NOTES AND CORRESPONDENCE

A New Mixing-Length Formulation for the Parameterization of Dry Convection: Implementation and Evaluation in a Mesoscale Model

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ABSTRACT

A realistic representation of the evolution of the dry convective boundary layer in mesoscale and large-scale atmospheric models has been an elusive goal for many years. In this paper the performance of a new mixing-length formulation for the dry convective boundary layer is evaluated in the context of the Coupled Ocean–Atmosphere Mesoscale Prediction System (COAMPS). In this new formulation, the mixing length is proportional to a time scale and to the square root of the turbulent kinetic energy. The model results are tested against observations from the Climate Impact of Changes in Land Use (CICLUS) field experiment in the south of Portugal. It is shown that COAMPS with the new formulation produces a more realistic simulation of the boundary layer growth. A data assimilation experiment performed with COAMPS shows that the improvements provided by the new formulation are significant, particularly in terms of the humidity vertical distribution. Finally, one-dimensional simulations are used to confirm that the new formulation provides more accurate results because of a more realistic representation of the entrainment and of the vertical mixing in general.

1. Introduction

The entrainment at the top of the planetary boundary layer (PBL) is a fundamental aspect of the dynamics of the dry convective boundary layer. A realistic parameterization of the entrainment and of the growth of the PBL in atmospheric models has been a major challenge in boundary layer research. It is well known that large-scale and mesoscale models have serious deficiencies in representing the development of the dry convective PBL (e.g., Ayotte et al. 1996; Beljaars and Betts 1993).

In Teixeira and Cheinet (2004, hereafter TC04) a simple mixing-length formulation for the eddy-diffusivity parameterization of dry convection was proposed, in order to realistically represent the PBL evolution. The new formulation relates the mixing length ($l$) to the square root of the turbulent kinetic energy ($e$) and a time scale ($\tau$): $l = \tau \sqrt{e}$. Two different ways of determining the time scale were analyzed in TC04: (i) calculated as proportional to the ratio between the boundary layer height ($h$) and the convective velocity scale ($w_*$), $\tau \propto h/w_*;$ or (ii) taken as a constant, equal to the typical mean eddy turnover time in a dry convective PBL, $\tau = 600$ s. The simulation of dry atmospheric convection events showed that the new formulation reproduces in a realistic way the top entrainment and the overall PBL evolution. Although the approach of assuming a constant time scale produced slightly worse results than the more physical one, it still showed a surprising robustness in its sensitivity to a spectrum of differing surface fluxes and tropospheric lapse rates. This new formulation has been generalized successfully for cloud-topped boundary layers, both in stratocumulus and cumulus cases (Cheinet and Teixeira 2003), in the context of one-dimensional (1D) models.

In this paper we test this new mixing-length formulation using the U.S. Navy Coupled Ocean/Atmosphere Mesoscale Prediction System (COAMPS), which is briefly described in section 2. The new formulation is introduced in section 3. The observations and the mesoscale model results are analyzed in section 4. A dis-
cussion using 1D simulations is presented in section 5 and some conclusions in section 6.

2. COAMPS

COAMPS (Hodur 1997; Hodur and Doyle 1998) is a mesoscale model with a finite-difference approximation to the fully compressible, nonhydrostatic equations. COAMPS can be used as an analysis/nowcast and short-term forecast (up to 72 h) tool, applicable for any given region on earth. COAMPS includes a full atmospheric data assimilation system with data quality control, analysis, initialization, and nonhydrostatic atmospheric model components, coupled with a hydrostatic ocean circulation model. COAMPS uses a terrain-following vertical coordinate and can be integrated on a system of nested grids that enables the highest resolution to be focused over a specific region of interest.

The boundary layer and turbulence parameterization uses a prognostic equation for the turbulent kinetic energy (TKE) based on Mellor and Yamada (1982). The surface fluxes are computed based on Louis et al. (1982), and the radiation parameterization follows Harshvardhan et al. (1987). The moist convection processes are parameterized following the approach of Kain and Fritsch (1993), and the cloud microphysics processes are parameterized based on Rutledge and Hobbs (1983). The boundary conditions are from the Navy Operational Global Atmospheric Prediction System (NOGAPS). The dynamics and numerics of NOGAPS are described in Hogan and Rosmond (1991), and the main physical parameterizations are described in Louis et al. (1982), Harshvardhan et al. (1987), Teixeira and Hogan (2002), and Emanuel and Zivkovic-Rothman (1999).

3. Mixing-length formulation

The boundary layer parameterization in COAMPS is based on the eddy-diffusivity closure with a prognostic equation for TKE. The eddy-diffusivity coefficients and the TKE dissipation are parameterized as follows:

\[ K_u = K_q = K_e = S_{a,e} l_h \sqrt{e}, \]  
\[ K_u = K_v = S_{m,m} l_m \sqrt{e}, \]  
\[ \varepsilon = C_e \frac{e^{3/2}}{l_e}, \]

where \( \theta \) is the potential temperature; \( e \) is the TKE; \( q \) is the water vapor mixing ratio; \( u \) and \( v \) are the horizontal wind components; \( \varepsilon \) is the TKE dissipation; \( l_h \) is the mixing length for potential temperature, water vapor, and TKE; and \( l_m \) is the momentum mixing length. In the control version of COAMPS, \( S_{a,e,m} \) are functions of the Richardson number (Chen et al. 2003). \( S_a \) is a constant, and the different mixing lengths are equal to a master length scale \( l_h = l_m = l \), with \( l \) being calculated using Blackadar’s formulation (Blackadar 1962, hereafter B62)

\[ \frac{1}{k} = \frac{1}{kz} + \frac{1}{l}, \]

where \( k \) is the von Kármán constant, and the length \( \lambda \) is calculated as

\[ \lambda = \alpha \frac{\int e \, dz}{\int e \, dz}. \]

The value of \( \alpha \) is often taken as constant: \( \alpha = 0.1 \) as used in Yamada and Mellor (1975) or \( \alpha = 0.2 \) as suggested by Moeng and Wyngaard (1989) (note that in these two studies the TKE is replaced by \( \sqrt{2}e \) inside the integrals). In COAMPS, \( \alpha = 0.1 \) for stable and neutral boundary layers and has a stability correction for the unstable PBL (Chen et al. 2003).

In a new version of COAMPS, the new formulation for the mixing length proposed in TC04 is used for potential temperature, water vapor mixing ratio, and TKE. In this new formulation the mixing length is proportional to the square root of the TKE multiplied by a time scale:

\[ l_h = \tau \sqrt{e}, \]

where \( \tau \) is the time scale.

For convective situations (positive surface buoyancy flux) we use in this study a constant time scale equal to 600 s that produced realistic results in TC04. For stable situations we combine TC04 with Deardorff (1976) by determining the time scale as \( \tau = \min(600, 0.76/N) \), where \( N \) is the Brunt–Väisälä frequency. Furthermore, \( S_{a,e,m} = 0.5 \) and \( C_e = 0.16 \).

Close to the surface the mixing length is a linear function of height, and the actual formulation used in the model is

\[ l_h = \tau \sqrt{e} + (kz - \tau \sqrt{e}) e^{-z/\mu}, \]

where \( \mu = 100 \) m is a crude approximation for the height of the surface layer. The exponential interpolating function (7) is used, instead of the approach of B62, in order to be able to represent the influence of the large eddies close to the surface in a convective PBL.

Since the B62 mixing-length formulation produces unrealistic neutral boundary layers and has been successfully used for a number of years (e.g., Louis et al. 1982; ECMWF 2000), we use it for the momentum mixing length with \( \lambda = 150 \) m (e.g., ECMWF 2000). In principle, there is no a priori physical reason to assume that the mixing lengths for momentum and heat must be the same. Also, 1D simulations using the new formulation as the mixing length for momentum-produced mixed-layer wind values that were too low when compared to observations (not shown).
We assume that the TKE dissipation can be divided in two terms, one related to the production of TKE due to shear and the other due to buoyancy, which leads to a dissipation length that is a combination of the heat and momentum mixing lengths:

$$\frac{1}{l_c} = \frac{1}{l_h} + \frac{1}{l_m}. \quad (8)$$

Note that in this particular version of the model, stability corrections to the surface-layer mixing length, based on Monin–Obukhov similarity, are not being taken into account. Sensitivity experiments for dry convection situations have shown that these corrections do not seem to have a significant impact on the results.

4. COAMPS simulations

a. CICLUS case study

The Climate Impact of Changes in Land Use (CICLUS) field experiment was performed between October 1997 and September 1999. It included two years of continuous surface observations in 16 automatic weather stations, installed at the Dejebe Valley, Alentejo, south Portugal. Between 16 and 31 July 1998, an intensive observation period was performed, consisting of radiosondes (at latitude 38.53°N and longitude 7.88°W), some tethered balloon ascents, continuous sodar operation, and near-surface turbulence measurements with an ultrasound turbulence sensor (eddy correlation system).

On 24 and 25 July 1998, two days with a clear-sky situation, radiosonde observations were performed every 3 h, providing a detailed picture of the boundary layer evolution. In Figs. 1a and 1b, the observed potential temperature and water vapor mixing ratio are plotted at 0600, 1200, and 1500 UTC (same local time), 24 July 1998. As expected, the PBL height increases throughout the day, reaching its maximum at 1500 UTC. During this time, the PBL develops from a stable boundary layer into a well-mixed PBL, toped by a sharp inversion, typical of dry convective situations. The shallow dry layer at around 1000-m height includes air that is advected horizontally southwestward from the interior of Spain. Above it lies a layer of moister air of Atlantic origin, in a flow with a clear westerly component. The implied vertical shear is associated with a transition from the cyclonic heat low near the surface to the anticyclonic flow aloft.

For this particular simulation, the atmospheric component of the COAMPS model was configured in a three-dimensional mode over an area around point 38.53°N, 7.88°W in a Lambert conformal projection with the standard parallels being 30° and 45°N. In this application COAMPS uses 30 vertical levels and three horizontal domains. The outer grid has 45-km horizontal resolution and uses 45 grid points in each horizontal direction. Nest 1 has 15-km resolution with 49 × 49 grid points. Nest 2 has 5-km resolution with 85 × 85 grid points in both horizontal directions. The initial and boundary conditions for the simulation are taken from NOGAPS. Two 24-h COAMPS forecasts were produced starting from 24 July 1998 at 0000 UTC: (i) a control version (CTRL) with the standard mixing length and (ii) a new version (NEW) with the new mixing-length formulation. The observations were taken at latitude 38.53°N and longitude 7.88°W, and the COAMPS model results were obtained in the nearest grid point, at latitude 38.529°N and longitude 7.904°W.

Figure 2a shows the potential temperature from the observations and the two model versions at 1500 UTC. It is clear that for this situation the current COAMPS parameterization is unable to realistically represent the
boundary layer height and mean potential temperature; the control experiment is almost 2°C too cold compared to the observations, and the PBL height is around 500–600 m, which is about half of the observed height. With the new formulation the simulation is strikingly better. Both the mean PBL potential temperature and the PBL height are very close to the observations, showing that the new mixing-length formulation is able to produce a realistic entrainment and PBL growth. These results confirm and generalize the findings of TC04 that were obtained in the context of 1D model simulations.

In Fig. 2b, the same is shown, but for the water vapor mixing ratio. Again the control version produces a PBL that is not realistic: the model PBL top is too low, leading to a value of the water vapor mixing ratio that is about 4 g kg\(^{-1}\) too high. The new formulation leads to values of the mixing ratio that are quite close to the observations. Notice that none of the COAMPS versions seems to be able to capture the large-scale dynamics associated with the moisture minimum around 1000 m. This may well be due to a lack of vertical resolution in order to resolve this type of feature.

The evolution of the boundary layer was analyzed in detail. The profiles of potential temperature and water vapor for the CTRL and NEW experiments at 0600, 1200, and 1500 UTC (not shown) confirm that the new formulation produces more entrainment than the control version, leading to a deeper and more realistic boundary layer.

Figure 3a shows a cross section of the water vapor mixing ratio at latitude 38.529°N, for the CTRL experiment at 1500 UTC. This cross section starts offshore in the west and crosses the south of Portugal and Spain, showing a deeper boundary layer over land. Figure 3b shows the differences in water vapor mixing ratio between the NEW and CTRL experiments. As expected, the new formulation produces deeper boundary layers, leading to higher values of the mixing ratio closer to the top (above the CTRL PBL height) and lower values closer to the surface, due to a more realistic vertical redistribution of the water vapor mixing ratio.

b. Data assimilation experiment

To assess the significance of the previous case study, results from a much wider data assimilation experiment are also analyzed. For the data assimilation/forecast simulation, COAMPS was configured over an area around point 37°N, 236°E in a Lambert conformal projection with the standard parallels being 15° and 60°N. Once again in this application, COAMPS uses 30 vertical levels and three horizontal domains. The outer grid has 81-km horizontal resolution and uses 61 grid points in each horizontal direction. Nest 1 has 27-km resolution with 91\(^3\) grid points. Nest 2 has 9-km resolution with 166 points in the east–west direction and 187 points in the north–south direction. The boundary conditions are derived from NOGAPS. Five days of data assimilation and ten 3-day COAMPS forecasts (starting at 0000 and 1200 UTC) were produced starting from 13 June 2000 at 0000 UTC for the CTRL and the NEW versions. The overall area includes the western part of the continental United States and a substantial part of the Pacific Ocean.

Figure 4a shows the temperature bias and root-mean-square (rms) error of 12-h forecasts verifying at 0000 UTC against radiosonde data for the 5 days of the data assimilation experiment. These results correspond to the local afternoon or early evening at about 1600 LT. The radiosonde sites are over land in the western part of the United States and, because of topography, include much less data at 1000 hPa (a total of about 25 observations) then at 850 hPa and above (about 100 observations).
This implies that the statistics at 1000 hPa are less reliable than above. The new model is clearly warmer at 1000 and 925 hPa, leading to a positive bias at 1000 hPa but correcting a negative bias at 925 hPa. The rms error from the new formulation is larger at 1000 hPa but slightly smaller above that.

Dewpoint temperature results are shown in Fig. 4b, with a substantial reduction of the CTRL moist bias in the lower PBL and of the dry bias above. These results confirm what was found in the previous section: the new mixing-length formulation leads to deeper boundary layers with a more realistic humidity vertical distribution. The rms error is also substantially reduced in the new model both in the lower PBL and above.

Figures 5a and 5b show the corresponding results for 1200 UTC, about 0400 LT. In terms of temperature, the new mixing length reduces a cold bias in the PBL with an additional slight decrease in the rms error at 925 and 850 hPa. In terms of dewpoint temperature, the new formulation is able to reduce a moist bias close to the surface and to substantially reduce the rms error at 925 and 850 hPa.

5. One-dimensional simulations

In order to further investigate the role of entrainment in the improved representation of the convective PBL using the new mixing length, we use a simple 1D model.
and compare its results to large-eddy simulation (LES) model results.

a. One-dimensional model

The 1D boundary layer model used in the present study has prognostic equations for the mean potential temperature and the TKE. Under horizontally homogeneous conditions, assuming a zero-mean vertical velocity and with no diabatic forcing, the energy conservation equation is

\[ \frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}(w' \theta'). \]  

In the absence of wind and moisture, the prognostic equation for TKE is (e.g., Stull 1989)

\[ \frac{\partial e}{\partial t} = -\frac{\partial}{\partial z}\left( w'e + \frac{w'p'}{\rho_0} \right) + \frac{\varepsilon}{\theta_0} w' \theta' - \varepsilon, \]  

where \( \varepsilon \) represents the TKE dissipation.

The parameterization of the turbulent terms uses the
eddy-diffusivity approach [Eqs. (1)–(3)] with \( S_v = S_s = 0.5 \) and \( C_r = 0.16 \), and assumes \( l_s = l/2.5 \), following Therry and Lacarrère (1983).

Several different mixing-length formulations are tested using the 1D model: (i) the new assumption where the mixing length is diagnosed as a function of TKE (with \( \tau = 600 \) s) and (ii) the classic formulation of B62 (originally used in COAMPS) with differing methods of calculating the asymptotic value \( \lambda \). The first two options use Eq. (5) to calculate \( \lambda \) with \( \alpha = 0.1 \) as in COAMPS or with \( \alpha = 0.4 \), but without the stability correction. The reason we ignore the stability correction is to make the comparison straightforward and more general, since stability corrections may be different from model to model. In any case, the impact of the stability corrections can be represented in our simulations by increasing \( \alpha \) or \( l \). A third option that was analyzed is to have \( \lambda = 150 \) m, as in the ECMWF model (ECMWF 2000). It should be noted, however, that the ECMWF model does not use this formulation for dry convective boundary layer situations.

b. Results

As a case study we use the dry convection intercomparison case from Nieuwstadt et al. (1992) where the surface heat flux is imposed as 0.06 K m s\(^{-1}\). The surface TKE is imposed as zero, and at the upper boundary (\( z = 3 \) km) the fluxes of both variables are set to zero. The spatial discretization of the equations uses a finite-difference method, and the time discretization is performed using a fixed stability coefficient method (Teixeira 1999). This method can be simply described as a semi-Lagrangian equivalent for the diffusion equation and has been shown to provide results that are more stable and accurate than the implicit method as is typically used (Teixeira 2000). The vertical resolution for the 1D model is 20 m, and the time step is 60 s.

The results from the 1D model are compared with results from a three-dimensional LES model. The resolution of LES models is usually such that the large eddies, which are responsible for most of the mixing within the convective PBL, are well resolved. In this test case the LES model uses a resolution of 20 m in the vertical and 78.125 m in the horizontal in a domain of \((64 \times 64 \times 200)\) points. This particular LES model has been used in many boundary layer convection studies, such as Siebesma and Cuijpers (1995). In particular it has been used in some recent studies of the dry convective boundary layer (Siebesma and Teixeira 2000; Soares et al. 2004).

The potential temperature profile for the different formulations of the mixing length is shown in Fig. 6a, together with the LES results after 8 h of simulation (hourly mean). The new formulation simulates the boundary layer properties quite well with a realistic PBL height and a well-mixed profile. The formulations with \( \alpha = 0.1 \) and \( \lambda = 150 \) m clearly show some major problems: the entrainment is unrealistically small and there is little mixing close to the surface, leading to a highly unstable layer. The version with \( \alpha = 0.4 \) shows a slightly larger entrainment and exhibits a somewhat more realistic PBL evolution.

It can be argued that just using larger values for \( \lambda \) may lead to a situation where the PBL growth is realistic. In fact, other common versions of the B62 formulation use values of \( \lambda = 450 \) m (as in NOGAPS) or proportional to the PBL height. The results with \( \lambda = 450 \) m are similar to the ones obtained with \( \alpha = 0.4 \),
and indeed using the PBL height for \( \lambda \) does improve the results. However, it can be shown in the framework of this simple 1D model that, whatever the value of \( \lambda \) may be, the PBL never grows deep enough. The best results are achieved when \( \lambda = 10^4 \text{ m} \), but even then the PBL growth is too weak and the lower part of the PBL is still too unstable. It is interesting to note that for values of \( \lambda > 10^4 \text{ m} \) the results virtually do not change. Values of this magnitude are physically unrealistic and not justifiable, and may also lead to unrealistically large values of the diffusivity coefficient above the PBL.

Similar results can be seen when analyzing Fig. 6b, where the corresponding evolution of the buoyancy flux profile is shown. The new formulation produces a realistic linear buoyancy flux profile with the correct amount of entrainment. The version of the old model with \( \alpha = 0.1 \) exhibits unrealistic fluxes, with no clear linear flux or entrainment. The other two versions indicate more realistic profiles of buoyancy flux, but still insufficient entrainment, as previously discussed. Note that the 1D simulation with the new mixing length leads to slightly stronger inversions and less entrainment above the PBL top. This feature was already apparent in TC04 and is not present in the 3D COAMPS simulations probably because of the lack of vertical resolution of the COAMPS model.

The different mixing-length profiles are shown in Fig. 7a. The new formulation leads to a much larger mixing length in the boundary layer that decreases naturally to a very small value above the PBL. The B62 formulations are all rather similar except in the magnitude of the mixing length. As expected, they all increase with height and are not able to distinguish between the PBL and the atmosphere above. These results clearly confirm that the traditional B62 formulation was not originally developed for convective boundary layers and that the new formulation provides a rather natural and simple way of representing the convective boundary layer mixing length.

The profiles of TKE from the different versions of the model are shown in Fig. 7b along with the vertical velocity variance from mixed-layer scaling (Stull 1989) using the LES PBL height. It should be noted that the model TKE can be compared directly with mixed-layer vertical velocity variance because, in general, it can be assumed (e.g., Therry and Lacarrère 1983) that \( e/\omega^2 = 2.5 \lambda/\omega^2 \) since \( \omega^2 = 2.5 \lambda/\omega^2 \) in our model. This leads to \( e = \omega^2 \omega' \) and hence \( \omega' = 2.5 \lambda/\omega^2 \).

Figure 7b shows that the TKE values produced by the new formulation are quite comparable with the results based on mixed-layer scaling. In fact, the results from the new formulation are within the range of uncertainty provided by previous studies (e.g., Garratt 1992; Stull 1989). On the other hand, the three versions of the old formulation clearly underestimate the TKE, which again shows that these versions are not capable of generating enough convective boundary layer mixing. It should also be noted that in TC04 direct comparisons are shown between the diffusivity coefficient from the new mixing length and from a k-profile closure (Troen and Mahrt 1986). K-profile schemes have been used with a certain degree of success in simulating the dry convective PBL, and TC04 show that the new mixing length leads to diffusivity coefficients that are similar to the ones from k-profile schemes.
6. Conclusions

A new physically based mixing-length formulation for the eddy-diffusivity parameterization was tested in COAMPS, in the simulation of a dry convective boundary layer observed during a field experiment in Portugal. The current COAMPS formulation produces boundary layers that are too shallow because of a lack of entrainment. As a consequence, the PBL is too cold and moist when compared to the observations.

The new formulation directly relates the mixing length to a time scale and the square root of the turbulent kinetic energy. This formulation, previously found to compare well with large-eddy simulation model results, dramatically improves the simulation of the dry convective boundary layer in COAMPS. The evolution of the vertical structures of both potential temperature and water vapor mixing ratio is much more realistic, with the new formulation producing boundary layers that are deeper, warmer, and drier than the current formulation. This implies a better representation of the dry boundary layer development process in general, and of the top entrainment in particular. A more realistic simulation of the dry convective PBL will also create better conditions for good forecasts of the onset of deep convection.

A data assimilation experiment showed that these results are significant and that the new formulation reduces the humidity biases in COAMPS. One-dimensional simulations showed that compared to traditional methods of calculating the mixing length (B62 formulations), the new formulation produces a more realistic top entrainment and vertical mixing in general. They also support the idea that it is actually not possible for B62 formulations to reproduce LES results for the dry convective PBL, however large the value of $\lambda$ may be.

These results overall suggest that this new simple parameterization could have a positive impact in the performance of numerical weather prediction models, with little or no additional computational cost.

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