

The standard error of the linear detrended series in this situation is 4.46, $N = 109$, and

$$\sum_{0}^{N-1} X(t) \cos\lambda t = 93.74$$

and

$$\sum_{0}^{N-1} X(t) \sin\lambda t = 58.86.$$

TABLE 5. Degree of fit by model A (27 h), B (20 h). First: Overall distribution of minimum error modulus by first/second half of series. Tabulated: number of occasions with minimum error modulus. Then: Distribution of minimum error modulus, when the difference of errors by the two models exceeds the stated threshold. Threshold exceeding 2, 3 or 4 knots.

	Model							
	A	B	A	B	A	B	A	B
First half	32	23	16	6	10	0	3	0
Second half	24	30	10	17	6	11	2	0

Evaluation gives

$$\frac{4\pi}{\sigma^2} I_N(\lambda) = 11.3.$$

This value is significant at the 0.5 percent level.

3. Discussion

The distribution of error over time was studied, to further characterise the model performance.

a. Trigonometric model fit

By hour, the error moduli of each model (20 h, 27 h) separately were compared. A count was taken of the number of hours each series had the smallest error modulus, for each half of the series. Table 5 gives the distribution of minimum error modulus by each model (A: 27 h; B: 20 h) and half-series. Inspection of the overall table showed a clear tendency for the 27 hour fit to be better early in the time series. The relative proportion of better fitting by the 20 hour harmonic increases later. Where the errors by two models were different, the 27 hour model fitted better in the first half of the series, and the 20 hour model fitted better in the second half of the series, except when the errors were most different. The performances of the two models were quite unusually different in the threshold results. A chi-squared test on the first 2 by 2 table in the threshold results gave $\chi^2 = 6.2$, significant at the 5% level, on a homogeneity test.

b. Maintenance of oscillation

The Richardson number R was calculated at the levels: surface, 900 mb, 850 mb, 800 mb, 700 mb, . . . , 100 mb, for Auckland, where

$$R = g\left(\Gamma + \frac{dT}{dz}\right) \Big/ T\left(\frac{du}{dz}\right)^2.$$

Table 6 gives the values by pressure level over 3–8 June 1978. Very large values occur, reflecting the very small wind shears present thoughout. The R are generally much greater than one, and largest at high altitudes initially. The atmosphere over Auckland was highly stable and therefore turbulent mixing, acting to dissipate energy in any oscillation, suppressed.

c. Possible mechanisms

Investigation of the inertial behavior of the boundary layer resulting from diurnal forcing requires wind observations at a finer resolution than the 6 hours available, ideally at 1–2 hours. However some simple considerations are possible, based on the available data.

The 6 hourly values of wind direction and speed, and 12 hourly temperature soundings, were examined. Table 7 shows the mean wind speed as a function of time and elevation in the lower atmosphere, observed at Auckland Airport. A definite maximum appears in the 1–3000 ft winds at 0600 local time, on average, and this at first sight appears consistent with a nocturnal wind maxima as discussed by Blackadar (1957). Also, however, an identifiable, well defined jet pattern was clearly present throughout the period, beginning early on the 5th.

Evidence for the Blackadar theory as a partial explanation of the dominant early morning maxima is inconsistent with the ocean being a heat sink, and hence a source of stability, during the afternoon. Then, one would expect an afternoon speed maxima. (This is

TABLE 6. Values of Richardson number at Auckland, as function of altitude and time.

	Date										
	3rd	noon	4th	noon	5th	noon	6th	noon	7th	noon	8th
200 mb–100 mb	229	53	757	3741	687	*	*	382	104	*	297
300 mb–200 mb	49	7	48	645	153	147	*	52	213	15	60
400 mb–300 mb	29	20	10	26	62	17	*	17	4	2	32
500 mb–400 mb	8	23	172	6	7	167	14	163	498	801	46
600 mb–500 mb	79	35	19	36	84	22	47	1036	84	1120	31
700 mb–600 mb	27	64	32	163	210	148	77	67	1793	17	180
800 mb–700 mb	17	25	52	88	742	182	31	51	26	27	19
850 mb–800 mb	240	33	23	152	82	29	*	26	14	19	2056
900 mb–850 mb	20	14	581	396	93	467	*	43	147	11	679
1000 mb–900 mb	20	5	72	223	6	5	*	4	6	5	6

TABLE 7. Mean wind speed (kt) over period, as function of height (1000 ft) and 6 hourly time interval, at Auckland, 4–8 June 1978.

Height (ft)	Time (UTC)			
	0000	0600	1200	1800
10 000	15	15	16.4	15
7 000	14.4	17.6	15	16.2
5 000	16.7	22.6	20	21
3 000	17.7	19.5	22.6	22.5
2 000	16.5	20.2	22.6	26.8
1 000	16.7	18	17.8	20.2
Surface	8.4	11.5	7.2	7.2

consistent with some observations which have been made in the Coral Sea.) If then the boundary layer at Auckland is subject to oceanic influences, then one would expect a reversed diurnal cycle with a speed maxima in lower levels in the afternoon. There is some evidence in the daily variations at some of the time consistent with this (as noted) but not when the mean jet maximum is.

When a diurnal variation, as defined by the data, is removed, a near 20 hour oscillation is still in evidence in the time series. This is not consistent with a diurnally fixed inertial variation still being present, as would be implied by the Blackadar mechanism, where a single inertial oscillation is triggered anew at a fixed time each day.

The presence of several oscillation periods at Auckland in the case examined in detail, only one of which is near the correct inertial latitude, would alone render the Blackadar theory insufficient. The wide latitude range of the observed 20 h oscillation (Table 3), (corresponding to an inertial latitude of 37°) is consistent with a source in only one latitude, whereas if it was a widespread boundary layer effect one might expect on the Blackadar theory, a corresponding range of analyzed inertial periods. Table 3 shows the 20 hour oscillation was of comparable strength at Kaitaia, New Plymouth and Ohakea. Finally, the presence of the jet, although of varying intensity, throughout most of the period suggests some alternative mechanism.

A possible alternate mechanism to consider is isallobaric tendency, as outlined in the introduction. Since inertial terms arise in simplified treatments of time varying pressure fields, since broadscale cyclogenesis occurred in the Tasman Sea, and since a low-level jet was present at Auckland from early in the period, it is suggested that the ideas of Uccellini (1980) may apply.

Uccellini related synoptic scale processes of cyclogenesis to the occurrence of low-level jets. In a review of observational studies, he considered positive evidence exists for low-level jets to be coupled with the upper-level jet through mass readjustment mechanisms, the link being through the associated synoptic scale processes of cyclogenesis:

> "Changes in the pressure gradient force related to leeside cyclogenesis and leeside troughing and the isallobaric wind response to these changes seem to be an integral part of the process that leads to the development of LLJ's observed in the Great Plains."

The link between the jet streams was seen by Uccellini as cyclogenesis, with the upper level jet giving rise to cyclogenesis, while the low level jet arose as an isallobaric response to these synoptic scale developments.

The link between inertial oscillations and the low-level jet is seen as the isallobaric tendencies which gave rise to the jet, as interpreted by Uccellini, and because they appear as components of the solution for simplified treatments of isallobaric development.

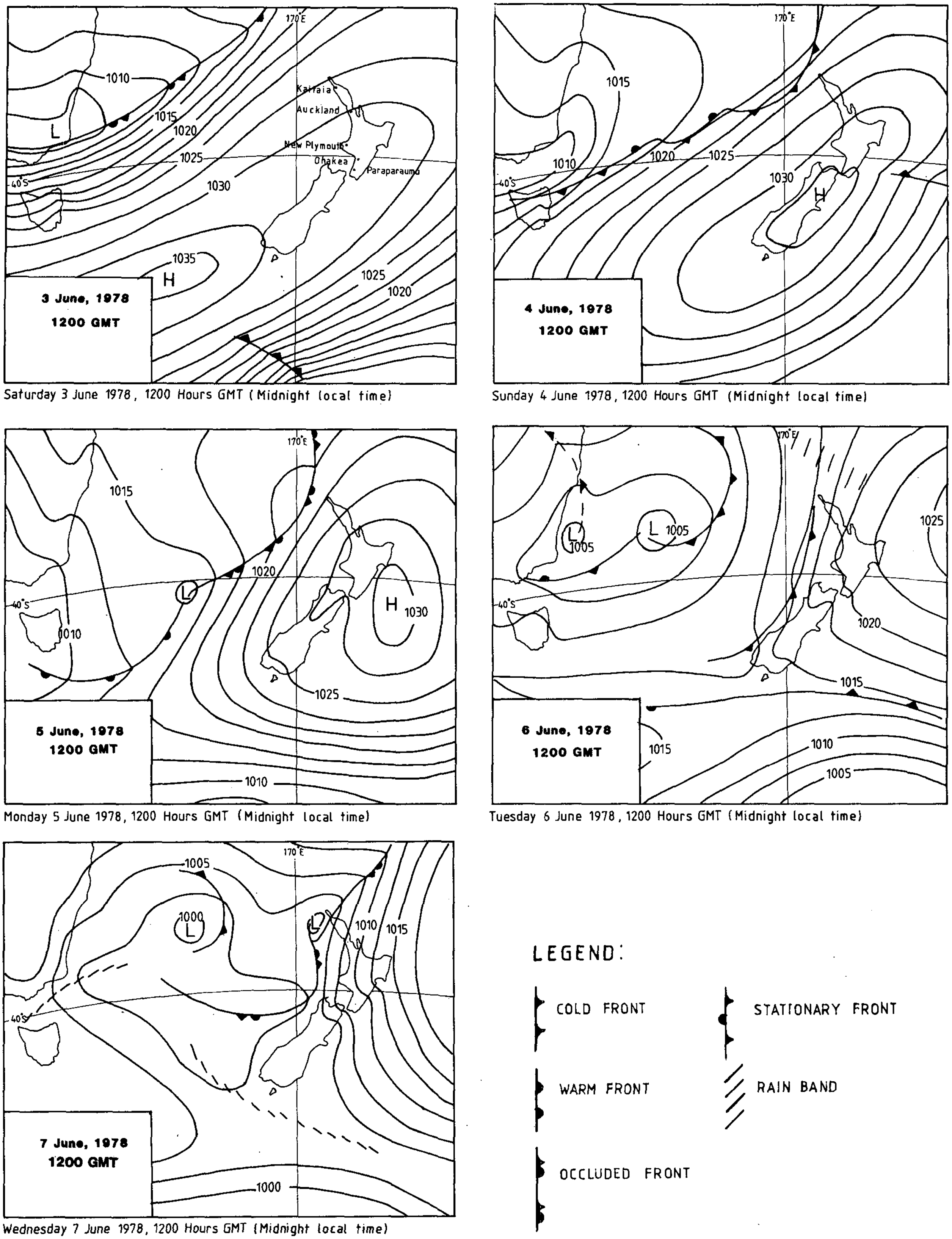
During 4–8 June there was evidence of a weak jet at the level 150–300 mb over Auckland in addition to the low-level jet. The presence of missing observations on the 5th and 6th, generally small time variations in winds at a given level, and the absence of other vertical soundings to the west, renders definite conclusions about vertical coupling impossible. The large stratification suggests that vertical exchange (turbulent mixing) was generally suppressed.

However, the large-scale changes in the trough in the western Tasman Sea (3a, 3b), the development of frontal waves (3b) in the first 24 hours of the period, the subsequent deepening of the surface trough (3d–3e) with upstream cyclogenesis affecting pressure gradients are all consistent in broad terms with the cyclogenetic cases considered by Uccellini.

The strength of the low-level jet was taken as the difference between the maximum wind (between the surface and 10 000 feet) and the wind at the surface, at a given time. This quantity had values of 3–5 knots from midday to midnight on the 4th, NZST (4th, 0000 to 1200 UTC) but early on the 5th strengthened by 10 knots [at 0600 local time on the 5th, NZST, 15 knots at 3000 feet, (4th, 1800 UTC): compare Fig. 3b]. It remained at this or greater strength for the remainder of the period, some 79 hours, as indicated by the 6-hour soundings.

The tightening pressure gradient over northern New Zealand (Figs. 3a–c) and strengthening northerly flow (Fig. 2a) observed at Auckland in the latter part of the period, when the 20 hour oscillation was in fact a better fit to the time series was similar to the tightening gradients found by Uccellini (1980) in the cyclogenetic cases giving low-level jets. Because of turbulent mixing with the strengthening flow, it was also not consistent with a strong boundary layer variation due to thermal forcing.

Although other mechanisms cannot be definitely ruled out on the basis of the above evidence, it is therefore suggested that the analyzed frequencies are inertial. The common analysed frequency at widely separated locations, suggests that the wind flow contains a large scale modification or control to the oscillation. One



Saturday 3 June 1978, 1200 Hours GMT (Midnight local time)

Sunday 4 June 1978, 1200 Hours GMT (Midnight local time)

Monday 5 June 1978, 1200 Hours GMT (Midnight local time)

Tuesday 6 June 1978, 1200 Hours GMT (Midnight local time)

Wednesday 7 June 1978, 1200 Hours GMT (Midnight local time)

FIG. 3. Midnight mean sea level pressure analyses through the period, 4–8 June. (a) 3/1200 UTC (4th, 0000 NZST). (b) 4/1200 UTC. (c) 5/1200 UTC. (d) 6/1200 UTC. (e) 7/1200 UTC (8th, 0000 NZST).

possibility for such broadscale control on these scales could be inertial.

The inertial interpretation then, is consistent with the development and presence of the low-level jet at Auckland, and the isallobaric interpretation of the low-level jet at Auckland (Uccellini). The isallobaric tendencies in sea-level pressure gradient from early in the period with the strengthening low level winds and later improvement of fit by the 20 hour model support this, for, given the isallobaric development, inertial components in the flow are expected by theory.

It is possible that part of the inertial variation was due to the Blackadar process, but overall it does not appear the most likely mechanism. The small magnitude of the analysed oscillation, of order several knots, limits the spatial amplitude of a given parcel's trajectory. A puzzling property of these observations, given they are inertial, is then the wide latitude range over which the same frequency is analyzed, suggesting a larger scale organization of the flow. This subject must remain a subject for future research.

4. Conclusion

An investigation of five years of meteorological situations identified 14 suitable sequences of slow moving frontal zones near Auckland. Detailed examination of the time series showed three situations with definite periodicities. In the situation considered in detail, the analyzed periodicity was consistent with an inertial period at a latitude within the same latitude range as those in which the oscillation was detected, and upstream cyclogenesis.

The atmosphere over Auckland was highly favourable to the maintenance of oscillations throughout the 5 day period and the 20-hour oscillation was more dominant late in the period, when earlier there was also evidence for isallobaric tendencies, and the development of a low-level jet. The oscillation was manifest over a large spatial region of the South Pacific during this time, comparable with the general synoptic scales of cyclones.

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Finally, I thank the reviewers for their detailed helpful comments.

APPENDIX

List of Symbols

α	Intercept in linear least squares fit
β	Gradient in linear least squares fit
bw	Bandwidth
δ_1	Semi-amplitude of harmonic least squares fit
δ_2	Semi-amplitude of harmonic least squares fit
df	Degrees of freedom
f^*	Frequency of oscillation
F	Fisher's F ratio
λ	Frequency of oscillation in periodogram analysis
MS	Mean square
SS	Sum of squares
N	Length of data series
P	Period of oscillation

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