Insensitivity of the Nonlinear Normal Mode Initialization of a Limited Area Model to the Inclusion of Nonstationary Rossby Modes

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ABSTRACT

Application of the nonlinear normal mode initialization method requires the construction of the normal modes of the linearized model equations.

In the case of a limited area model the Coriolis parameter in this linear system is usually assumed to be constant, so that the Rossby modes are stationary. Since inclusion of the beta terms might be important in practical applications of nonlinear normal mode initialization, we investigate this effect by including specific beta terms so that the Rossby modes are nonstationary. Results of the methods with stationary and nonstationary Rossby modes are compared.

They show that the two methods are virtually identical in their effect upon the initial fields and upon their development during the first hours of the forecast.

1. Introduction

The purpose of initialization of a primitive equation forecast model is to balance the initial fields of velocity and mass in order to suppress the high frequency wave oscillations in subsequent model integrations. Application of the nonlinear normal mode initialization method requires the construction of the normal modes of the linearized model equations. In the case of a limited area model the Coriolis parameter in this linear system is usually assumed to be constant, so that the Rossby modes are stationary. However, the inclusion of beta terms in these equations might be important in practical applications of the nonlinear normal mode method (Ballish 1979; see also Kasahara 1982, p. 394).

In order to investigate this effect we consider a simple modification of the linear model equations, by including specific beta terms so that the Rossby modes are nonstationary.

An outline of the method is given in section 2. Results of the method, compared with those with a constant Coriolis parameter in the linear system, are given in section 3. Conclusions are presented in section 4.

2. Outline of the method

We consider the shallow water equations on the sphere in differentiated form

$$\frac{\partial}{\partial t}\nabla^2\chi - 2\Omega\sin\theta\nabla^2\psi + \frac{2\Omega}{r^2}\left(\frac{\partial\chi}{\partial\lambda} - \cos\theta\frac{\partial\psi}{\partial\theta}\right) + \nabla^2\phi = Q_\chi,$$

$$\frac{\partial}{\partial t}\nabla^2\psi + 2\Omega\sin\theta\,\nabla^2\chi + \frac{2\Omega}{r^2}\left(\frac{\partial\psi}{\partial\lambda} + \cos\theta\frac{\partial\chi}{\partial\theta}\right) = Q_\psi,$$

$$\frac{\partial}{\partial t}\phi + d\nabla^2\chi = Q_\phi,$$

where χ, ψ, ϕ and d are the velocity potential, streamfunction, geopotential and mean free geopotential height, λ and θ longitude and latitude, r and Ω the radius and angular velocity of the earth and Q_χ, Q_ψ and Q_ϕ the nonlinear terms.

The equations used in the nonlinear normal mode method under consideration, on a discrete grid $\lambda_m = \lambda_0 + m\Delta\lambda$, $\theta_n = \theta_0 + n\Delta\theta$, with $M \times N$ interior grid points, are

$$\frac{\partial}{\partial t}\nabla_d^2\chi - \bar{f}\nabla_d^2\psi + \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\chi + \nabla_d^2\phi = Q'_\chi, \quad (1)$$

$$\frac{\partial}{\partial t}\nabla_d^2\psi + \bar{f}\nabla_d^2\chi + \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\psi = Q'_\psi, \quad (2)$$

$$\frac{\partial}{\partial t}\phi + d\nabla_d^2\chi = Q_\phi, \quad (3)$$

where $\bar{f} = 2\Omega\sin\bar{\theta}$ is a constant Coriolis parameter, ∇_d^2 the usual five-point discrete Laplacian operator in spherical coordinates and $(\partial/\partial\lambda)_d$ the centered difference operator in the λ-direction. The linearization of system (1) to (3) has been chosen because it admits nonstationary Rossby modes *and* because gravitational and Rossby modes can be obtained in a relatively simple way. In the case of a limited area model inclusion of other linear terms will complicate things to such an extent that no such simple solution *in terms of normal modes* exists. Deviations of the linear terms are ab-

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sorbed into the nonlinear terms on the right-hand side. In order that the linear system admits nonstationary Rossby modes, it is sufficient to include only the term $(\partial/\partial\lambda)_d\psi$. However, for the sake of symmetry we also include $(\partial/\partial\lambda)_d\chi$.

Because of the presence of the terms $(\partial/\partial\lambda)_d\chi$ and $(\partial/\partial\lambda)_d\psi$ in Eqs. (1) and (2), the eigenfunctions of the linear spatial operator on the left-hand side of Eqs. (1) to (3) are complex exponential functions in the E–W direction. Therefore, we would like to write the solution in the form

$$\chi = \chi_0 + \hat{\chi}, \quad \psi = \psi_0 + \hat{\psi}, \quad \phi = \phi_0 + \hat{\phi}, \quad (4)$$

where the fields $\hat{\chi}$, $\hat{\psi}$ and $\hat{\phi}$ at the intial time, have periodic boundary conditions in the E–W direction and zero boundary conditions in the N–S direction, and where the fields χ_0, ψ_0 and ϕ_0 satisfy the discrete Laplace equations

$$\nabla_d^2\chi_0 = 0, \quad \nabla_d^2\psi_0 = 0, \quad \nabla_d^2\phi_0 = 0 \quad (5)$$

and are equal to the initial values of $\chi - \hat{\chi}$, $\psi - \hat{\psi}$ and $\phi - \hat{\phi}$ on the boundary.

In order to facilitate computations, the time tendencies of χ, ψ and ϕ at the boundaries are set equal to zero.

Including the terms $(\partial/\partial\lambda)_d\chi_0$ and $(\partial/\partial\lambda)_d\psi_0$ in the nonlinear terms on the right-hand side, we could solve the remaining system of equations for the fields $\hat{\chi}$, $\hat{\psi}$ and $\hat{\phi}$.

However, because the periodic boundary values of $\hat{\chi}$, $\hat{\psi}$ and $\hat{\phi}$, at the initial time, are not known beforehand but follow from an iterative procedure, as we see at the end of this section, we proceed as follows. As a first approximation, we introduce the (time-independent) functions χ_0, ψ_0 and ϕ_0, which satisfy the discrete Laplace equations (5) and are equal to the initial values of χ, ψ and ϕ on the boundary. Then, using (4) and removing the primes of Q'_χ and Q'_ψ, Eqs. (1) to (3) become

$$\frac{\partial}{\partial t}\nabla_d^2\chi - \bar{f}\nabla_d^2\hat{\psi} + \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\hat{\chi} + \nabla_d^2\hat{\phi} = Q_\chi - \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\chi_0, \quad (6)$$

$$\frac{\partial}{\partial t}\nabla_d^2\psi + \bar{f}\nabla_d^2\hat{\chi} + \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\hat{\psi} = Q_\psi - \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\psi_0, \quad (7)$$

$$\frac{\partial}{\partial t}\phi + d\nabla_d^2\hat{\chi} = Q_\phi. \quad (8)$$

Let the normal modes of the linear operator on the left-hand side of Eqs. (6) to (8) have the following spatial behavior

$$S_{kl}(m, n) = f_{kl}(n)\exp(2\pi ikm/(M+1)),$$

satisfying $\nabla_d^2 S_{kl}(m, n) = -\alpha_{kl}^2 S_{kl}(m, n)$ with $f_{kl}(0) = f_{kl}(N+1) = 0$.

Then, substitution shows that the Rossby and gravity modes are determined by the eigenvectors and eigenvalues of the matrix

$$\mathsf{M}_{kl} = \begin{pmatrix} -i\epsilon_{kl} & -\bar{f} & 1 \\ \bar{f} & -i\epsilon_{kl} & 0 \\ -\alpha_{kl}^2 d & 0 & 0 \end{pmatrix}$$

with

$$\epsilon_{kl} = \frac{2\Omega}{r^2\Delta\lambda\alpha_{kl}^2}\sin\frac{2k\pi}{M+1}.$$

The eigenvectors of M_{kl} are proportional to (if T denotes the transpose)

$$(i(\sigma + \epsilon_{kl}), \bar{f}, \bar{f}^2 - (\sigma + \epsilon_{kl})^2)^{\mathrm{T}}$$

with eigenvalues $\nu = i\sigma$, where σ satisfies the equation

$$\sigma(\sigma + \epsilon_{kl})^2 - \bar{f}^2\sigma - (\sigma + \epsilon_{kl})\alpha_{kl}^2 d = 0. \quad (9)$$

We solve this equation for $\epsilon_{kl} \geqslant 0$; for $\epsilon_{kl} < 0$ we find the complex conjugate eigenvalues (see, for instance, van der Waerden 1955, p. 187 for the solution of a cubic equation). The values of σ corresponding to Rossby waves and westward and eastward gravity waves are (we consider geopotential heights so that $3\alpha_{kl}^2 d - 6\bar{f}^2 + \frac{2}{3}\epsilon_{kl}^2 > 0$)

$$\sigma_{kl1} = -\frac{2}{3}\epsilon_{kl} - \frac{2}{3}(3p)^{1/2}\cos(\pi + \mu)/3,$$

$$\sigma_{kl2} = -\frac{2}{3}\epsilon_{kl} - \frac{2}{3}(3p)^{1/2}\cos(\pi - \mu)/3,$$

$$\sigma_{kl3} = -\frac{2}{3}\epsilon_{kl} + \frac{2}{3}(3p)^{1/2}\cos\mu/3,$$

where

$$\mu = \arctan((12p^3 - q^2)^{1/2}/q), \quad 0 < \mu \leqslant \pi/2,$$

$$p = \bar{f}^2 + \alpha_{kl}^2 d + \frac{1}{3}\epsilon_{kl}^2,$$

$$q = 3\epsilon_{kl}\alpha_{kl}^2 d - 6\epsilon_{kl}\bar{f}^2 + \frac{2}{3}\epsilon_{kl}^3.$$

Note that $\epsilon_{kl} = 0$ for $k = 0$ and $k = (M+1)/2$ (if M is odd), so that $\sigma_{kl1} = 0$, $\sigma_{kl2} = -(\alpha_{kl}^2 d + \bar{f}^2)^{1/2}$ and $\sigma_{kl3} = (\alpha_{kl}^2 d + \bar{f}^2)^{1/2}$. Let us write the solution in the form $\hat{\eta}(m, n) = (\hat{\chi}(m, n), \hat{\psi}(m, n), \hat{\phi}(m, n))^T$ and define the scalar product

$$\langle\hat{\eta}_1, \hat{\eta}_2\rangle = \frac{1}{M+1}\sum_{m=0}^{M}\sum_{n=1}^{N}[\hat{\phi}_1\hat{\phi}_2^* - d(\hat{\chi}_1\nabla_d^2\hat{\chi}_2^* + \hat{\psi}_1\nabla_d^2\hat{\psi}_2^*)]\cos\theta_n,$$

where the asterisk denotes complex conjugation. Then the normalized Rossby mode and westward and eastward gravity modes are given by

$$\mathbf{P}_{klr} = N_{klr}^{-1/2}\mathbf{A}_{klr}S_{kl}(m, n), \quad r = 1, 2, 3 \quad (10)$$

where

$$\mathbf{A}_{klr} = (i(\sigma_{klr} + \epsilon_{kl}), \bar{f}, \bar{f}^2 - (\sigma_{klr} + \epsilon_{kl})^2)^T, \quad (11)$$

$$N_{klr} = (\bar{f}^2 - (\sigma_{klr} + \epsilon_{kl})^2)^2 + \alpha_{kl}^2 d((\sigma_{klr} + \epsilon_{kl})^2 + \bar{f}^2). \quad (12)$$

We may expand $\hat{\eta}$, at the initial time, in the normal modes

$$\hat{\eta}(m, n) = \sum_{k=0}^{M}\sum_{l=1}^{N}\sum_{r=1}^{3}\hat{\gamma}_{klr}\mathbf{P}_{klr},$$

where

$$\hat{\gamma}_{klr} = \langle\hat{\eta}, \mathbf{P}_{klr}\rangle.$$

Substituting this into Eqs. (6) to (8) and projection on the gravity modes yields,

$$\dot{\gamma}_{klr} = -\nu_{klr}\hat{\gamma}_{klr} + F_{klr}, \quad r = 2, 3$$

where

$$F_{klr} = \frac{1}{M+1}N_{klr}^{-1/2}\sum_{m=0}^{M}\sum_{n=1}^{N}\Bigg\{(\bar{f}^2 - (\sigma_{klr} + \epsilon_{kl})^2)Q_\phi - d\left[\bar{f}\left(Q_\psi - \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\psi_0\right) - i(\sigma_{klr} + \epsilon_{kl})\left(Q_\chi - \frac{2\Omega}{r^2}\left(\frac{\partial}{\partial\lambda}\right)_d\chi_0\right)\right]\Bigg\}S_{kl}^*\cos\theta_n,$$

$$\dot{\gamma}_{klr} = \frac{1}{M+1}N_{klr}^{-1/2}\sum_{m=0}^{M}\sum_{n=1}^{N}\left[(\bar{f}^2 - (\sigma_{klr} + \epsilon_{kl})^2) \times \frac{\partial\phi}{\partial t} - d\left[\bar{f}\frac{\partial}{\partial t}\nabla_d^2\psi - i(\sigma_{klr} + \epsilon_{kl})\frac{\partial}{\partial t}\nabla_d^2\chi\right]\right] \times S_{kl}^*\cos\theta_n.$$

The boundary values of the forcing terms are assumed to be zero. Following Machenhauer (1977), the initial tendencies of gravity wave components are set to zero, yielding

$$-\nu_{klr}\hat{\gamma}_{klr} + F_{klr} = 0, \quad r = 2, 3.$$

This nonlinear equation can be solved iteratively, as follows

$$\hat{\gamma}_{klr}^{(q+1)} = F_{klr}^{(q)}/\nu_{klr}, \quad r = 2, 3.$$

The new fields are constructed from

$$\chi^{(q+1)} = \chi_0^{(q+1)} + \hat{\chi}^{(q+1)},$$

$$\psi^{(q+1)} = \psi_0^{(q+1)} + \hat{\psi}^{(q+1)}$$

and

$$\phi^{(q+1)} = \phi_0^{(q+1)} + \hat{\phi}^{(q+1)},$$

where the fields $\hat{\eta}^{(q+1)}$ are given by the equations

$$\hat{\eta}^{(q+1)} = \hat{\eta}^{(q)} + \sum_{k=0}^{M}\sum_{l=1}^{N}\sum_{r=2}^{3}(F_{klr}^{(q)}/\nu_{klr} - \hat{\gamma}_{klr}^{(q)})\mathbf{P}_{klr}$$

and where the fields $\chi_0^{(q+1)}$, $\psi_0^{(q+1)}$ and $\phi_0^{(q+1)}$ satisfy the discrete Laplace equations

$$\nabla_d^2\chi_0^{(q+1)} = 0, \quad \nabla_d^2\psi_0^{(q+1)} = 0, \quad \nabla_d^2\phi_0^{(q+1)} = 0$$

and are equal to the initial values of $\chi - \hat{\chi}^{(q+1)}$, $\psi - \hat{\psi}^{(q+1)}$ and $\phi - \hat{\phi}^{(q+1)}$ on the boundary.

These new fields may be used to evaluate $F_{klr}^{(q+1)}$. Since $\hat{\gamma}_{M+1-klr} = \hat{\gamma}_{klr}^*$, for $k = 1, \cdots, (M-1)/2$ if M is odd or $k = 1, \cdots, M/2$ if M is even, it is sufficient to consider the wave numbers $k = 0, \cdots, (M+1)/2$ if M is odd or $k = 0, \cdots, M/2$ if M is even.

3. Results

In order to test the initialization method described above, a test run was made with the five-level limited area version of the ECMWF grid point model employed by Temperton and Williamson (1981), on a grid hav-

TABLE 1. Frequencies and modified frequencies of the gravity waves of the f-plane approximation and the method of section 2, respectively, for the external and first internal mode. For further explanation, see section 3.

k	$(\alpha_{k1}^2 d + \bar{f}^2)^{1/2}$		$\sigma_{k12} + \epsilon_{k1}$		$\sigma_{k13} + \epsilon_{k1}$	
0	4.2233×10^{-4}	1.9154×10^{-4}	-4.2233×10^{-4}	-1.9154×10^{-4}	4.2233×10^{-4}	1.9154×10^{-4}
1	8.0448×10^{-4}	3.1674×10^{-4}	-8.0242×10^{-4}	-3.1494×10^{-4}	8.0654×10^{-4}	3.1855×10^{-4}
2	1.3736×10^{-3}	5.1824×10^{-4}	-1.3722×10^{-3}	-5.1695×10^{-4}	1.3749×10^{-4}	5.1954×10^{-4}
3	1.9186×10^{-3}	7.1563×10^{-4}	-1.9176×10^{-3}	-7.1468×10^{-4}	1.9196×10^{-3}	7.1658×10^{-4}
4	2.4247×10^{-3}	9.0025×10^{-4}	-2.4239×10^{-3}	-8.9953×10^{-4}	2.4254×10^{-3}	9.0097×10^{-4}
5	2.8836×10^{-3}	1.0682×10^{-3}	-2.8830×10^{-3}	-1.0677×10^{-3}	2.8841×10^{-3}	1.0688×10^{-3}
6	3.2873×10^{-3}	1.2163×10^{-3}	-3.2869×10^{-3}	-1.2158×10^{-3}	3.2878×10^{-3}	1.2167×10^{-3}
7	3.6286×10^{-3}	1.3415×10^{-3}	-3.6283×10^{-3}	-1.3412×10^{-3}	3.6289×10^{-3}	1.3418×10^{-3}
8	3.9010×10^{-3}	1.4415×10^{-3}	-3.9008×10^{-3}	-1.4413×10^{-3}	3.9012×10^{-3}	1.4418×10^{-3}
9	4.0994×10^{-3}	1.5144×10^{-3}	-4.0992×10^{-3}	-1.5143×10^{-3}	4.0995×10^{-3}	1.5146×10^{-3}
10	4.2199×10^{-3}	1.5587×10^{-3}	-4.2198×10^{-3}	-1.5586×10^{-3}	4.2200×10^{-3}	1.5588×10^{-3}
11	4.2603×10^{-3}	1.5736×10^{-3}	-4.2603×10^{-3}	-1.5736×10^{-3}	4.2603×10^{-3}	1.5736×10^{-3}

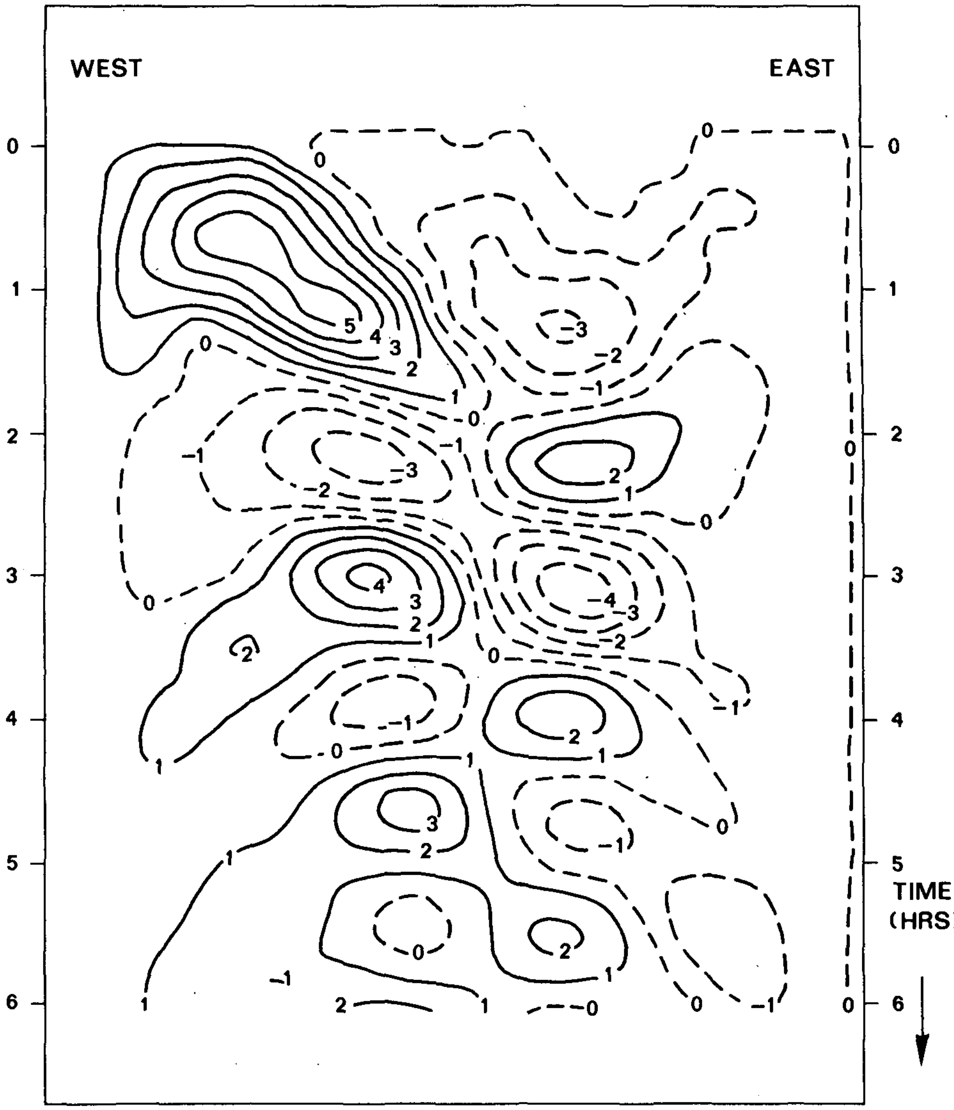

FIG. 1. Longitude–time diagram of surface pressure tendencies along the latitude line of 55°N, starting at 1200 UTC 5 January 1982, before initialization. Units are millibars per hour.

ing a mesh spacing $(\Delta\lambda, \Delta\theta) = (2°, 1°)$ and covering approximately the area between 45° and 65°N and 20°W and 20°E.

By a vertical decomposition of the model equations, initialization of a baroclinic model becomes equivalent to the initialization of the resulting system of shallow water equations. For details the reader is referred again to the paper of Temperton and Williamson.

Initial data are obtained from analyses of the geopotential at pressure levels. Temperatures are derived hydrostatically. Wind components u and v at sigma levels are calculated with the linear balance equation. Orography is included.

Initialization starts with the computation of divergence and vorticity. From the divergence and vorticity the stream function ψ and velocity potential χ are derived. After initialization the velocity components are obtained from ψ and χ.

The retrieval of the temperature and pressure changes at each iteration step of the initialization procedure is accomplished by a variational method (Daley 1979).

The effect of initialization is made visible by means of a longitude–time diagram of the surface pressure tendencies along the latitude line 55°N, from 22°W to 20°E.

An experiment with the initialization method of section 2 was performed on initial data valid at 1200

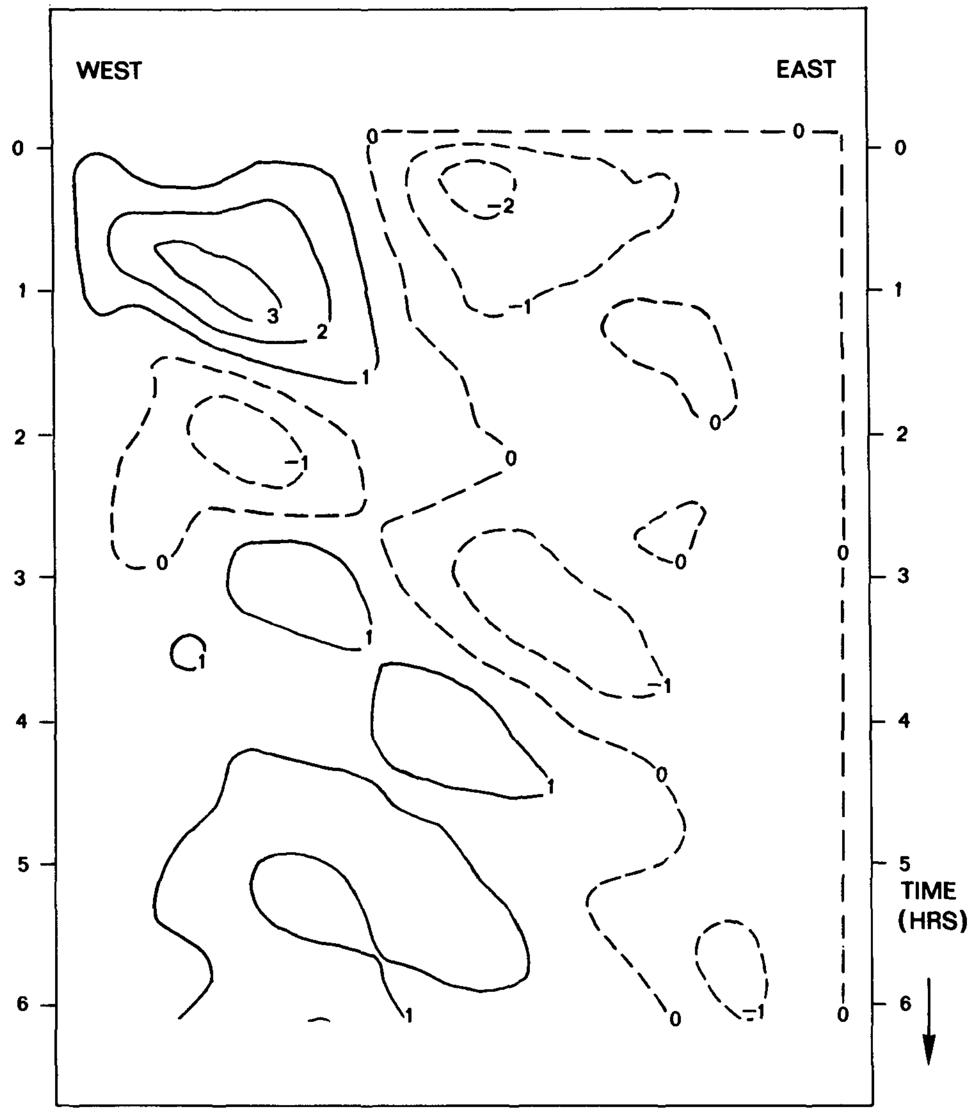

FIG. 2. As in Fig. 1, but after initialization with the method of section 2 of the external mode, with two iterations.

UTC, 5 January 1982. The same data were used for a comparison between the nonlinear normal mode and bounded derivative methods in Bijlsma and Hafkenscheid (1986). For details, for instance concerning the weights that were used in Daley's variational method, the reader is referred to that paper.

At this point let us first examine how the gravity modes, which in fact determine the initialization results, are changed by the inclusion of the beta terms.

In the case of an f-plane approximation the Rossby mode and westward and eastward gravity modes (denoted by $\mathbf{R}_{kl1}$, $\mathbf{G}_{kl2}$ and $\mathbf{G}_{kl3}$) are given by relations (10), (11) and (12) with 0, $-(\alpha_{kl}^2 d + \bar{f}^2)^{1/2}$ and $(\alpha_{kl}^2 d + \bar{f}^2)^{1/2}$ substituted for $\sigma_{kl1} + \epsilon_{kl}$, $\sigma_{kl2} + \epsilon_{kl}$ and $\sigma_{kl3} + \epsilon_{kl}$ respectively. By inclusion of the beta terms the last two expressions satisfy

$$(\sigma_{klr} + \epsilon_{kl})^2 = \alpha_{kl}^2 d + \bar{f}^2 + \frac{\epsilon_{kl}}{\sigma_{klr}} \alpha_{kl}^2 d, \quad r = 2, 3,$$

where we used Eq. (9). It can be readily shown that $\epsilon_{kl} \ll \sigma_{klr}$ ($r = 2, 3$), if we consider a midlatitude approximation for the Coriolis parameter and sufficiently large equivalent depths, so that $\sigma_{kl2} + \epsilon_{kl}$ and $\sigma_{kl3} + \epsilon_{kl}$ may be approximated by $-(\alpha_{kl}^2 d + \bar{f}^2)^{1/2}$ and $(\alpha_{kl}^2 d + \bar{f}^2)^{1/2}$ respectively. In Table 1 these expressions are compared for the first two vertical modes (with geopotential heights satisfying $3\alpha_{kl}^2 d - 6\bar{f}^2 + \frac{2}{3}\epsilon_{kl}^2 > 0$),

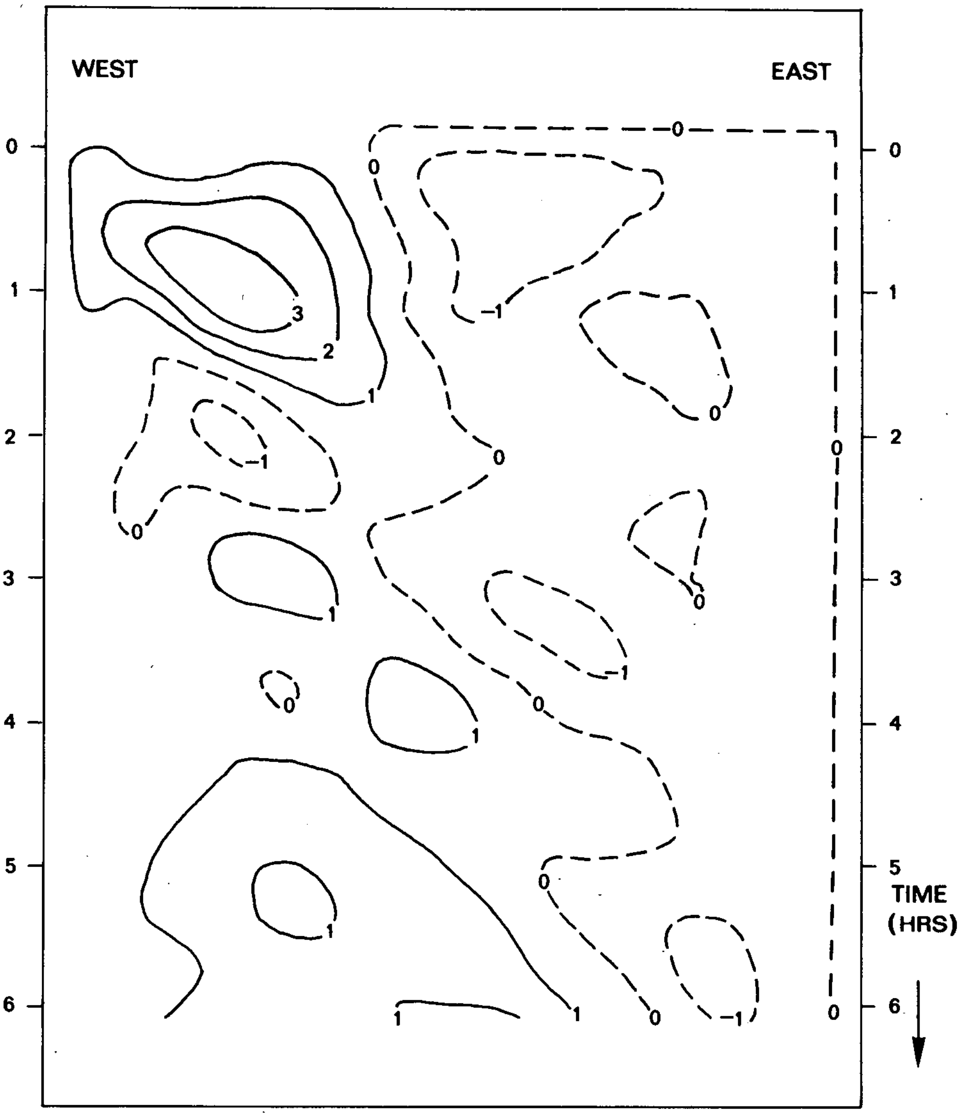

FIG. 3. As in Fig. 2, but after initialization with the method of section 2 of the external and first internal mode, with two iterations.

zonal wavenumbers $k = 0, \cdots, (M+1)/2$ and meridional wavenumber $l = 1$ (for a given value of k: largest meridional scale, smallest value of $\alpha_{kl}^2 d$).

In that case $\epsilon_{kl}[k = 1, \cdots, (M-1)/2]$ is as large as possible and so is the difference between $(\sigma_{klr} + \epsilon_{kl})^2$ and $\alpha_{kl}^2 d + \bar{f}^2 (k = 1, \cdots, (M-1)/2\,; r = 2, 3)$. The parameters used have the following values: the geopotential heights are $d = 91932.53$, $12478.39\ \mathrm{m^2\,s^{-2}}$ respectively, the angular velocity $\Omega = 7.29 \times 10^{-5}\ \mathrm{s^{-1}}$, the radius of the earth $r = 6367 \times 10^3$ m, $M = 21$, $N = 20$ and $\bar{\theta} = 55\frac{1}{2}°$N.

In order to investigate the effect of the change of the normal modes due to the linearization more accurately (Errico 1984), let us consider the projections of the $\mathbf{R}_{kl1}$ and $\mathbf{G}_{klr}$ $(r = 2, 3)$ modes onto the modified gravity modes $\mathbf{P}_{klr}$ $(r = 2, 3)$. We note that only projections of modes with corresponding zonal and meridional wavenumbers are nonzero. Using the scalar product, defined in section 2, we compute the coefficients $\langle \mathbf{R}_{kl1}, \mathbf{P}_{klr} \rangle$, $\langle \mathbf{G}_{kl2}, \mathbf{P}_{klr} \rangle$ and $\langle \mathbf{G}_{kl3}, \mathbf{P}_{klr} \rangle$ $(r = 2, 3)$ for the external and first internal mode, zonal wavenumbers $k = 1, (M-1)/2$ and meridional wavenumber $l = 1$. We note again that this value of the meridional wave number gives the most unfavorable result for a given value of k. We start with $k = 1$. For the external mode the coefficients are 5.4811×10^{-4}, 1.0000 and -1.2812×10^{-3} for $r = 2$ and -5.4683×10^{-4}, 1.2815×10^{-3} and 1.0000 for $r = 3$. For the first internal mode these

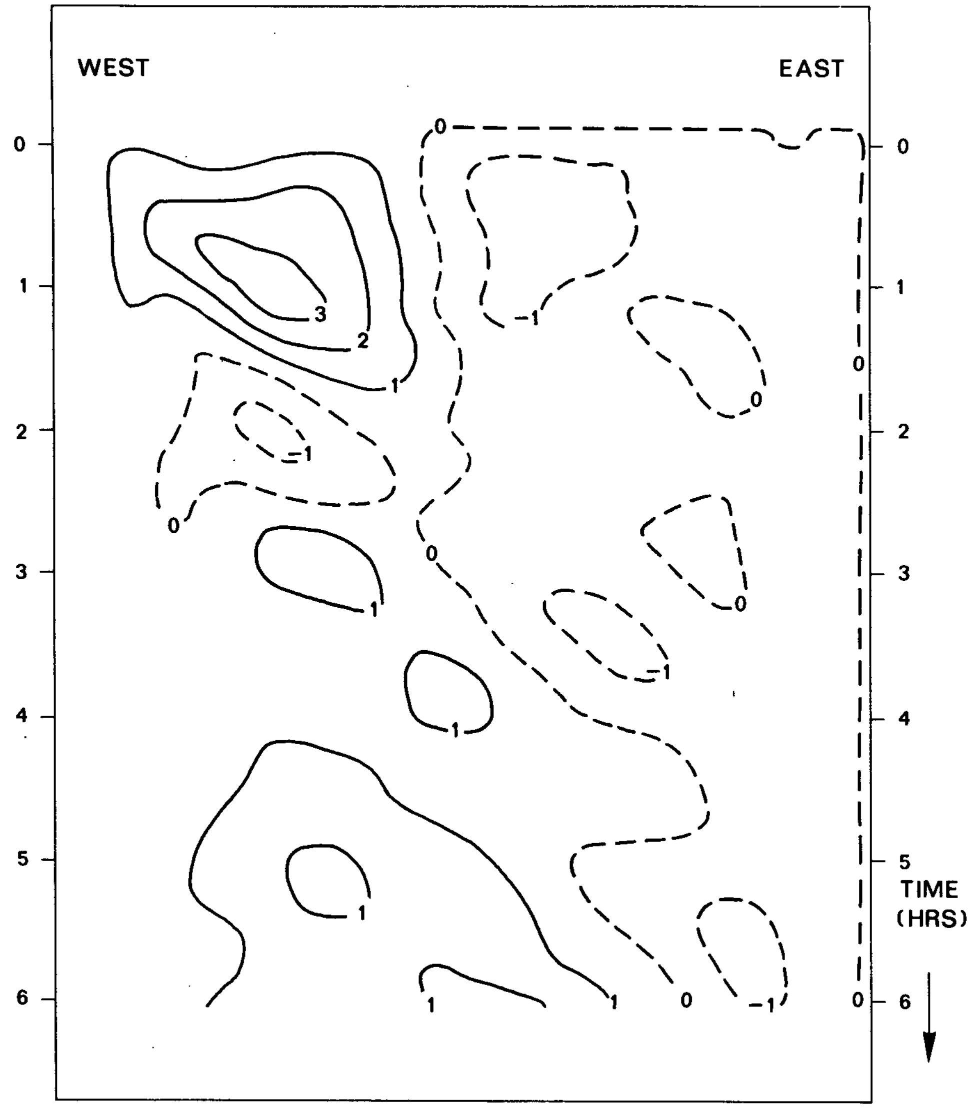

FIG. 4. As in Fig. 3, but after normal mode initialization with constant Coriolis parameter of the external and first internal mode, with two iterations.

numbers are 3.3078×10^{-3}, 9.9999×10^{-1}, -2.8442×10^{-3} and -3.3016×10^{-3}, 2.8552×10^{-3}, 9.9999×10^{-1} respectively.

For $k = (M - 1)/2$ the coefficients are 7.1531×10^{-7}, 1.0000, -8.8781×10^{-6} and -7.1530×10^{-7}, 8.8781×10^{-6}, 1.0000 (external mode; $r = 2, 3$) and 5.2294×10^{-6}, 1.0000, -2.3912×10^{-5} and -5.2292×10^{-6}, 2.3912×10^{-5}, 1.0000 (first internal mode; $r = 2, 3$).

These results show that the gravity wave expansion coefficients of a field $\hat{\eta}(m, n)$ expanded in the normal modes of the f-plane approximation are very similar to those obtained from the beta-plane approximation. In fact, for the faster gravity modes these coefficients are nearly identical.

This applies in particular to the time tendency field. Let $\eta = \eta_0 + \hat{\eta}$ obtained after two nonlinear iterations, be the initialized field in the f-plane approximation, so that the gravity wave expansion coefficients ($\dot{\gamma}_{klr}$; $r = 2, 3$) of the time tendency field in the f-plane approximation have certain small nonzero values. Then, in view of the values of the scalar products of the normal modes of the two approximations, we may expect that these coefficients are very similar (for slow gravity waves) or nearly identical (for fast gravity waves) in the beta-plane approximation. This means that the field η is also an initialized field in the sense of the beta-plane approximation. Further, the initialization result is affected by the number of initialized vertical modes (one or two) rather than by the order of smallness of the coefficients $\dot{\gamma}_{klr}$, $r = 2, 3$ (two or three nonlinear iterations).

Therefore, we may conclude that the initialization results resulting from the two approximations must be almost the same, if the same number of vertical modes is initialized.

Figure 1 gives the longitude–time diagram for the first 6 hours of the limited area model run, showing the gravity wave pattern before initialization.

In Fig. 2 the longitude–time diagram is shown after initialization of the first vertical mode with two iterations using the initialization method of section 2. Figure 3 shows results of this method after initialization of the first two vertical modes with two iterations. The diagram for normal mode initialization with constant Coriolis parameter, after initialization of the first two vertical modes with two iterations, is shown in Fig. 4.

From these experiments we may conclude that the method described in section 2 is almost identical to the nonlinear normal mode method with constant Coriolis parameter in its effect on the initial field and in its success in suppressing noise in the early forecast. This result is consistent with results of Lynch (1985, section 4).

4. Conclusions

A nonlinear normal mode initialization method is applied to a baroclinic limited area forecast model, in the case that specific beta terms are included in the linearized model equations, so that the Rossby modes are nonstationary.

A result of the initialization method has been compared with a result using nonlinear normal mode initialization with a constant Coriolis parameter in this linear system, leading to stationary Rossby modes.

These results show that the inclusion of the beta terms in the linearized model equations has no significant effect upon the development of the fields during the initial forecast hours.

This is in agreement with the results of section 3, where we showed that the initialized field in the case of the midlatitude beta-plane approximation is very similar to that obtained from the midlatitude f-plane approximation.

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