The Environment of Hurricane Debby (1982). Part II: Thermodynamic Fields

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ABSTRACT

A three-dimensional analysis of temperature and relative humidity in the environment of Hurricane Debby (1982) has been completed. Observations from Omega dropwindsondes (ODWs) within 1000 km of the storm have been combined with rawinsondes over the continental United States and the Caribbean and with observations from surface ships and aircraft data where possible.

The temperature and relative humidity analyses, together with wind analyses from a previous study, form a dataset that can be used as an initial condition in a multilevel prognostic model when combined with analyses over areas larger than our analysis domain. In this paper a series of diagnostic tests has been applied to the dataset to evaluate its performance without using a prognostic model. These tests include horizontal maps of the moist convective instability, calculation of the heat and moisture budgets in the vicinity of Bermuda, which was 350 km to the northeast of the storm center, and diagnosis of precipitation from these budgets and from the Arakawa–Schubert cumulus parameterization.

Results show that the horizontal distribution of moist convective instability is strongly affected by the low-level moisture field upstream of the main inflow region to the storm. The total surface heat flux, estimated with a bulk aerodynamic method, matches the vertically integrated eddy flux of moist static energy to within observational errors. Precipitation estimates from the budgets give rates of approximately 20 mm day\(^{-1}\), which are consistent with an estimated rate from radar. Partition of the rainfall rate into convective scale and resolvable scale (stratiform) shows about equal contributions.

Our results lead us to believe that, within the limitations determined by the horizontal distribution of the observations, the final dataset for Hurricane Debby provides a realistic depiction of the various physical processes that were occurring in Debby's environment. Future work will include data sensitivity experiments with a three-dimensional forecast model.

1. Introduction

Insufficient observations in the hurricane environment and inadequate initial analysis of these observations are widely accepted factors limiting the performance of hurricane track forecast models. If official track forecasts are to improve substantially, model guidance for these forecasts must lead the way by showing consistently smaller forecast errors.

As an initial effort to improve hurricane track forecasts by increasing data availability in the mid- and lower troposphere, the Hurricane Research Division (HRD) of the National Oceanic and Atmospheric Administration/Atlantic Oceanographic and Meteorological Laboratory (NOAA/AOML) flew research missions on 15–16 September 1982 to measure the synoptic-scale wind and thermodynamic fields in the environment of Hurricane Debby. At this time Debby was located roughly 350 km southwest of Bermuda, moving to the north-northeast at about 10 m s\(^{-1}\). United States Air Force (USAF) reconnaissance reports at the time of the HRD flights indicated a minimum central pressure of 968 mb and maximum flight-level (700 mb) winds of about 50 m s\(^{-1}\) (Clark et al. 1983).

Omega dropwindsondes (ODWs) were used during the experiment to obtain soundings of temperature, humidity, and winds from midtropospheric levels to the surface. These data were transmitted to the National Hurricane Center (NHC) and the National Meteorological Center (NMC) in real time, used operationally in the SANBAR hurricane track prediction model (Goldenberg et al. 1985), and postprocessed for research purposes at HRD. Lord and Franklin (1987, hereafter LF87), using a nested-grid technique, analyzed ODW wind observation combined with data from rawinsondes over the continental United States and Caribbean, surface ships, USAF reconnaissance aircraft, and the VISSR Atmospheric Sounder (VAS) to produce three-dimensional wind fields. Their results showed some dynamical consistency between the rotational wind fields and the storm motion at the analysis time using a vorticity advection model. Furthermore, forecasts with their deep-layer mean winds in

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the SANBAR model had substantially reduced short-
range (12–36 h) track errors.

In this paper, which is a sequel to LF87, we complete
the description of Debby’s environment by presenting
the analyzed temperature and relative humidity fields.
These analyses, when combined with the wind analyses
of LF87, form a dataset that can be used to initialize
a multilevel prognostic model in the region of the
storm. It is therefore of considerable interest to evaluate
the physical content of the entire dataset before its in-
sertion into the prognostic model. To this end, we apply
a series of diagnostic tests that measure the physical
consistency of the analyzed wind, temperature, and
moisture fields. The results of these diagnostics may
be used to interpret the eventual results of using the
analyzed fields in a prognostic model.

Since the observed wind and thermodynamic vari-
ables have very different distributions both horizontally
and vertically, and since the observations were analyzed
univariately and with no physical constraints, there is
no reason a priori to expect the same physical consis-
tency in the analyzed fields as indicated in the raw
observations or from theoretical considerations. Two
equations will illustrate this point, the first being some-
what hypothetical in nature, and the second more real-
istic. First, assume that the observations indicate an
atmosphere in geostrophic balance and that the obser-
vations are error free and sufficiently abundant that
the analysis is perfect; i.e., it represents the observations
faithfully on all observed scales. The analyses would
then show a vertical wind shear proportional to the
horizontal temperature gradient. However, if the hori-
Zontal and vertical filters used in our analysis do not
preserve this relationship or if there are distortions
caused by erroneous or insufficient observations, these
factors will create an inconsistency between the wind
and mass fields and the geostrophic balance will be
destroyed. As a second example, if the divergent wind
and thermodynamic fields are inconsistent, diagnosed
precipitation may not agree with observed cloud cover.

Our primary interest in this paper is the interaction
and consistency of the analyzed divergent wind and
thermodynamic fields, since the dynamical consistency
of the rotational winds has been assessed in LF87.
Therefore, three diagnostic tests are applied to the an-
alyzed wind and thermodynamic fields. First, the hori-
Zontal distribution of moist convective instability is
calculated over the analysis domain. We show that the
values are in a typical range given by other studies and
that the horizontal distribution of low-level moisture
has a strong control over the distribution of convective
instability. Second, heat and moisture budgets are cal-
culated in the region of Bermuda, where local changes
in temperature and moisture may be estimated. Here,
we show that the vertically integrated eddy flux of total
heat agrees with the total surface heat flux within ob-
servational error. Third, we apply the Arakawa–
Schubert cumulus parameterization over the analysis
domain and find that diagnosed precipitation rates are
reasonably distributed geographically and have mag-
nitudes similar to those of area-averaged rates in the
outer rain areas of hurricanes. The results of these di-
gnostic tests give strong evidence that, within the lim-
itations determined by the horizontal distribution of
the observations, the final dataset for Hurricane Debby
provides a realistic depiction of some of the various
physical processes that were occurring in Debby’s en-
vironment.

At the time of our analysis, Hurricane Debby was
embedded in a synoptic-scale trough and appeared to
be in a transitional state between a tropical system and
an extratropical system. Therefore, the results of our
thermodynamic budget calculations should not be
taken as representative of a classical tropical cyclone.
While analysis of the processes leading to the extra-
tropical system is of considerable interest, it is beyond
the scope of this paper and will be left for further study.

In section 2 we summarize the data processing and
analysis procedures and compare some observed and
analyzed soundings to summarize the effects of vertical
filtering in the vertical cross-section analysis. In section
3 we review the synoptic situation at the analysis time
with the aid of radar depictions of Debby’s environ-
ment that were not shown in LF87, review some wind
fields from that paper, and present some examples of
the analyzed temperature and relative humidity fields.
Results of the diagnostic calculations for moist con-
vective instability, thermodynamic budgets, and the
precipitation rates are discussed in sections 4–6, re-
spectively.

2. The analysis

a. Preliminary processing

Preliminary processing procedures for all observa-
tions have been described in detail by LF87. In brief,
al soundings (ODWs, rawinsondes) are processed with
a digital low-pass filter with a half power wavelength
of 100 mb to prevent aliasing of vertically small-scale
fluctuations into the final soundings sampled at 50 mb
intervals. Flight-level data from the WP-3D aircraft are
subjected to a low-pass filter and sampled at approxi-
mately 70 km intervals. All suspicious data are removed
subjectively as described in the Appendix of LF87.

b. Analysis method

The analysis method has been described in detail by
LF87; it employs a two-dimensional, least-squares fit-
ing algorithm combined with a derivative constraint
term that acts as a spatial low-pass filter on the analyzed
field (Ooyama 1987). The filter cutoff is user specified
to remove observational noise while retaining infor-
mation in the scales resolvable by the data distribution.
The analyzed field is presented continuously through-
out the domain as a bilinear combination of local basis
functions (cubic splines) centered at a two-dimensional array of nodal points at 1° lat-long intervals. Homogeneous, analytically defined boundary conditions allow extrapolation of the analysis to the borders of a limited domain. The analysis domain may be divided into nested subregions (LF87) if the data density permits analysis on smaller scales closer to the storm center. Within each nested domain, observations are analyzed univariately in the horizontal at each level and then vertically in cross sections using the gridded horizontal analyses as input. Temperature is converted to potential temperature for the vertical cross-section analysis to facilitate diagnosis of superadiabatic lapse rates. Interjection of samples from the vertically analyzed latitudinal and longitudinal cross sections into a second set of horizontal analyses results in three-dimensional continuity of the final analyzed fields. There are no dynamical constraints applied in this analysis algorithm.

c. Analysis domain and parameters

As in LF87, the horizontal analysis domain is 16.5°–38.5°N and 60.5°–83.5°W. There are 19 vertical levels: 18 constant pressure surfaces at 50 mb intervals from 100 to 950 mb, and the surface. LF87 used three horizontally nested analysis regions to allow resolution of smaller scales in the region of more dense ODW coverage near the hurricane. For the humidity fields analyzed here we use two nested regions, the inner region with boundaries at 21.5° and 38.5°N, 60.5° and 74.5°W. These boundaries allow increased resolution of the sharp moisture gradients in the near-storm environment. The nesting procedure was unnecessary for temperature due to the relatively weak observed horizontal gradients. The distribution of ODWs and rawinsondes used in the analysis is shown in Fig. 1. Within 200 km (2° latitude) of Debby’s center, all analyses (winds, temperature, and relative humidity) should be considered unreliable due to deficiencies in the data coverage. For this reason we confine our discussions to Debby’s environment outside the 200 km radius. For all variables the cutoff wavelength \( l_e \) for outer nested domain was 6° latitude (LF87); the inner nest had \( l_e = 4° \) for relative humidity. For the vertical cross-section analysis, \( l_e = 300 \) mb for winds, and \( l_e = 200 \) mb for temperature and relative humidity.

d. Comparison of analyzed and observed soundings

Figure 2 presents two examples of analyzed and observed soundings from the Debby environment. Filtering in the cross section analysis removes rapid vertical changes such as those commonly seen in moisture data (Fig. 2a). The analysis (solid line) does not represent the observed dry layer just above 850 mb or the moist layer at 650 mb (dashed lines); however, the general nature of the sounding is well preserved. Horizontal filtering may also result in the analysis sounding differing from the observed sounding, as illustrated at Bermuda (Fig. 2b). Above 400 mb, where there is very little data other than the rawinsonde, the analysis (solid line) and observation (dashed) agree almost exactly. The divergence of the two soundings near 500 mb results from reports of near saturation from the two ODWs on either side of Bermuda. When features are of broad scale both horizontally and vertically, as they are throughout most of the analysis domain, the analysis tends to represent them well.

3. Hurricane Debby on 16 September 1982

Time composited reflectivity maps from the NOAA Office of Aircraft Operations (NOAA/OAO) WP-3D airborne radar over approximately a 4 hour period centered on 16 September 0000 UTC (Fig. 3), analyzed winds from LF87 (Fig. 4), and some examples of the analyzed temperature and relative humidity fields are presented here for orientation and reference in the following sections. The NOAA/OAO aircraft passed within \(~55\) km of Bermuda, traveling eastward at \(~7400\) m height (Fig. 1). Reflectivity data from the lower fuselage radar (Jorgensen 1984) were composited as described in Marks (1985). The large-scale composite (Fig. 3a) shows the large convective band to the east and southeast of the storm center and convective features to the north and northeast. Bermuda is located in a region of relatively uniform horizontal reflectivity.
September, 24 h after this analysis time (Burpee et al. 1984). Large-scale anticyclonic curvature can be found along the U.S. East Coast, as well as north and east of Puerto Rico. The low-level circulation of Debby (Fig. 4b) is better defined than at 500 mb, but is still embedded in generally southerly flow east of the trough axis. Convection southeast and east of the storm and, in particular, the convection band near 27°N, 65°W, lies generally along a confl uence line between westerly flow from behind the trough and southwesterlies curving anticyclonically north of Puerto Rico.

The 900 mb temperature analysis (Fig. 5a) shows cool air extending from 70°W at the northern domain boundary to just off the northern Florida coast. This cool region is related primarily to a shallow pool of cold air confined to the lowest 100 mb, but is also associated with the synoptic-scale trough. At 700 mb (Fig. 5b), the coolest air is found underneath the cloud cover to the east and northeast of Debby. Both the north-south temperature gradient east of the storm and the poleward increase of relative humidity in the same region (Fig. 6b, below) suggest that evaporative cooling from precipitation is an important thermodynamic process here.

The thermodynamic signature of the cutoff low is very weak at 700 mb, being visible only as an extension of the 6°C contour to the northwest away from the major region of cloudiness (Fig. 5b). Between 600 and 450 mb (not shown), the analyses do show a distinct temperature minimum located near 37°N, 67°W associated with the cutoff low. By 400 mb, though (Fig. 5c), the thermodynamic signature of the cutoff has disappeared, as a new pocket of cold, very dry air has developed west of Debby. The latter feature is well documented by ODW, NOAA/OAR WP-3D aircraft, and USAF observations. Although data above this level are scarce, the cold pocket appears to be no more than 100 mb deep. Its origin and significance could not be determined from our analyses. This relatively incoherent vertical distribution of temperature probably contributes to the broad and somewhat ragged appearance of the large-scale trough (Fig. 4a).

The low-level relative humidity (RH) field (Fig. 6a) shows a moisture maximum near 26°N, 70°W—upwind of the convective bands southeast of the hurricane (Fig. 3a). Dry air can be found along the East Coast of the United States and in a tongue extending north-northeast of Puerto Rico into the cloud-free area east of 65°W and south of 30°N. The 700 mb level (Fig. 6b) is characterized by strong humidity gradients throughout the domain. The driest air at both levels is seen to have recent histories within anticyclonic circulations presumably accompanied by large-scale descent. The vertical velocity fields shown in LF87 Fig. 10, do, in fact, show descent north-northeast of Puerto Rico. Rising motion, however, is diagnosed in the dry air just off the U.S. East Coast. This discrepancy is discussed in LF87 and below (section 6). The highest

with several convective features nearby, the largest of which is approximately 100 km to the northeast (Fig. 3b). Due to the large distance between the aircraft and the radar echoes, and the high flight altitude, the radar depiction should significantly underestimate the intensity of convective-scale echoes.

The 500 mb wind field (Fig. 4a) shows Debby as a small circulation located slightly east of a synoptic-scale trough axis. A cyclonic (cutoff low) circulation to the north-northwest of Debby is centered at 36°N, 68.9°W. This circulation was hypothesized to play an important role in Debby’s abrupt slowdown on 17
values of relative humidity can be found near Debby, the cutoff low, and in a cloudy region over Cuba and Jamaica.

The 400 mb RH field (Fig. 6c) shows maximum values in the cloud cover to the east of Debby and in the cloudy region south of Cuba. The very dry air just northwest of Debby is consistent with an analyzed region of strong descent in the middle troposphere (LF87 Fig. 10). In general, the distribution of RH is consistent with the observed horizontal distribution of cloud cover from the satellite.

A height–latitude cross section of RH along 65.5°W (2° east of the hurricane center) is shown in Fig. 7a. Results above 400 mb are, in general, not reliable due to a lack of thermodynamic data at these levels over the ocean. However, this cross section passes very close to the Bermuda rawinsonde (32.4°N, 64.7°W), which had humidity data up to 300 mb, so that results should be reliable in this neighborhood. The flow is generally along this section from south to north. It should be noted that ODWs may tend to overestimate relative humidity in high-moisture environments, particularly at lower levels. This is probably due to the thermal lag of the ODW hygrostat causing condensation on the sensor element. No attempt has been made to correct for this measurement bias.

A layer of moist air at ~500 mb extends northward from the southernmost part of the trailing convective band (25°N) to the north edge of the analysis domain (see Figs. 3 and 4 for the radar depiction and a horizontal representation of the cloud cover). This layer becomes thicker along the northward flow until it almost reaches the surface at 36°N. The RH maximum above 400 mb from 30°–32°N suggests the presence of moist outflow from the hurricane center, although there are no observations indicating a saturated cirrus layer in the upper troposphere. However, since this cross section is parallel to southerly flow passing through the band, and is intersected by upper level streamlines that pass through or very near Debby’s
center (not shown), outflow of rain and precipitating ice from the storm may make substantial contributions to the high moisture content in this region. South of 20°N, and from 400–800 mb, the air is very dry with RH < 30% throughout. Vertical velocities diagnosed from the analyzed wind fields (LF87, Fig. 10) indicate subsidence throughout the troposphere to the east of this cross section.

Below 800 mb, the lower tropospheric moist layer becomes deeper and more moist with increasing latitude from the southern boundary of the domain up to the latitude of the storm (30°N). This trend is seen in all height–latitude RH cross sections east of 72°W. Inspection of height–longitude RH cross sections south of 30°N shows an analogous west-to-east RH increase. The section at 28.5°N is typical (Fig. 7b). Together, these cross sections show the large horizontal extent of moisture accumulation at low levels by the generally west-southwesterly flow south of Debby at ~27°N. Bearing in mind that the analyzed winds are undoubtedly underestimated within 300 km of Debby’s center, we hypothesize that this flow is overtaking the storm from the southeast and providing a moisture source to the low-level circulation. This source was probably becoming less effective as Debby was accelerating northward under the influence of the trough. Evaluation of the advective terms in the moisture budget (4, below) in this region shows that both horizontal and vertical advection make significant positive contributions to this moisture accumulation within a radius of ~800 km of the storm center. To the north and northwest of the storm, subsidence (LF87, Fig. 10) behind the trough is acting to dry the middle troposphere.

4. Horizontal distribution of moist convective instability

In this section we investigate the vertical thermodynamical structure of Debby’s environment by calculating the horizontal distribution of moist convective instability. Following the procedures of Lord and Arakawa (1980) and Arakawa and Chen (1986), we use the Arakawa-Chubert cloud work function \( A(\lambda) \) as a measure of the most convective instability. Here \( \lambda \) is a parameter that identifies a particular cumulus subensemble; we use \( \lambda = \beta \), where \( \beta \) is the cloud top (de-trainment) pressure. When virtual temperature effects are neglected, the cloud work function is given by

\[
A(\lambda) = g \int_{z_B}^{\hat{z}(\lambda)} \eta(z, \lambda) \frac{T_c(z, \lambda) - T(z)}{T(z)} \, dz, \quad (1)
\]

where \( z_B \) and \( \hat{z}(\lambda) \) are the cloud base and cloud top heights, \( \eta(z, \lambda) \) is the cloud subensemble mass flux normalized at cloud base, and \( T_c(z, \lambda) \) and \( T(z) \) are
temperature and moisture fields. These correlated changes tend to maintain a relatively constant value of moist convective instability and result primarily from the interaction of cumulus convection and the large-scale forcing of clouds by various physical processes (Arakawa and Schubert 1974). For example, using datasets from the tropics and subtropics, Lord and Arakawa (1980) showed that time and spatially averaged cloud work functions fall into a well-defined, narrow range for each cloud subensemble even though the thermodynamic vertical structures for each dataset are quite different. The cloud work function is an extremely sensitive measure of moist convective instability. Lord and Arakawa have shown that changes of 10% in relative humidity more than double cloud work function values for all subensembles. Therefore, comparing the moist convective instability in Debby's environment with values from other datasets provides a good test for the analyzed thermodynamic fields. Moreover, since positive values of the cloud work function are a necessary condition for diagnosed precipitation from the Arakawa–Schubert cumulus parameterization (see section 6), it is interesting to compare the horizontal distribution of positive cloud work function values with the observed cloud coverage. From analyzed temperature and moisture at $1^\circ$ lat./long. intervals over the analysis domain and the method of Lord and Arakawa (1980), we calculated cloud work functions for various values of $\tilde{p}$ using cloud base values of $z_B = z_{950}$ and $h_M = h_{950}$, where the subscript 950 refers to values at $p = 950$ mb, the moist static energy $h = C_p T + gz + L_q$, and other notations are standard.

The horizontal distribution of $A(\tilde{p})$ for deep clouds with $\tilde{p} = 375$ mb (Fig. 8) shows a distinct region of maximum $A$ at $26.5^\circ$N, $68.5^\circ$W, immediately upstream of the inflow region for the convective band. This maximum will be discussed in more detail later in this section. A second local maximum is on the western edge of the cloudy area at $38^\circ$N, $67^\circ$W; it derives from the cool, almost saturated, midtropospheric air in that region (Figs. 5 and 6). The highly stable area over the eastern United States results from the cool low-level air behind the trough (Fig. 5a); the stable area over central Florida results from dry air in the anticyclonic flow along the whole east coast (Fig. 6a).

The stable region north of Debby's center derives from a small area of cool and dry air below 750 mb (not shown), which results from subsiding air behind the trough that is being advected southward.

Statistics of $A(\tilde{p})$ for all $\tilde{p}$ over the analysis domain are shown in Fig. 9. Also shown are statistics from twice daily spatially averaged soundings over the Florida peninsula (Burpee and Lahiff 1984) during the summer sea-breeze regime (June–September 1973–1976; Lord 1987, unpublished results) and results from Marshall Islands data and VIMHEX data (Lord and Arakawa 1980). The mean work function values appear to be scale dependent; the smallest mean values belong to the largest scale observations (Marshall Is-
lands) for each cloud type, and the smallest scale observations (VIMHEX, Florida sea breeze and the Debby environment) have larger mean values. We note that the results from the Debby environment are not very different than those from the other highly convective regimes.

As shown by Arakawa and Chen (1986), the cloud work function depends on both the low level moisture distribution and the lapse-rate between the lower and middle troposphere. We show how these two factors contribute to the horizontal changes of the cloud work function south of Debby's center as follows. Following Arakawa and Chen, we rewrite the cloud work function for the simple nonentraining case ($\lambda = 0$) in terms of moist static energy $h$, and saturation moist static energy $h^* = s + Lq^*$, where $s = C_p T + g z$ is the dry static energy, $q$ is the mixing ratio, and $(\quad)^*$ denotes a saturation value,

$$ A(0) = \int_{z_b}^{z_0} \beta(z) \left( h_M - h^*_M - \int_{z_{b*}}^{z} \frac{\partial h^*(z')}{\partial z'} dz' \right) dz $$

$$ = \int_{z_b}^{z_0} \beta(z) (h_M - h^*(z)) dz. \quad (2a) $$

FIG. 5. Temperature analysis at (a) 900, (b) 700 and (c) 400 mb. The contour interval is 1 K.
Figure 6. Relative humidity analysis at (a) 900, (b) 700 and (c) 400 mb. The contour interval is 10%.

where $\beta(z) = C_p T(z) / [1 + L / C_p (\partial q^* / \partial T)_p]$ is a slowly varying function of height and the subscript $B+$ indicates a level just above the boundary layer $z = z_B$. Since we use a cloud base value $h_M = h_{950}$, we rewrite (2a) as

$$A(0) = \int_{z_B}^{z(0)} \beta(z)(R_{950} + S(z)) dz,$$

where $R_{950} = h_{950} - h^*_{950} = -Lq^*_{950}(1 - RH_{950})$, $RH_{950} = q_{950} / q^*_{950}$ is the 950 mb relative humidity and $S(z) = h_{950} - h^*(z)$. Note that $R_{950} \leq 0$ is constant with height and approaches zero as the 950 mb relative humidity approaches 100%. Here $S(z)$ is related to the vertically integrated lapse rate between cloud base and the level $z$ (Arakawa and Chen 1986).

Figure 10 shows $R_{950}$ and $S(z)$ for $z = 5850$ m, or approximately 500 mb, along 26.5°N. At this latitude the maximum cloud work function is at 68.5°W, which corresponds to the maximum value of $R_{950}$. Note that lapse rate changes are very small east of 70.5°W, so that low level moisture increases are directly responsible for the increase of the cloud work function in this region. West of 70.5°W the decrease of $S$ reflects stabili-
and

$$Q_2 = -L \left( \frac{\partial \tilde{q}}{\partial t} + \tilde{\mathbf{V}} \cdot \nabla \tilde{q} + \frac{\tilde{\omega}}{\omega} \frac{\partial \tilde{q}}{\partial p} \right)$$

$$= L(c - e) + L \frac{\partial}{\partial p} (q' \tilde{\omega}),$$  \hspace{1cm} (4)

where \( \mathbf{V} \) is the horizontal wind, \( \tilde{\omega} \) the vertical \( p \)-velocity, \( Q_R \) the radiative heating, \( c \) and \( e \) the condensation and evaporation rates per unit mass of air, \( L \) the latent heat of condensation, \( (\quad) \) denotes a horizontal average and primes denote a deviation therefrom. From (3) and (4), a vertical eddy flux of moist static energy may be defined as

$$F(p) = -\frac{1}{g} \tilde{H} \tilde{\omega} = \int_p^{p_r} (Q_1 - Q_2 - Q_R) \frac{dp}{g},$$ \hspace{1cm} (5)

where we assume that eddy fluxes are confined below the upper boundary, \( p_r = 100 \) mb. It is easily shown (Yanai et al. 1973) that the eddy flux of moist static energy vertically integrated over the entire atmospheric column must be in balance with the surface flux of total heat,

$$F(p_0) = S_0 + LE_0,$$ \hspace{1cm} (6)

where \( p_0 \) is the surface pressure, and \( S_0 \) and \( E_0 \) are the surface sensible heat flux and evaporation respectively.

Although the local time changes, \( \partial \tilde{s}/\partial t \) and \( \partial \tilde{q}/\partial t \), in (3) and (4) are unknown over most of the analysis.

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5. Thermodynamic budgets near Bermuda

The apparent heat source and moisture sink \( (Q_1, Q_2, \text{ Yanai et al. 1973; Johnson 1984; and many others}) \) are defined by

$$Q_1 = \frac{\partial \tilde{s}}{\partial t} + \nabla \cdot \tilde{\mathbf{S}} + \frac{\tilde{\omega}}{\omega} \frac{\partial \tilde{s}}{\partial p}$$

$$= Q_R + L(c - e) - \frac{\partial}{\partial p} (\tilde{s}' \tilde{\omega})$$ \hspace{1cm} (3)

Fig. 7. Height–latitude cross section of relative humidity at (a) 65.5°W, and height–longitude section at (b) 28.5°N. In each panel, the contour interval is 10%. Values greater than 90% are stippled.

Fig. 8. Horizontal distribution of the Arakawa–Schubert cloud work function (J kg\(^{-1}\)) for clouds detraining at \( \tilde{\beta} = 375 \) mb. The contour interval is 150 J kg\(^{-1}\). The zero contour (heavy solid line) marks regions where there is no net buoyancy for these clouds.
Fig. 9. Statistical distributions of the cloud work function for all detrainment levels (P). Mean values are denoted by the symbols and ±1 standard deviation are denoted by error bars. Distributions for Marshall Islands are VIMHEX data are at P = 150, 200, ..., 800 mb, those for Hurricane Debby and the Florida sea breeze are at P = 175, 225, ..., 825 mb. See text and Lord and Arakawa (1980) for procedural details.

domain, the complete Q1 and Q2 may be determined near Bermuda (32.4°N, 64.7°W) from soundings at 1800 UTC 15 September and 0600 UTC 16 September,

\[
\frac{\partial \bar{s}}{\partial t} = C_p \left( \frac{T_s - T_a}{12h} \right)
\]

and

\[
\frac{\partial \bar{q}}{\partial t} = \left( \frac{q_s - q_a}{12h} \right),
\]

where the subscripts refer to the times relative to the analysis time of 0000 UTC 16 September. Calculations of the remaining terms for Q1 and Q2 were made from analyzed fields and their derivatives at 32.5°N, 64.5°W, the grid point nearest Bermuda. These values represent averages over a 1° lat-long square (Fig. 3b), which will be referred to as the Bermuda area. Crude estimates of the surface fluxes were made from the bulk aerodynamic method,

\[
S_0 = \rho A C_D |V_A|(T_s - T_a) \tag{7a}
\]

and

\[
E_0 = \rho A C_D |V_A|(q_s^* - q_a), \tag{7b}
\]

where \(\rho\) is density, \(C_D = 1.2 \times 10^{-3}\) is the drag coefficient, the asterisk denotes a saturation value, and the subscripts S and A refer to sea surface and surface air respectively. The sea surface temperature (SST) at Bermuda (27°C) was taken from the NOAA weekly SST analysis for 14 September 1982 and is considered accurate to ±0.5°C. We calculated \(S_0\) and \(E_0\) based on SSTs of 26.5°C and 27.0°C (Tables 1-2); uncertainties of order 0.5°C do not change the conclusions of this budget study substantially.

Longwave radiative heating \(Q_L\) was calculated with the commonly used broad-band emissivity method (see Stephens 1984) and the empirically fitted emissivity

Fig. 10. The two components of (2b), \(R_{\text{eff}}\) and \(S(z)\) at \(z = 5850\) m and 26.5°N (J kg\(^{-1}\)).
functions of Rodgers (1967), as implemented by Dudhia (1988, personal communication). Calculations were made for both a clear sky sounding \( (Q_R) \) and one with an assumed distribution of water \( (Q^w_R) \). The water content was derived from the lower fuselage airborne radar during the Debby flights (Fig. 3b) and the following empirical relationship between reflectivity (dBZ) and ice water content \( m \) (g m\(^{-3}\)) derived for the hurricane environment (Black 1990),

\[
\text{dBZ} = 17.9 \log_{10}(m) + 28.3. \tag{8}
\]

The estimated reflectivity in the Bermuda area was 16 dBZ (Fig. 3b), which gives \( m = 0.21 \) g m\(^{-3}\). This value should be considered an average value over a vertical slice from \( z = 5 \) km to \( z = 9 \) km that was covered by the radar beam (Marks 1988, personal communication). The cloud layer was defined from 375 to 675 mb, where the observed relative humidity from the Bermuda sounding exceeded 80% (Fig. 11). Above the 0°C isotherm (~600 mb), the water was treated as precipitating snow and below the water was treated as suspended liquid.

The eddy flux of moist static energy \( F(p) \) and the corresponding profile of total (sensible plus latent) heating by eddies from (3)–(5) are shown in Fig. 12. Both profiles are calculated with the clear sky radiative heating \( (Q_R) \) and the liquid water and ice profile \( (Q^w_R) \). In both cases \( F(p) \) is positive (upward flux), shows a rapid increase from the surface to 850 mb, and decreases to 450 mb. Above 450 mb, \( F(p) \) remains upward but is larger for the \( Q^w_R \) case to compensate for the strong cooling at cloud top. The estimated surface fluxes \( F(p_0) \) are 433 W m\(^{-2}\) (clear sky) and 357 W m\(^{-2}\) (liquid water and ice) which compare to \( S_t + LE_0 = 207 \) W m\(^{-2}\) for \( T_s = 26.5°C \) and 241 W m\(^{-2}\) for \( T_s = 27.0°C \) (Tables 1, 2). Thus, the existence of clouds reduces the estimated surface total heat flux by approximately 20% but does not change the overall shape of the flux profile below about 650 mb. As we shall see below, the discrepancy between the surface flux estimates from the vertically integrated \( Q_1 - Q_2 - Q_R \) budget and the independent bulk aerodynamic method 116–226 W m\(^{-2}\), is substantially less than any of the major components of either \( Q_1 \) or \( Q_2 \) and is probably the same order as errors in the individual components.

The strong divergent moist static energy flux below 850 mb indicates that eddies are removing heat from the boundary layer and below 850 mb and depositing it in the layer from 500–850 mb (Fig. 12b). The total heating is decomposed into its component parts \( Q_1, Q^w_R(Q^w_R) \) and \( Q_2 \) in Figs. 13 and 14. These figures show that the predominant heat transport at low levels is latent heat, which is transported upwards from below 900 mb to the 700–800 mb layer. From Fig. 11, we note the very moist layer at 900 mb and below and the relative humidity minimum at 700–800 mb. Thus, eddies are acting to redistribute moisture from relatively moist to relatively dry layers in the lower troposphere. Redistribution of sensible heat in the layer from 800 mb to the surface is very small compared to the moisture sink \( Q_2 \).

From (3)–(4) we note that mesoscale circulations are characterized by \( Q_1 - Q^w_R \approx Q_2 \approx Q_1 - Q_2 - Q_R \) (Cheng and Yanai 1989). Figures 12–14 show that mesoscale circulation appears to predominate over small scale eddies from 350–500 mb, which is the upper half of the nearly saturated region in the upper troposphere (Fig. 11). In the lower half of the region, small scale eddies appear to be significant contributors to both the latent and sensible heat budgets.

The vertically integrated heat source \( Q_1 \) (Table 1) is 495 W m\(^{-2}\) and is composed of positive contributions due to local warming (299 W m\(^{-2}\)) and vertical advection (708 W m\(^{-2}\)) and a negative contribution due to horizontal advection (−512 W m\(^{-2}\)). The radiative heating is −236 W m\(^{-2}\) for the clear sky case and −160 W m\(^{-2}\) for the case with liquid water and ice. The vertically integrated \( Q_2 \) (Table 2) shows strong local moistening (−602 W m\(^{-2}\)) which is opposed by drying from both horizontal and vertical advection (453 and 447 W m\(^{-2}\), respectively) to give a net moisture sink of 298 W m\(^{-2}\). The estimates for \( Q_2 \) should span the range of possible contributions by radiative heating. We note again that the discrepancies in the heat balance equations (5)–(7), 116–226 W m\(^{-2}\), are substantially less than any of the major components of either \( Q_1 \) or
$Q_2$ and are probably the same order as errors in the individual components. The close balance in the vertically integrated budgets gives confidence to our analysis of the fields, in particular the divergence field at all levels.

It is interesting to break down the vertical profiles of the $Q_1$ and $Q_2$ budgets into contributions by horizontal and vertical advection and the local time changes as presented in (3)-(4). While the total quantities $Q_1$ and $Q_2$ are, of course, invariant under a Galilean transformation, our interpretation is from the point of view of a stationary observer at 32.5°N, 64.5°W. The vertical distribution of $Q_1/C_P$ (Fig. 13) shows net warming throughout most of the troposphere with a maximum of 14 K day$^{-1}$ at 600–650 mb and a minimum at 850–900 mb of −2.5 K day$^{-1}$. Vertical advection and $Q_1$ are approximately equal in the middle troposphere, but only because the local change and horizontal advection are about equal and of opposite sign. Strong warm horizontal advection at low levels that is not completely compensated by vertical advection or local warming results in a net heat sink below 800 mb. The general shape of the $Q_1$ profile bears a strong resemblance to the mesoscale heating profile of Johnson (1984, Fig. 6) except that the levels of maximum heating and cooling are 100–150 mb lower in our case.

The vertical distribution of $Q_2/C_P$ (Fig. 14) shows a maximum moisture sink at low levels, a secondary maximum at 400–450 mb and a strong moisture source at 650–850 mb. The moisture sink at low levels is determined primarily by strong horizontal advection (28 K day$^{-1}$) but the local moistening is approximately one-third of that amount. At 700–800 mb, the moisture source is approximately equal to the local moistening and the horizontal and vertical advective contributions are approximately equal and opposite in sign. Above, both the horizontal and vertical advection contribute to the moisture sink and the local moistening is about 40% of each and of opposite sign. Our $Q_2$ profile strongly resembles the mesoscale drying profile of

### Table 1.
Components of the vertically integrated sensible heat budget ($Q_1$) derived from (3) and (7a). The respective contributions of the local time change, and horizontal and vertical advection are $L$, $H$, and $V$. Units are W m$^{-2}$, except the precipitation rate, $P_0$ (mm day$^{-1}$).

<table>
<thead>
<tr>
<th></th>
<th>$Q_1$</th>
<th>$L$</th>
<th>$H$</th>
<th>$V$</th>
<th>$Q_0$</th>
<th>$S_0$</th>
<th>$P_0$</th>
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<td>−512</td>
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<td></td>
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<td>21.6*</td>
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### Table 2.
Components of the vertically integrated latent heat budget ($Q_2$) derived from (4) and (7b). Notation and units as in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>$Q_2$</th>
<th>$L$</th>
<th>$H$</th>
<th>$V$</th>
<th>$E_0$</th>
<th>$P_0$</th>
</tr>
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<tbody>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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thermodynamic and vertical velocity fields. This cumulus parameterization has been used to diagnose precipitation rates from the wind and thermodynamic fields over the GATE A/B-scale area by Lord (1982) with reasonable agreement between rates from thermodynamic budgets and radar both in the time average and at individual observation times. Since the calculations for Debby are at a single observation time, and are produced from independent wind and thermodynamic analyses without any consideration of time continuity, they provide a stringent test for the physical realism of the vertical velocity fields (hence, the divergent component of the horizontal wind fields) and the vertical thermodynamic structure as illustrated in the above examples of the temperature and RH fields.

The Arakawa–Schubert precipitation rate at 32.5°N, 64.5°W, the grid point nearest Bermuda, is 9.7 mm day⁻¹ compared to the budget and radar estimates of ~20 mm day⁻¹. However, the possibility of “resolvable scale” (stratiform) rain must also be considered for the following reasons: 1) the observed dense overcast from satellite (Fig. 4); 2) the nearly saturated air from p2 = 400 mb to p1 = 650 mb of both the Bermuda sounding (Figs. 2b, 11) and the ODW sounding nearest Bermuda at 31.8°N, 64.0°W (2157 UTC, 15 September, not shown); and 3) the diagnosed upward vertical velocity (Fig. 10, LF87) in a broad region surrounding Bermuda. Therefore, we make a crude estimate of the resolvable scale rain rate Pr from

\[
P_r = \int_{p_1}^{p_2} \omega \frac{\partial \tilde{q}}{\partial p} \frac{dp}{g} = 9.8 \text{ (mm day}^{-1}) \tag{9}\]

6. Diagnosed precipitation rate

The precipitation rates at Bermuda diagnosed from the thermodynamic budgets (Tables 1–2) are 24.2 mm day⁻¹ (Q₁ - Q₄, clear sky radiation), 21.6 mm day⁻¹ (Q₁ - Q₈, liquid water and ice radiation), and 16.5 mm day⁻¹ (Q₂). An independent estimate from the airborne radar (Fig. 3b) using the average water content \( m = 0.23 \text{ g m}^{-3} \) and an average fall velocity of 1 m s⁻¹, typical of precipitating ice above the melting layer, gives a rate of 19.1 mm day⁻¹. This radar estimate should be considered as a lower bound on the actual rainfall rate due to the large distance between the aircraft and Bermuda, and the obviously convective nature of the distribution of radar echoes in the vicinity of Bermuda.

The close agreement between the budget-derived precipitation estimate and the radar estimates at Bermuda is probably fortuitous, given the scattered distribution of convective precipitation seen by the radar (Fig. 3b) and the crudity and uncertainty inherent in the budget estimates. However, we may calculate another precipitation estimate by applying the Arakawa–Schubert cumulus parameterization to the analyzed
Since the Bermuda lapse rate is somewhat more stable than moist adiabatic from $p = 500$ to $p = 650$ mb (6–8 km), (9) probably underestimates $P_r$. The contributions by convective scale rain (from the Arakawa–Schubert parameterization) and resolvable scale rain are approximately equal, and their total, 19.5 mm day$^{-1}$, is in good agreement with both the budget-derived rate and that from radar.

The distribution of cloud base mass flux for the above Arakawa–Schubert cumulus parameterization calculation (not shown) shows that only clouds with cloud tops $\hat{p}$ in the range $375 \leq \hat{p} \leq 475$ mb have positive mass flux. These cloud top values correspond well with the upper limits of the nearly saturated layer in the Bermuda sounding and our assumed cloud layer (Fig. 11).

Diagnosed precipitation rates over the analysis area (Fig. 15) show a maximum of $\sim 110$ mm day$^{-1}$ just southeast of the hurricane center. The displacement of the maximum rain area from Debby’s center is, as noted before, a direct consequence of insufficient observations. Typical rainband-averaged rainfall rates estimated from radar reflectivity data in hurricanes are 50–100 mm day$^{-1}$ (Burpee and Black 1989). In general, for the entire hurricane and trough circulation north of 25°N and east of 70°W, there is good agreement between the area of deep cloud indicated by satellite (Fig. 4), the observed radar reflectivity (Fig. 3a), and the diagnosed precipitation distribution Fig. 15.

Precipitation rates in the cloudy area north of 33°N and east of 67°W are much more uniform horizontally than in the area south and east of Debby. As discussed earlier in connection with the Bermuda sounding, the radar reflectivity in this area is characteristic of stratiform rain areas with embedded cumulus precipitation and mesoscale ascent in the middle to upper troposphere (Houze and Betts 1981; Marks and Houze 1987). Moreover, ODW soundings in this area show almost saturated air in the middle troposphere with lapse rates near moist adiabatic.

The diagnosed maximum in precipitation rate over the Bahamas has some verification in the observed cloud cover, but the rainy area extends too far north. This discrepancy is most likely a result of errors in the analyzed wind field that result in diagnosed upward motion all along the East Coast from North Carolina to South Florida. Note, however, that the parameterization does not show any rainfall in the cool, stably stratified northerslies west of the cutoff low (Fig. 8), even though there are upward vertical velocities in this area (LF87).

7. Conclusions

A three-dimensional analysis of thermodynamic data from ODWs in the environment of Hurricane Debby (1982) has been completed and various diagnostic calculations have been made on the results. Accumulation of low-level moisture over regions within $\sim 1000$ km from the hurricane center and upstream of the main convective features in the storm periphery is clearly indicated (Figs. 7, 10). There are considerable geographical variations in conditional instability in the hurricane environment that appear to be brought about by many processes including low-level moisture accumulation, low-level cooling and mid tropospheric moistening and cooling (Fig. 8).

Complete thermodynamic budgets ($Q_i$ and $Q_e$) have been calculated for the region near Bermuda, where the local thermodynamic changes could be estimated from six hourly rawinsonde soundings. Both budgets show an interesting vertical distribution of sources and sinks (Figs. 12–14). The vertically integrated budgets, together with estimates of radiative heating, agree with estimates of the total surface heat flux to well within observational errors. Several rainfall rate estimates from the $Q_i$ and $Q_e$ budgets and radar reflectivity data give values of $\sim 20$ mm day$^{-1}$. Using the Arakawa–Schubert cumulus parameterization, and estimates of the resolvable scale (stratiform) rainfall rate, we have shown that convective and stratiform rain contribute about equally to the total rate.

Diagnosis of precipitation rates over the analysis area using the Arakawa–Schubert cumulus parameterization gives quite reasonable agreement with the horizontal distribution of satellite-observed cloudiness. Maximum precipitation rates diagnosed near the storm
are in the range of representative radar estimates averaged over rainband scales. Since diagnosed rainfall rates from the Arakawa–Schubert cumulus parameterization depend critically on the vertical distribution of upward motion, our results lend some credence to the analyzed divergent wind component.

The large contributions by local temperature and moisture changes to the $Q_1$ and $Q_2$ budgets (Tables 1 and 2) pose a formidable problem for construction of reliable analyses of the hurricane environment and thermodynamic budgets. Experiments using aircraft typically last for ~9 h during which the steady state assumption implicit in our analysis method could not be expected to hold. Extreme care must be taken to ensure that observations nearest the storm center are taken as close together in time as possible. The ODW soundings closest to Debby were taken within ~2 h of the Bermuda sounding. This arrangement may help to explain the reasonable consistency between the different budget calculations in this paper.

While the vertical resolution of our thermodynamic analyses is too coarse for boundary layer studies, the accumulation of moisture in the Debby environment should be studied further using the full resolution of the ODW soundings. The changing boundary layer thermodynamic structure in different quadrants around the storm offers many opportunities to explore interactions between the hurricane and its environment that extend to distances an order of magnitude greater than the radius of maximum wind. With the completion of this fully three-dimensional dataset, we look forward to further study of hurricanes with prognostic numerical models.

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