A Comparison of Extended and Quasigeostrophic Dynamics for a Case of Small–Rossby Number Extratropical Cyclone Development

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18 July 1996 and 20 April 1997

ABSTRACT

This note compares the extended (EXT) and quasigeostrophic (QG) dynamics of a small–Rossby number extratropical cyclone using the Zwack–Okossi (ZO) equation. Applied to a cyclone that occurred on 8–9 November 1985 over the North Atlantic Ocean, results show that although differences exist, both the EXT and QG forms of the ZO equation provide very adequate estimates of the large-scale forcing processes associated with this case.

1. Introduction

The coupling of observational evidence with dynamical understanding of large-scale extratropical circulations has been greatly aided by the introduction and application of quasigeostrophic (QG) theory, a concise summary of which can be found in Holton (1992, 150–180). While QG theory has been remarkably useful, it is also clear that for more intense and/or shorter-wave synoptic circulations it has limitations. In particular, geostrophic constraints are most applicable for circulations with Rossby number Ro of order 0.1 (Phillips 1963). When the order of Ro exceeds 0.1, QG representations are less likely to provide reliable quantitative estimates of atmospheric dynamics, although they may still provide qualitatively useful insights.

Thus, it is understandable that examinations of the limitations of QG theory have focused on case studies of more intense large-scale circulation events. For example, Tsou et al. (1987) found that a more generalized form of the height tendency equation more accurately diagnosed the height changes in a rapidly developing continental cyclone with mid- and upper-tropospheric Rossby numbers between 0.3 and 0.6. Further, Pauley and Nieman (1992) examined the application of QG theory to estimates of vertical motion, noting that in cases of rapid cyclone development the generalized omega equation was better able to consistently represent both the large latent heating rates and the heating-induced variations in relative vorticity than was the QG omega equation. Even for anticyclonic circulations, Bengtsson (1981) and Tsou and Smith (1990) found that using QG theory can be inadequate for diagnosing and predicting blocking events.

Of course, many important weather-producing systems are not of the intensity (and larger Ro) of the events described in the above-cited papers. Thus, keeping in mind that, as suggested by Gall (1977), even some small Ro baroclinic waves possess significant non-QG components, the objective of this note is to compare QG with a more generalized representation of large-scale dynamics for a small–Rossby number (Ro < 0.4) extratropical cyclone. A favorable comparison means that QG mechanisms dominate the surface pressure changes associated with the cyclone system, while a superior performance by the extended (EXT) equation means that non-QG contributions exhibit a significant influence on the pressure changes.

2. Methodology

The diagnosis described in this note applies a methodology that contains two primary features. The first is the utilization of a relatively new diagnostic expression known as the Zwack–Okossi (ZO) equation. Originally formulated in QG form by Zwack and Okossi (1986), this equation has also been developed in more generalized forms by Lupo et al. (1992) and in addition has been successfully applied to cyclone and anticyclone diagnoses in Lupo et al. (1992), Uhl et al. (1992), King et al. (1995), Rausch and Smith (1996), and Rolfsen and Smith (1996). The second is the application of the “extended” equation approach in the specification of the more generalized form of the ZO equation. Origi-
nally applied to the height tendency equation by Tsou et al. (1987), this approach simply generalizes the QG equation by replacing the geostrophic quantities with observed or analyzed values and allowing three-dimensionally varying static stability. Thus, the new equation retains the basic simplicity of the corresponding QG equation, but at the same time “extends” the QG equation by permitting the inclusion of non-QG forcing.

The extended (EXT) ZO equation (see Lupo et al. 1992 for derivation) is

\[
\frac{\partial \xi_e}{\partial t} = \frac{1}{p_s - p_t} \int_{p_t}^{p_s} (-\nabla \cdot \nabla \xi_e) \, dp - \frac{R_d}{f(p_s - p_t)} \int_{p_t}^{p_s} \nabla \cdot \left( \frac{\nabla T}{c_p} + S \omega \right) \, dp,
\]

where \( \xi_e \) is the near-surface geostrophic vorticity tendency, \( p_s \) the near-surface pressure level, \( p_t \) the upper pressure level, \( \mathbf{V} \) the horizontal wind vector, \( \xi \) the absolute vorticity, \( f \) the Coriolis parameter, \( R_d \) the dry-air gas constant, \( T \) the absolute temperature, \( \dot{Q} \) the diabatic heating/cooling rate per unit mass, \( c_p \) the specific heat at constant pressure, \( S \) the static stability parameter \( [S = -(T/\partial \theta/\partial p)] \), where \( \theta \) is potential temperature\), and \( \omega \) the vertical motion in isobaric coordinates \( (\omega = dp/dt) \). In this study, the near-surface pressure is the 50-hPa-interval pressure level nearest to the surface pressure (generally 950 hPa), and the upper pressure level is 50 hPa. The QG form of (1) is obtained by replacing \( \mathbf{V} \) and \( \xi \) with their geostrophic values, setting \( f \) to a constant \( f_c \), and regarding \( S \) as a function of pressure only. The latter two parameters were determined as averages over the display domain (see section 3a). Differences between the extended and QG forms of (1) represent the contributions of non-QG processes.

The basic diagnostic quantity is the near-surface geostrophic vorticity tendency, which in turn can be relaxed to obtain the surface pressure tendency (described in section 3e). In (1), dynamical forcing is provided by horizontal vorticity advection (term b), while thermal forcing is provided by temperature advection (term c), diabatic heating (term d), and adiabatic temperature change (term e).

3. Computational methods

a. Data

The data for this study were global 2.0° latitude by 2.5° longitude analyses in 6-h intervals provided by the National Aeronautics and Space Administration (NASA)/Goddard Laboratory for Atmospheres (Schubert et al. 1993). Data for the entire Northern Hemisphere were retained for the computations, thus mini-

b. Vertical motion and diabatic heating

The vertical motion was calculated using the extended (Rolfson and Smith 1996) and QG (Holton 1992, 167) forms of the omega equation. The diabatic heating quantities in this study are convective and stable latent heat release, boundary layer sensible heating, and longwave radiation. The stable latent heat release component was set proportional to the vertical advection of saturation specific humidity, following Krishnamurti and Moxim (1971) and Vincent et al. (1977). The convective latent heat release component was calculated using the Kuo (1965, 1974) parameterization technique with the suggested modifications of Edmon and Vincent (1976), Lin and Smith (1979), and Smith et al. (1984). Boundary layer sensible heating was calculated as in Lupo et al. (1992), while infrared radiation was determined using the longwave radiation parameterization of Sasamori (1968), with the corrections and modification of Ramathan et al. (1983) and Harshvardhan et al. (1987).

c. Finite differencing and filtering

Horizontal (vertical) derivatives were calculated using fourth-order (second-order) finite differencing. Vertical integrals were estimated using the trapezoidal role. The pressure tendencies obtained by relaxing the geostrophic vorticity tendencies were filtered using a fourth-order filtering scheme developed by Shapiro (1970) to remove small-scale features. The response function for this filter is similar to that shown in Rolfson and Smith (1996, see their Fig. 3), retaining less than 10% of the information below 1400 km and over 95% above 2500 km.

d. Relaxation

Since sea level pressure changes are used as the primary measure of cyclone evolution, development is given by the surface pressure tendency at the center of the cyclone. To obtain pressure tendencies from the near-surface geostrophic vorticity tendencies obtained from (1), sequential overrelaxation was used to produce a near-surface height tendency field. This relaxation iteration was terminated when the absolute difference between two successive height tendency solutions at all grid points was less than 10^{-5} m s^{-1} for the total height tendency and for the tendency forced by all of the forcing terms except the smaller sensible heating and long-
Table 1. Rossby numbers at selected levels at two representative map times during the cyclone development.

<table>
<thead>
<tr>
<th>Pressure level (hPa)</th>
<th>1200 UTC 8 November 1985</th>
<th>1200 UTC 9 November 1985</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.34</td>
<td>0.36</td>
</tr>
<tr>
<td>500</td>
<td>0.27</td>
<td>0.28</td>
</tr>
<tr>
<td>700</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>850</td>
<td>0.19</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The cyclone studied here originated in Quebec on 5 November 1985, moved eastward across the Atlantic Ocean south of Greenland and Iceland to 53°N, 15°W at 1800 UTC 8 November. It then turned northeastward and propagated through Scotland into Norway over the
next 36 h. The period of primary interest in this study is from 0000 UTC 8 November, when the cyclone was at 51°N, 27°W at a central pressure of 980 hPa, through 1800 UTC 9 November, when the cyclone was at 59°N, 5°E at a central pressure of 966 hPa. This 42-h pressure decrease of 14 hPa at a mean latitude of 55°N corresponds to a deepening rate of 0.35 bergerons (see Sanders and Gyakum 1980; Sanders 1986).

This case was chosen because it is a prominent cyclone exhibiting a very modest deepening rate and a consistently small Rossby number, though not quite 0.1. This latter point is exemplified in Table 1, which shows Ro at four levels and at two representative map times, 1200 UTC 8 and 9 November. Here Ro was calculated as the ratio of the average (over the display area, which encompasses the synoptic wave system being studied) of the relative vorticity to the Coriolis parameter. Note that Ro did not exceed 0.36 and in fact was less than 0.3 at all but the 300-hPa level, thus satisfying the objective stated in the introduction. The fact that the Ro values for this case are relatively small is further exemplified by comparing the present values with those for the case of intense cyclone development described by Tsou et al. (1987). In the latter, Ro achieves values in excess of 0.5 (with a maximum of 0.63) at 300 hPa and also approaches a value just under 0.5 at 500 hPa.

In the interest of brevity, synoptic maps, as well as later maps displaying diagnostic results, are restricted to the two map times included in Table 1 and to sea level and 300 hPa. By 1200 UTC 8 November 1985,
the cyclone was located due west of Ireland at 53°N, 18°W (Fig. 1a) with a central pressure of 977 hPa. At 300 hPa (Figs. 1b,c), the cyclone was positioned southwest of the surface cyclone, reflecting the baroclinic structure of the system. The isotherm field was about 90° out of phase with the height field at 850 hPa (not shown). Also, 300 hPa shows two jet streaks, one southeast and one southwest, in both the analyzed and QG isotach fields, with the QG being the stronger of the two. As a consequence of the wind similarities, the analyzed and QG vorticity fields (not shown) were also similar. By 1200 UTC 9 November 1985, the central pressure of the cyclone had decreased to 966 hPa (Fig. 2a) and had moved northeastward to the southwest coast.

Fig. 4. The ZO surface pressure tendencies [4 hPa (12 h)⁻¹] for 1200 UTC 8 (a), (c), and (e) and 9 (b), (d), and (f) November 1985; EXT ZO (a), (b), QG ZO (c), (d), and difference between EXT and QG ZO (e), (f).
TABLE 2. Mean absolute values, correlation coefficients, and $S_1$ scores for the cyclone case display area.

<table>
<thead>
<tr>
<th>Mean absolute value [hPa (12 h)$^{-1}$]</th>
<th>Correlation coefficient</th>
<th>$S_1$ score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EXT</td>
<td>QG</td>
</tr>
<tr>
<td>0000 UTC</td>
<td>7.65</td>
<td>6.67</td>
</tr>
<tr>
<td>8 November 1995</td>
<td>7.81</td>
<td>7.42</td>
</tr>
<tr>
<td>0600 UTC</td>
<td>6.82</td>
<td>8.01</td>
</tr>
<tr>
<td>1200 UTC</td>
<td>8.11</td>
<td>8.49</td>
</tr>
<tr>
<td>8 November 1995</td>
<td>8.49</td>
<td>7.38</td>
</tr>
<tr>
<td>1800 UTC</td>
<td>8.05</td>
<td>7.05</td>
</tr>
<tr>
<td>9 November 1995</td>
<td>6.81</td>
<td>7.80</td>
</tr>
<tr>
<td>1200 UTC</td>
<td>8.10</td>
<td>8.69</td>
</tr>
<tr>
<td>1800 UTC</td>
<td>7.73</td>
<td>7.69</td>
</tr>
<tr>
<td>Averages</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Results

a. Comparison of ZO and observed pressure tendencies

Observed sea level pressure tendencies at 1200 UTC 8 and 9 November 1985 are depicted in Fig. 3. These results indicate that the observed pressure tendencies were less intense on 8 November than 9 November, reflecting the fact that the cyclone central pressure fell only 5 hPa from 0000 UTC 8 November 1985 to 0000 UTC 9 November 1985, but fell 9 hPa from 0000 to 1800 UTC 9 November. As time progressed, the cyclone propagated farther into the region of pressure falls.

The corresponding EXT and QG ZO tendencies are shown in Fig. 4. Both forms compare reasonably well with the observed tendency patterns (Fig. 3), though the magnitudes of both ZO tendencies exceed the observed. Comparison of the two ZO fields shows very strong pattern comparability and that, generally, the magnitudes of the QG ZO values are larger than the EXT ZO at these two map times. Pattern comparability was also very strong in all of the individual forcing fields (not shown). Difference fields for the ZO tendencies (Figs. 4e,f) show that, aside from the relatively large tendencies over Greenland on 8 November, the non-QG contributions, which increased with time, were largest in the pressure fall regions northeast of the cyclone center. Pressure falls (integrated divergence) forced by terms b (cyclonic vorticity advection), c (warm-air advection), and d (diabatic heating, primarily latent heat release) and pressure rises (integrated convergence) by term e (adiabatic cooling) in (1) (fields not shown) also maximized in these same regions. Thus, in this case the non-QG contributions were most significant in the portion of the cyclone wave system that experienced the most vigorous integrated divergence and upward motion.

The comparative statistics for all map times are shown in Table 2. The mean absolute values (MAs) confirm that both the EXT and QG forms of the ZO equation yielded stronger pressure tendencies than the observed finite difference (FD) at every map time. This may be attributed to the fact that the ZO pressure tendencies are instantaneous values, while the observed are finite-difference average values over 12-h periods. On the average, the EXT and QG MAs were comparable, with each exceeding the other in four of the eight map times.

The correlation coefficients show that in general the EXT and QG patterns compared well with the observed patterns and particularly well with each other. On average, both the EXT and QG patterns correlated with the observed at near 0.75 and with each other at 0.90. However, examination of individual map times reveals an interesting contrast between the first four and the last four map times. Recall that over the last four times the cyclone central pressure decreased at a rate nearly twice that observed over the first four times. During the first four, the EXT fields correlated better with the observed than did the QG, and the two correlated less well with each other. The reverse was true for the last four times. A similar contrast is seen in the $S_1$ scores. Thus, it appears that the EXT and QG patterns were more comparable with each other and the QG was closer to the observed during the period of most rapid development.

The greater comparability between the EXT and QG fields during the latter times may be related to the greater similarity in the upper-tropospheric analyzed and QG
Fig. 5. Vertical profiles at 54°N, 17.5°W, for 1200 UTC 8 November 1985 of (a) horizontal absolute vorticity advection ($10^{-9} \text{s}^{-2}$), (b) temperature advection ($10^{-4} \text{K s}^{-1}$), (c) static stability parameter $S$ ($10^{-1} \text{K hPa}^{-1}$), (d) vertical motion $\omega$ ($10^{-3} \text{hPa s}^{-1}$), (e) $S\omega$ ($10^{-4} \text{K s}^{-1}$), and (f) latent heat release ($10^{-5} \text{K s}^{-1}$) for analyzed (solid) and QG (dashed) data fields.

jet streak structure during this period (see Figs. 1 and 2), thus rendering less significant differences in upper-level advection fields. Results of Lupo et al. (1992), Rausch and Smith (1996), and Rolfson and Smith (1996) show that surface pressure tendencies are strongly influenced by upper-level vorticity and temperature advections. The better correlation of EXT fields with the observed during the first four map times suggests a greater relative importance of nonquasigeostrophic forcing during earlier cyclone development, when the upper-
level jet structure was more complex (i.e., a double jet streak on 8 November versus a single jet streak on 9 November).

The $S_I$ scores show that the EXT gradients compared better with the observed than did the QG gradients at all map times, although differences between the two sets of $S_I$ scores are small. Earlier it was noted that a lower-bound "perfect" $S_I$ score could not be specified for the

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**Fig. 6.** As in Fig. 5 but for 1200 UTC 9 November 1985 at 60°N, 5°E.
ZO surface pressure tendencies. However, it is possible to speculate on this bound by focusing attention on the period 0600–1800 UTC 9 November. The very high correlation coefficients and closeness of the QG and EXT MAV statistics suggest that an $S_1$ score of 35–40 might be considered “near perfect.”

b. Vertical profiles

In order to examine the forcing processes responsible for the integrated EXT and QG fields displayed in Fig. 4, vertical profiles of these processes—horizontal vorticity advection, (VA), horizontal temperature advection (TA), the static stability parameter $S$, vertical motion $v_0$, the adiabatic term $S_0 v$, and latent heat release (LHR) at the cyclone center—are presented for the two map times (Figs. 5 and 6). The cyclone center was chosen because pressure tendencies at this point are representative of cyclone development.

At both map times there is considerable similarity in the analyzed and quasigeostrophic profiles, with differences being primarily in magnitude rather than sign. Both show that cyclonic and warm-air advections, mitigated by adiabatic cooling, dominated the pressure fields at the cyclone center. The advections maximized at 300 hPa or above, while adiabatic cooling maximized 50–150 hPa above the upward motion maximum in the region of increasing static stability. Latent heat release was only a minor contributor to development in this case. The profiles are similar to those prescribed for weak to moderate development in Rolfson and Smith (1996).

6. Summary and conclusions

A comparison of the extended (EXT) and quasigeostrophic (QG) forms of the ZO equation for a case of small–Rossby number extratropical cyclone development has been presented. The cyclone occurred in November 1985 with the focus on the 42-h period prior to decay (8–9 November). The diagnosis was based on analyses provided by NASA/Goddard Laboratory for Atmospheres. The statistics in Table 2 reveal that the overall magnitudes of both the EXT and QG surface pressure tendencies were nearly the same and exceeded the observed by about 30%, perhaps reflecting the 12-h interval average pressure change used for the observed values. In addition, the overall EXT and QG patterns were nearly the same and compared well with the observed, although an examination of individual map times shows that the EXT and QG tendencies fields compared more favorably with each other and the QG more favorably with the observed during the period of most vigorous development. This may have been associated with the upper-level jet structure during this period, which, in contrast to a double-jet configuration during the least vigorous development period, was a simple single jet located east of the upper-level trough.

The only statistic that suggests consistently superior performance by one of the sets of calculations is the $S_1$ score, which indicates that the EXT pressure tendency gradient magnitudes were consistently closer (but by less than 10% on the average) to the observed than were the QG gradients.

The overall general similarities seen in Table 2, the maps, and the vertical profiles lead us to conclude that for this small Rossby number case both the EXT and QG forms of the ZO equation provide very adequate estimates of the large-scale forcing processes, thus confirming the dominance of QG mechanisms for this case. By contrast, the results of Tsou et al. (1987) indicate that if $Ro$ exceeds 0.50, even if such values are restricted to the upper troposphere, significant differences between EXT and QG diagnostic quantities can occur. In such cases, the EXT values appear to be distinctly better.

REFERENCES


