

CORRIGENDUM

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There are several changes to the article “A General Pressure Gradient Formulation for Ocean Models. Part I: Scheme Design and Diagnostic Analysis,” which appeared in *Monthly Weather Review*, Vol. 126, No. 12, 3213–3230.

The attached appendix is one of several contributions to the article made by Daniel G. Wright of Fisheries and Oceans Canada, Bedford Institute of Oceanography, Dartmouth, Nova Scotia. During the revision process, both the appendix and the name of DGW were removed. We agree that the appendix should be restored to the paper as appendix B and Dr. Wright’s name should be added as coauthor. DGW is grateful to Dr. David Dietrich for stimulating discussions and the provision of an independent derivation of the content of appendix B. We acknowledge financial support from the Atmospheric Environment Service of Canada through the Canadian Institute of Climate Studies and from the National Aeronautics and Space Administration.

Future references to this work should appear as follows:

Song, Y. T., 1999: A general pressure gradient formulation for ocean models. Part I: Scheme design and diagnostic analysis. *Mon. Wea. Rev.*, **126**, 3213–3230; Corrigendum, *Mon. Wea. Rev.*, **128**, 2608–2609.

APPENDIX B

An Alternative Derivation of the WJ Scheme

In this appendix we consider an alternative approach to the derivation of the weighted Jacobian scheme, which is less general, but perhaps somewhat more transparent.

Consider a single cell of a numerical model, with vertices (x_i, z_1) , (x_{i+1}, z_2) , (x_{i+1}, z_3) , (x_i, z_4) , where, for convenience, we have introduced the notation $z_1 = z_{i,k}$, $z_2 = z_{i+1,k}$, $z_3 = z_{i,k+1}$, $z_4 = z_{i+1,k+1}$, and we will use analogous notation for the density at these locations.

Now, consider a density field that varies linearly with

z at each horizontal position. The change in the horizontal pressure difference over the distance δx between the top and the bottom of the cell is given by

$$\begin{aligned} \delta_x p|_{z=\bar{z}_b^x} - \delta_x p|_{z=\bar{z}_t^x} &= \int_{\bar{z}_b^x}^{\bar{z}_t^x} \frac{\partial \rho}{\partial x} \delta x dz = [\rho|_{(x_i, \bar{z}_c)} - \rho|_{(x_i, \bar{z}_c)}] \delta_s \bar{z}^x \\ &= \left[\frac{\rho_4(\bar{z}_c - z_2) + \rho_2(z_4 - \bar{z}_c)}{z_4 - z_2} \right. \\ &\quad \left. - \frac{\rho_3(\bar{z}_c - z_1) + \rho_1(z_3 - \bar{z}_c)}{z_3 - z_1} \right] \delta_s \bar{z}^x \\ &= c_1 \rho_1 + c_2 \rho_2 + c_3 \rho_3 + c_4 \rho_4; \end{aligned}$$

where $\bar{z}_B^x = 0.5(z_1 + z_2)$, $\bar{z}_T^x = 0.5(z_3 + z_4)$, $\bar{z}_c = 0.5(\bar{z}_T^x + \bar{z}_B^x)$, and we have used linear interpolation or extrapolation to obtain the third equality. The coefficients c_1 through c_4 are given by

$$\begin{aligned} c_1 &= -\frac{\delta_s \bar{z}^x (z_3 - \bar{z}_c)}{(z_3 - z_1)}, & c_2 &= \frac{\delta_s \bar{z}^x (z_4 - \bar{z}_c)}{(z_4 - z_2)}, \\ c_3 &= \frac{\delta_s \bar{z}^x (z_1 - \bar{z}_c)}{(z_3 - z_1)}, & \text{and } c_4 &= -\frac{\delta_s \bar{z}^x (z_2 - \bar{z}_c)}{(z_4 - z_2)}. \end{aligned}$$

Making use of the relations

$$\begin{aligned} z_1 &= \bar{z}_c - 0.5(\delta_x \bar{z}^s + \delta_s z_i), \\ z_2 &= \bar{z}_c + 0.5(\delta_x \bar{z}^s - \delta_s z_{i+1}), \\ z_3 &= \bar{z}_c - 0.5(\delta_x \bar{z}^s - \delta_s z_i), \\ z_4 &= \bar{z}_c + 0.5(\delta_x \bar{z}^s + \delta_s z_{i+1}); \end{aligned} \tag{B1}$$

it is now a straightforward exercise in algebraic manipulation to show that this result is equivalent to (2.11).

Clearly this approach is very straightforward, but it does leave some false impressions. First, it leaves the impression that extrapolation in z is required. In actuality, one may expand the buoyancy as a bilinear function of x and z over each cell and obtain exactly the same result for the pressure gradient at $x = x_c$, $z = z_c$. Thus, extrapolation is not required. However, bilinear interpolations in x and z still do not guarantee that the resulting buoyancy field will be continuous across the intersections of the cells. The continuity of the bilinear

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interpolations in x and s is more pleasing from a physical point of view. This “weakness” of the weighted Jacobian is also easily overcome. Simply expanding the buoyancy field as a bilinear function of x and s and then substituting for s from (2.11) gives a fit to the buoyancy field in terms of x and z that is continuous between cells.

This approach achieves continuity between the model cells through the addition of a term proportional to $(x - x_c)^2(z - z_c)$, which clearly gives no contribution to the horizontal pressure gradient at x_c . Thus, the final estimate of the pressure gradient is exactly equivalent to the simpler procedure outlined above.