Short-Term Forecasting of a Midlatitude Convective Storm by the Assimilation of Single–Doppler Radar Observations

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ABSTRACT

The McGill University radar data assimilation system is used to initialize a convective storm at high resolution (1 km) from single–Doppler radar observations. In this study, the background term in the assimilation system is improved. Specifically, by assuming the correlation of the errors of the control variables to be isotropic and homogeneous, the background error covariance matrix is modeled by a recursive filter. In addition, a 3-h-prior high-resolution model forecast is used as the background field. The analysis fields from the assimilation system successfully trigger the convective storms in the radar-observed regions from a single assimilation window. Without data assimilation, the modeled storms did not occur at the right time and place. To account for the rapid evolution of the convective storms and to correct the forecast errors with time, a cycling process is applied for a very short-term forecast. It is found that the first assimilation window can maintain the prediction of the storms for less than 1 h. The cycling process helps to maintain the intensity of the storm cells for a longer period of time. However, a comparison of radar observations with the 90-min simulation indicates an error in the position of the convective cells. The error of the radial component of the wind field between the observation and the simulation is larger at the upper levels. A wavelet analysis between the observation and simulated reflectivities indicates that the forecast is able to adequately predict the convective scale (10–20 km) during the first 20 min, whereas the simulation has more predictability at the longer scale (>30 km) beyond 20 min.

1. Introduction

The prediction of the initiation and development of convective storms represents a great challenge in the atmospheric sciences. The Lagrangian advection of radar echoes (called radar nowcasting) does not account for initiation, development, and dissipation. Forecasts using numerical weather prediction models are problematic because conventional synoptic-scale observations contain little information on the mesoscale. The initial conditions at the convective scale are therefore poorly described and the assimilation of high-resolution data is necessary to improve the forecast of convective systems.

Weather radar can sample the structures of convective storms with good resolution both in time and in space. However, the assimilation of radar data is not straightforward because radar-observed variables are not model variables. For example, a Doppler radar can measure only the radial component of a three-dimensional wind. Various techniques have therefore been developed to obtain the model variables from radar measurements. Qiu and Xu (1992), Laroche and Zawadzki (1994, 1995), Shapiro et al. (1995), and Gao et al. (2001) developed three-dimensional variational data assimilation (3D-Var) algorithms to retrieve the 3D winds from single–Doppler radar observations. Gao et al. (1999) and Protat and Zawadzki (1999) obtained the 3D wind field using multiple Doppler radars. Once the complete wind field is known, the thermodynamic variables like pressure and potential temperature can also be calculated (Gal-Chen 1978; Liou et al. 2003).
Methods other than 3D-Var have also been tested. The 4D-Var technique has been proposed to obtain the best analysis for all prognostic model variables by combining the information from the observations over a time window and the domain of the numerical model in a single analysis step. In particular, Sun and Crook (1997, 1998) introduced warm rain microphysics into their cloud model to retrieve the complete set of control variables. Wu et al. (2000) included ice phase microphysics in their algorithm and also assimilated the differential reflectivity from polarimetric radars. Recently, an alternative method (the ensemble Kalman filter, EnKF) has become quite popular in data assimilation. Snyder and Zhang (2003) used the simulated Doppler radar wind field to demonstrate the potential of EnKF to assimilate radar data at the convective scale.

The newer 4D-Var assimilation system with a perfect model assumption and the ensemble Kalman filter techniques are, however, computationally expensive. In the 4D-Var system, the use of the tangent linear model and its adjoint requires a large number of calculations. In addition, because the microphysics processes at the convective scale are highly nonlinear, their implementation in the adjoint model remains difficult. While the ensemble Kalman filter is easier to implement, it nevertheless demands a large number of ensemble members to span the most likely atmospheric state and to generate the probability distribution function (pdf) for a particular forecast event. For these reasons, the McGill University radar assimilation system (Laroche and Zawadzki 1994; Caya 2001) does not use a tangent linear or adjoint model but employs a cloud-resolving model as a weak constraint, as suggested by Sasaki (1970) and Zupanski (1997). In this way the retrieved control variables are consistent with the observed data over a given assimilation period and they also satisfy the governing model equations. This treatment reduces the computational load in finding the optimal analysis, and allows for the imperfection of the model. The McGill data assimilation system does not require a tangent linear and adjoint model. Hence, it naturally avoids the problems of the linear assumption and of the discontinuities in the 4D-Var algorithm. Montmerle et al. (2001, 2002) demonstrated the utility of the McGill assimilation system. They used bistatic Doppler radar observations to retrieve the wind, pressure, and temperature fields in three dimensions and showed that if the moisture is reasonably specified, the model can be successfully initialized and the convective storm can be predicted for a period of 30 min.

Recently, the assimilation of radar data has gradually moved from research into operations. Examples include 1) the Swedish High-Resolution Limited Area Model (HIRLAM) 3D-Var system to assimilate Doppler wind (Lindskog et al. 2004); 2) the Météo-France 3D-Var system, which assimilates the observations of the French radar network to improve the local forecast of severe convective events (Montmerle and Faccani 2008); and 3) the Korean Meteorological Administration (KMA) system, which has adopted the Weather Research and Forecasting (WRF) model 3D-Var algorithm to assimilate Doppler radar observations (radial velocity and reflectivity) to improve rainfall forecasting (Xiao et al. 2008).

This study represents an extension of the work of Caya (2001) and Montmerle et al. (2001, 2002). The main objective is to demonstrate that, with a better specification of the background term, the updated McGill radar assimilation system can deduce an optimal analysis for the prediction of convective storms and, thereby, provide a proper formulation for the problem of radar data assimilation. A secondary objective is to examine the predictability of very short-term forecasts. Because only single S-band scanning radar data are available over Montreal, Quebec, Canada, at present, we have used the radial velocity and the reflectivity from a single Doppler radar in our study. In section 2, we present the algorithm of the McGill assimilation system, and its improvement in specifying the background term. Section 3 describes the details of the data processing and the implementation of the cycling process. The effects of assimilating radar data, the impacts of the background field, and the results of the forecasts in a case study are presented in section 4. The quantitative verifications of the forecast are presented in section 5. A summary and suggestions for future work are given in section 6.

2. Methodology of radar data assimilation

The McGill data assimilation is based on the variational formalism. The cost function $J$ measures the discrepancy between the analysis from the background fields and that from the observations and also includes the residual errors of the model equations. The form of the cost function is

$$J = \frac{1}{2} (x - x^b)^T B^{-1} (x - x^b)$$

$$+ \frac{1}{2} \sum_{n=1}^{N} [H(x) - y_n]^T R^{-1} [H(x) - y_n] + \frac{1}{2} e^T Q^{-1} e$$

$$= J_b + J_o + J_m,$$  \hspace{1cm} (1)

where $J_b$ is the background term, $J_o$ is the observation term, and $J_m$ is the model term. The vector $x$ is the vector of all the control variables over the assimilation window.
The vector $y$ represents the observations and $H$ stands for the observational operator that projects the model space into the observational space. The index $n$ denotes the observational times. The vector $e^n$ represents the residual errors of the model equations. The superscripts $-1$ and $T$ represent, respectively, the inverse and the transpose of the matrix. The symbols $B$, $R$, and $Q$ denote the background, observation, and model-error covariance matrices, respectively.

### a. Background term

The background term plays an important role in the variational scheme by providing information in the data-void regions of the analysis domain. In addition, the background error covariances determine the filtering and propagation characteristics of the observed information. However, the true background errors are unknown because the true state of the atmosphere at any given instant is unknown. Therefore, assumptions have to be made to determine the background error covariance matrix. Caya (2001) assumes that the background error covariances are uncorrelated and can thus be represented by a diagonal matrix. This assumption, however, prevents information from the observation point from propagating into neighboring points resulting in large gradients that may hinder a proper minimization of the cost function. Caya (2001) found that a penalty term, which acts as a low-pass filter to smooth the noise, is required to obtain the optimal solution. However, the weighting coefficients of the penalty term have to be determined from experience. To avoid such difficulties and to modify the assumption of the background error matrix in Caya (2001), we have improved on the McGill assimilation system by modeling a background error matrix with the proper error statistics. Filtering is subsequently accomplished through the use of the appropriate error correlation length scales. Our strategy for introducing a proper background term is as follows:

(i) Following Courtier et al. (1994), a preconditioning procedure is applied by assuming that the optimal analysis, $x^a$, is obtained by adding an increment $\delta x^a$ to the background $x^b$, $x^a = x^b + \delta x^a$. Then, the cost function can be shown to be

$$J_{\text{inc}}(\delta x) = \frac{1}{2}(\delta x)^T B^{-1} (\delta x) + \frac{1}{2} (H \delta x - d)^T R^{-1} (H \delta x - d) + J_m(\delta x),$$

(2)

The subscript “inc” denotes “incremental” and $d = y_0 - H x^b$ is the innovation vector. The background error covariance matrix $B$ can be considered to be an operator and is formulated using its square root: $B = \sqrt{B_0} \sqrt{B} = C^T C$, where $C$ is a unique symmetric matrix with the same eigenvectors as $B$ and the same eigenvalues of the square root of $B$. Given a control variable $\chi$, we can write

$$\delta x = C \chi \quad \text{or} \quad \chi = C^{-1} \delta x,$$

(3)

and (2) becomes

$$J_{\text{inc}}(\chi) = \frac{1}{2} \chi^T C \chi + \frac{1}{2} (H C \chi - d)^T R^{-1} (H C \chi - d).$$

(4)

By comparing (2) and (4), it is clear that in the rewritten form of the cost function (4), the inversion of the background error covariance matrix, $B^{-1}$, is not required.

(ii) To minimize (4), the matrix $C$ is required. This is achieved by decomposing $C$ as

$$C = D F,$$

(5)

### Table 1. Variables in Eq. (12).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$, $v$, $w$</td>
<td>Wind component along the x, y, and z coordinates of the model</td>
</tr>
<tr>
<td>$T$</td>
<td>Atmospheric temperature ($=T^* + T'$)</td>
</tr>
<tr>
<td>$T'$</td>
<td>Temperature departure from the basic state</td>
</tr>
<tr>
<td>$q$</td>
<td>Liquid precipitation concentration</td>
</tr>
<tr>
<td>$q'$</td>
<td>Departure of $q$ from the isothermal hydrostatic basic state</td>
</tr>
<tr>
<td>$M$</td>
<td>Cloud–humidity</td>
</tr>
<tr>
<td>$m$</td>
<td>Source of precipitation</td>
</tr>
<tr>
<td>$\epsilon^b$</td>
<td>Model residuals</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
</tr>
<tr>
<td>$e$</td>
<td>Water vapor pressure</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>Cloud concentration</td>
</tr>
<tr>
<td>$Q$</td>
<td>Vapor concentration</td>
</tr>
<tr>
<td>$Q_{av}$</td>
<td>Vapor concentration at saturation</td>
</tr>
<tr>
<td>$L$</td>
<td>Latent heat released</td>
</tr>
<tr>
<td>$G$</td>
<td>Generating term</td>
</tr>
</tbody>
</table>

### Table 2. Constants in Eq. (12).

<table>
<thead>
<tr>
<th>Constant</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>Coriolis parameters ($10^{-4} \text{ s}^{-1}$)</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity constant (9.81 m s$^{-2}$)</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Constant temperature of the basic state (273.16 K)</td>
</tr>
<tr>
<td>$p_0$</td>
<td>Reference pressure (1000 hPa)</td>
</tr>
<tr>
<td>$\mathcal{C}$</td>
<td>Gas constant of dry air (286.86 J kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$H$</td>
<td>Scale height (10 km)</td>
</tr>
</tbody>
</table>
where \( D \) is a diagonal matrix of the standard deviation of the background error covariance matrix \( B \) and \( F \) is the square root of a matrix, with unit diagonal elements and off-diagonal elements given by the background error correlation coefficients. The full \( F \) is too large to compute directly, and assumptions and approximations are made to simplify the calculation of \( F \). Huang (2000) demonstrated that the effects of applying the operator \( F \) on the control variable \( q \) in (4) can be achieved through the use of equivalent spatial filters. We follow the methodology in Purser et al. (2003) by assuming that the error covariance matrix \( B \) is isotropic and homogeneous, and recursive filters are applied to the control variables to model the effects of \( F \) as

\[
 X'_i = \alpha X'_{i-1} + (1 - \alpha)X'_i \quad \text{for} \quad i = 1, \ldots, n \\
 X''_i = \alpha X''_{i+1} + (1 - \alpha)X'_i \quad \text{for} \quad i = n, \ldots, 1,
\]

where \( X_i \) is the initial value of the control variable at the grid point \( i \), \( X'_i \) is the value after filtering for \( i = 1 \) to \( n \), \( X''_i \) is the initial value after one pass of the filter in each direction, and \( \alpha \) is the filter coefficient defined as

\[
 \alpha = 1 + E - \sqrt{E(E + 2)} \quad \text{and} \quad E = \frac{2N\Delta x^2}{\epsilon},
\]

where \( \epsilon \) is the correlation length scale, \( \Delta x \) is the grid spacing, and \( N \) is the number of filter passes to be applied. In this work, \( \Delta x = 1 \) km, \( N = 4 \), and the error correlation lengths are set to 3.0 and 0.6 km in the horizontal and vertical dimensions, respectively, values that are comparable with the correlation lengths of the reflectivity field itself. These values are basically chosen to smooth the fields while maintaining a sufficient structure at convective scale. Here, we have considered no cross correlation between the errors in different control variables. This is a topic that is still under research and the first preliminary results (Snyder and Zhang 2003; Tong and Xue 2005) are not very conclusive. In the modified assimilation system, once the recursive filters are applied to estimate the background error covariances, the penalty term in the cost function is no longer needed.

Previously, Caya et al. (2002) used a linear wind analysis to specify the 3D environmental flow surrounding the precipitation area. Their method allowed an estimate of the environmental wind when the background fields from a numerical model are not available. One should notice that the algorithm is using the same source in both the observation and background terms. Since, nowadays, high-resolution cloud-resolving model outputs are more affordable, and they represent reliable information from an alternate independent source other than radar, the present study adopts the previous model forecast as the background, and hence the large-scale analysis plays a role in the assimilation system. The impacts of the background term will be discussed in section 4.
b. Observation term

The McGill assimilation system assimilates both Doppler radial velocity and radar reflectivity as follows. First, the radar reflectivity $Z$ (dBZ) is used to estimate the rainwater mixing ratio, $q_r$ (g kg$^{-1}$), a model control variable with the relationship

$$Z = 43.1 + 17.5 \log(\rho q_r), \quad (9)$$

where $\rho$ is the air density. The radial velocity $V_r$ is related to the Cartesian velocity components as

$$V_r = u_r \frac{x}{r} + v_r \frac{y}{r} + (w + V_t) \frac{z}{r}, \quad (10)$$

where $u$, $v$, and $w$ are, respectively, the wind components along the $x$, $y$, and $z$ directions, and $r = (x^2 + y^2 + z^2)^{1/2}$. The mass-weighted mean terminal velocity, $V_t$, of raindrops is parameterized by assuming a Marshall–Palmer drop size distribution with the following result:

$$V_t = 5.94 \times M^{1/8} \exp\left(\frac{l}{2h}\right), \quad (11)$$

where $l$ is the altitude, $h$ is the scale height (10 km), and $M = \rho q_r$ (g m$^{-3}$) is the precipitation concentration deduced from (9).

\begin{align*}
\text{1800 UTC} & \quad \text{1805 UTC} & \quad \text{1810 UTC} & \quad \text{1830 UTC} & \quad \text{1835 UTC} & \quad \text{1840 UTC} \\
\text{obs} & \quad \text{obs} & \quad \text{obs} & \quad \text{obs} & \quad \text{obs} & \quad \text{obs}
\end{align*}

Assimilation window 1  Forecast  Assimilation window 2  Forecast

Cycling process start

Fig. 2. Target area for the simulation domain.

Fig. 3. Schematic of the cycling process.
To interpolate radar observations onto the model grid, the path of the radar beam has been computed assuming normal propagation conditions in a standard atmosphere, and the observation operator assumes a perfectly narrow radar beam (i.e., the effects of beam broadening are not yet considered in our assimilation system). The ice crystal content is initially set to 0, as in the work of Montmerle et al. (2001), because the current configuration of the system only includes the warm rain–cloud microphysics.

As in most existing data assimilation systems, the observation errors are assumed to be uncorrelated in space and time. The covariance matrix \( \mathbf{R} \) of the observation errors is given by

\[
\mathbf{R} = \begin{pmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{pmatrix}
\]

FIG. 4. Reflectivity (dBZ) of the convective storms detected by the McGill S-band radar at (a) 1810, (b) 1840, (c) 1910, (d) 1940, and (e) 2010 UTC 12 Jul 2004. The red dot in (a) indicates the location of the radar. The dotted line in (b) indicates the location of the cross section in Figs. 10 and 14. The maps are at an altitude of 2.5 km.
error is then diagonal. The observation term in the cost function can thus be written as

$$J_o = \sum_{n=1}^{N} \left[ \frac{(V_r - V_{r,obs})^2}{\sigma_{rad}^2} + \frac{(Z - Z_{obs})^2}{\sigma_{ref}^2} \right],$$

where $\sigma_{rad}$ and $\sigma_{ref}$ are the standard deviation of the observation errors of the radial velocity and reflectivity, respectively. In the current configuration of the system, we use values of $\sigma_{rad} = 1$ m s$^{-1}$ and $\sigma_{ref} = 3$ dB.

We point out that the assumption of uncorrelated observation errors, acceptable at large scales when observations are from radiosondes that are launched far apart, may not be applicable for high-resolution radar data. Recent progress has been made in understanding the structure of errors affecting radar observations. For example, Berenguer and Zawadzki (2008) analyzed cases of stratiform precipitation over Montreal and obtained the structure of the error covariance matrix of the surface radar rainfall, showing how these errors are significantly correlated in time and for several grid points in space. To fully characterize the three-dimensional error structure of volumetric radar observations, especially in convective precipitation, much work is yet to be done. Data thinning is a possible alternative for dealing with this problem [see, e.g., Montmerle and Faccani (2008) who “thinned” Doppler winds for their assimilation over France]. In our case, we opted for the very usual alternative of assimilating all available high-resolution observations. However, achieving this goal with a diagonal $R$ matrix (i.e., assuming no correlation between observation errors) may result in too much weight being given to the observation term and, thus, to a suboptimal analysis (as discussed by Liu and Rabier 2003). Currently, when the description of the observation error covariance matrix is still an unsolved topic, this approximation would, at least, guarantee a good depiction of the convective activity, which is crucial at these scales, especially when no convection is present in the background term (as it is the case for the case study presented in section 4).

c. Model term

The Mesoscale Compressible Community (MC2) atmospheric model (Laprise et al. 1997), coupled with the Kessler (1969) microphysics scheme, is used in the McGill data assimilation system as a weak constraint. The governing equations are

$$\frac{du}{dt} = -fv + R(T' + T^*) \frac{\partial q'}{\partial x} = \varepsilon_{mx},$$

$$\frac{dv}{dt} = fu + R(T' + T^*) \frac{\partial q'}{\partial y} = \varepsilon_{my},$$

$$\frac{dw}{dt} = -g \frac{T'}{T} + g \frac{(M + Q_s)}{\rho} + R(T' + T^*) \frac{\partial q'}{\partial z} = \varepsilon_{mz},$$

where
\[
\frac{dq'}{dt} - \frac{wg}{RT^*} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} - \frac{1}{(T^* + T')} \frac{dT'}{dt} = T_c^q,
\]
\[
\frac{dT'}{dt} - a(T^* + T') \left( \frac{dq'}{dt} - \frac{wg}{RT^*} \right) - L = T_h^q,
\]
\[
\frac{dM}{dt} + \frac{Mw}{H} + M \frac{\partial V_z}{\partial z} - S(M, m) = T_c^q,
\]
\[
\frac{dm}{dt} + \frac{mw}{H} - wG + S(M, m) = T_c^q.
\]

where \(Q_{sv}\) is the saturated water vapor density; \(Q_c\) is the cloud water density, and \(Q_v\) is the water vapor density. The negative values of \(m\) correspond to a deficit of water vapor relative to saturation. The generating function \(G\) in (12) is given by \(G = -[Q_{sv}/H + (\partial Q_{sv}/\partial z)]\).

The source term \(S\) includes three processes: the autoconversion of cloud droplets to precipitation (AC), the collection of cloud droplets by precipitation (CC), and the evaporation of precipitation (EP). The forms of the three processes are

\[
AC = k_1 (m - a) \hat{H}(m - a),
\]

\[
CC = k_2 EN_0^{1/8} m M^{7/8} e^{d/2h} \hat{H}(m),
\]

\[
EP = k_3 N_0^{7/20} m M^{13/20} \hat{H}(-m),
\]

All variables and constants in (12) are given in Tables 1 and 2. The total derivatives following the fluid are defined as \(d/dt = \partial/\partial t + u(\partial/\partial x) + v(\partial/\partial y) + w(\partial/\partial z)\) and \(d_M/dt = \partial/\partial t + u(\partial/\partial x) + v(\partial/\partial y) + (w + V_z)(\partial/\partial z)\). The total derivative for rain contains part of the sedimentation term \((\partial V_z/M/\partial z)\).

The cloud water content \(m\) is defined as

\[
m = Q_c + Q_v - Q_{sv},
\]
and $k_3 \times 10^{-6}$ s$^{-1}$ are the constants for AC, CC, and EP, respectively.

**d. Minimization of the cost function**

Since the model is used as a weak constraint, the assimilation period determines the size of the control variables in the analysis. In the current study, three time levels are used, so the size of the control variables is the number of control variables multiplied by the number of grid points and the number of time levels.

An iterative conjugate-gradient minimization algorithm is used. The iteration terminates when the following convergence criterion is achieved:

$$\frac{J_{i-1} - J_i}{J_i} < 10^{-7},$$
where $J_i$ is the cost function at the $i$th iteration. The analysis fields are the optimal solution within the assimilation period.

Previous studies in Caya (2001) obtained the optimal analysis in the McGill data assimilation system by using a three-step approach in the minimization of (1). First, the full three-dimensional wind field in space and time is obtained by the assimilation of Doppler radar reflectivity and wind observations using the anelastic mass-continuity equation and the linear wind analysis as constraints. Second, by making use of the momentum equations as a constraint, the corresponding pressure and temperature fields are retrieved. Third, with the help of the microphysics equations, the humidity–cloud variables consistent with the dynamics and the observed precipitation rate are obtained. Once all the control variables are retrieved, a good first guess for minimizing (1) can be obtained. In contrast to Caya’s previous study, here we use a high-resolution model forecast as the background field to provide the first guess for the minimization. Furthermore, since the background fields furnish information on all the control variables in the first guess, it is not necessary to use a three-step approach and the optimal solution for all the control variables can be obtained simultaneously by minimizing (4).

e. Single-observation experiment

To understand the modified assimilation system and to illustrate how the background term spreads the effects of each radar observation over the analyzed grid

\[\text{FIG. 8. Simulated reflectivity at a height of 2.5 km from (a) 5-min forecast valid at 1815 UTC, (b) 30-min forecast valid at 1840 UTC, and (c) 60-min forecast valid at 1910 UTC.}\]
points, a single-observation test with a reflectivity of 50 dBZ and a Doppler velocity of 20 m s\(^{-1}\) at 2 km in the center of the analyzed domain is performed. Figure 1 shows the analysis increment response to this single radar observation at a height of 2 km. The horizontal wind component, \(u\), illustrates the obvious increment from the center grid (observation) to its neighboring grid points, and this is due to the assimilation of the Doppler wind observations. In the current implementation, although we did not apply cross correlation between the control variables in the background term, the thermodynamic fields such as the temperature field still acquired some of the increment (Fig. 1b), and this is created via the governing equations (model term) in the cost function.

3. Data preprocessing

a. Radar data

The McGill S-band radar requires 5 min to complete a volume scan with 24 elevation angles for reflectivity and radial velocity. Before the assimilation, the radar data
are interpolated into a Cartesian coordinate. The projection of the observations on a model grid point is performed by using weighted averages of the data in neighboring points and a prescribed radius of influence. Ground echoes are eliminated using a predetermined ground echo mask obtained during nonprecipitation conditions. In addition, in order to account for the evolution of a precipitation system, a linear interpolation between two successive volume scans is made to yield the data at a single reference time, corresponding to a model time level. An interpolated measurement can be expressed as

\[ c^*(t*) = \frac{c(t_1) + c(t_2)}{t_2 - t_1} (t_2 - t_1)/c_0, \]

where \( c \) is the measured variable (reflectivity and radial velocity), \( c^* \) and \( t^* \) are the reference value and time for the linear interpolation, and \( t_1 \) and \( t_2 \) are the times for the successive scans, respectively.

**b. Background field**

A short-term 3-h forecast using the MC2 model with a resolution of 1 km is used as the initial background field. This short-term forecast is obtained as follows. First, the MC2 model is run at 15-km resolution (over domain A in Fig. 2) for 12 h from 1200 UTC 12 July to 0000 UTC 13 July 2004 using the initial conditions (at 1200 UTC 12 July) and boundary conditions (every 3 h) provided by a 15-km Global Environmental Multiscale (GEM) operational model forecast. Then, using a one-way nesting strategy, a second MC2 run with a resolution of 2.5 km (over domain B in Fig. 2) is performed for the same 12-h forecast period. Finally, the 3-h forecast from the 2.5-km MC2 run at 1500 UTC 12 July is interpolated onto the 1-km grid (domain C in Fig. 2) as the background field.

c. Implementation of the cycling process

Since phenomena on the convective scales change quickly with time, a single assimilation window may not capture the evolution of the convective storms. It is desirable to use a cycling process to assimilate the observed radar data over more than one time window and thus obtain better analyses at different reference times. A better forecast may be expected by periodically reinitializing the numerical model using the updated analyses from the cycling process. However, Xiao et al. (2005) and Sun (2005a) found that due to the spinup problem of the forecast model, a too frequent assimilation of the observations is not recommended. The McGill assimilation system therefore adopted a cycling process consisting of two 10-min assimilation windows, as illustrated in Fig. 3. In the second assimilation window, the background fields are first updated from the model 30-min forecast, and radar observations are assimilated for another 10 min. The new analyses from the assimilation system are then used to initialize the model at 1840 UTC 12 July to perform the forecast again.

4. Case study and the results

a. Case description

The case of 12 July 2004 is selected because of its strong, long-lasting convective-scale storms. The cells are located about 60–70 km south of the McGill radar site. Figure 4 depicts the evolution of the storm at 2.5-km altitude every 30 min starting at 1810 UTC 12 July. The storm is almost stationary, with the individual cells undergoing the developing and dissipating stages very rapidly.

b. Impact of the background fields

In the study of Montmerle et al. (2001, 2002), the sounding data are used and modified to retrieve the thermodynamic fields of pressure and temperature. In addition, the refractivity is used to estimate the humidity field in one lower level. However, the refractivity field is not available for this case study, and the thermodynamic fields are provided from the previous forecasting data to the current assimilation system. The background fields such as temperature and moisture cover the whole domain, and one can examine the properties of the background term in the data assimilation by estimating the convective available potential energy (CAPE). The CAPE is computed by mixing the air in the lowest 60 hPa. Figure 5 shows that the high values of CAPE
from the reference background field (without data assimilation) are distributed over almost the entire domain of the analysis. Since the observed values of CAPE in thunderstorm environments often exceed 1000 J kg\(^{-1}\), the results indicate that if the dynamic field is correct, the background field from the large-scale analysis has the potential to support convective storms.

c. Effect of assimilation on the optimal analyses

After assimilating the radar data for 10 min, the MC2 model is first initialized at 1810 UTC. Figure 6 depicts the horizontal wind fields at 2 km at 1810 UTC. Note that far away from the radar echoes (Fig. 4a), because the impacts of the error covariance are localized, the \(u\) component of the background wind (Fig. 6a) is similar to that of the optimal analysis from the assimilation system (Fig. 6b). A similar conclusion can be drawn regarding the \(v\) component (Figs. 6c and 6d). Note also that the \(u\) and \(v\) components are substantially different in the neighborhood of the radar echoes, a direct consequence of setting the horizontal error correlation length scale to 3 km, which induces the increment (analysis–background)
at the observational points to propagate to the neighboring grid points.

d. Discussion of results

The optimal analysis fields, obtained from the assimilation system, are used as the initial conditions in the MC2 model. In addition, even though only warm rain microphysics is used in the data assimilation, both warm and cold microphysical processes (Kong and Yau 1997) are included in the model forecast at 1-km resolution over a domain of area $250 \times 200$ km$^2$. The MC2 model has 25 nonuniform vertical levels with the top of the model at an altitude of 25 km. In this case study, even though the model runs with a cloud-resolving high-resolution scheme, it could not predict the storms at the right locations. However, after assimilating radar observations, the model is able to trigger the storm in the right location and predict its evolution reasonably well.

1) FORECASTS WITH ONE ASSIMILATION WINDOW

For the forecast with one assimilation window, the MC2 model initiated at 1810 UTC using the optimal analyses from the assimilation system is integrated forward for 1 h until 1910 UTC. Figures 7a and 7b show the divergence and vorticity fields at a height of 1 km for the 5-min forecast. The low-level convergence is very apparent and vorticity couplets are also present. These results illustrate the characteristics of the early stage of convective development and indicate that, by assimilating radar observations, the convective storm is readily
triggered by the model. The rain mixing ratio from the model output has been converted to predicted reflectivity using Eq. (9). Figure 8 demonstrates the simulated reflectivity during 1-h prediction. The pattern and the location of the 5-min forecast of the simulated reflectivity (Fig. 8a) are in good agreement with the observation at 1815 UTC (not shown). However, the intensity of the simulation is smaller than that of the observation, and this is due to the spinup problem of the model.

The intensity of the low-level convergence for the 30-min forecast in Fig. 9a is compatible to that for the 5-min forecast, and many cellular patterns are evident in the storm system. The low-level cyclonic vorticity (Fig. 9b) also shows several small structures at this stage. Figure 10 depicts the cross section for the 30-min forecast of the vertical velocity field at 1840 UTC along the path shown in Fig. 4b. The updraft between 180 and 200 km corresponds to the main convective cell observed in Fig. 4b. The simulated secondary updrafts around 100 and 130 km

FIG. 13. The 30-min model forecasts obtained with the cycling process: (a) the divergence field at 1 km and (b) the vorticity field at 1 km (zero contour dotted, positive values solid, and negative value dashed; \(10^{-5} \text{ s}^{-1}\)).
also show agreement with the locations of radar echoes around these locations. Note that the main updraft is not completely vertical but slants upward with height, and this tilting character is a favorable condition for severe storm development. It is quite remarkable that the simulated reflectivity at this stage (Fig. 8b) reproduces well the situation observed by the radar (Fig. 4b).

In contrast to the early simulation, the 1-h forecast for 1910 UTC in Fig. 11 shows the intense divergence when the cyclone vorticity is present at the low level. Although the simulated reflectivity in Fig. 8c still predicts the storm in the analysis domain, it no longer matches the radar observations well (Fig. 4c). In fact, the observed storm cells continue to develop but the forecast storm cells are dissipating. We conclude from this result that with one assimilation window the forecast skill is limited to a forecast time of less than 1 h.

We have also analyzed the impacts of assimilating only one radar parameter, either the radial velocity or the reflectivity. The results of these experiments (not shown) are the following: when only radial velocity is assimilated, the model needs a longer time to generate precipitation (20 min after simulation); alternatively, when only reflectivity data are assimilated, the model fails to accurately capture the dynamics of the storms (e.g., areas of convergence), and the precipitation areas last for a shorter period of time than were observed by the radar.

2) FORECAST WITH A CYCLING PROCESS

After assimilating radar data during a second assimilation window, the new analyses are used to reinitialize the model for a second period of forecasts. Figure 12 depicts the simulated reflectivity after this cycling process. Figure 12a reveals that the analysis time of 1840 UTC better resembles the radar observations in Fig. 4b. Via the cost function, the control variable (rain mixing ratio here) minimized the differences between the background (previous simulation Fig. 8b) and the new observation (Fig. 4b).

The low-level divergence and the vorticity fields of the 30-min forecast (Figs. 13a and 13b, respectively) depict many small-scale structures. Comparison with the same valid time (1910 UTC) reveals that many locations have a stronger convergence with the cycling process than with the simulation when using one assimilation window (Fig. 11a). The results underscore the impacts of the cycling process: the dynamics and thermodynamics are reconstructed via the second assimilation window. Figure 14 depicts the cross section of the vertical velocity at 1910 UTC. The updrafts still tilt upward with height. Furthermore, the magnitudes of the updrafts and downdrafts are much stronger compared with those obtained with one assimilation window at 1910 UTC (not shown).

For the 1-h forecast at 1940 UTC, the simulated (Fig. 12b) and the observed (Fig. 4d) reflectivity fields compare rather well, although the model fails to reproduce the small cell in the west but captures the location and the intensity of the main storm in the southeast. We can thus say that, on the basis of this one example, the McGill data assimilation system with the cycling process may improve the dynamic and thermodynamic environment of the storm leading to a convective storm predictability of at least 1 h. The 1.5-h forecast of simulated reflectivity at 2010 UTC (Fig. 12c) correctly maintains the intensity of the storm system but moves it farther downwind from the actual location of the observed radar echoes in (Fig. 4e). A possible reason for this the position error is discussed in the next section.

5. Quantitative verification of the results

How best to quantitatively evaluate the model forecast at the convective scale is not obvious. In this case study, since the lifetime of the storm remains under the McGill radar coverage, a quantitative verification is performed using the high spatial and temporal resolutions of the radar observations.

a. Radial component of the wind

With single-Doppler radar measurements, one can verify only the radial component of the wind field. From Eq. (10), the simulated radial velocity can be constructed from the model output and a comparison made with the observed radial velocity. As a measure of the error, we use the normalized root-mean-square error defined as
Wavelet analysis of predictability at different scales

The observed (Fig. 4d) and simulated (Fig. 12b) reflectivities at 1940 UTC display similar patterns in cell structure, but discrepancies are also evident. To determine at what scale the model exhibits strong predictability in the forecast lead time, we have performed an analysis using the wavelet transform technique described in Turner et al. (2004).

Wavelet analysis decomposes data into different scales or resolutions. Compared with Fourier analysis, the localized pulse functions in the wavelet transform can provide a better representation of localized, intermittent fields. In this study, the 2D Haar wavelet and its associated smoothing function are used. The total spectrum represents the sum of the contributions from the observations (or forecasts) from each length-scale interval, and the cospectrum presents the product of the wavelet coefficients of observations and forecasts at each scale. To compute the wavelet spectrum $S$ and cospectrum $(Co)$, coefficients from the two-dimensional wavelet transform are used:

$$
NRMS = \sqrt{\frac{1}{N} \sum (V_{r,\text{obs}} - V_{r,\text{simu}})^2 / \sqrt{\frac{1}{N} \sum (V_{r,\text{obs}})^2}},
$$

where $N$ is the total number of analysis points and the subscripts “obs” and “simu” indicate the observed and simulated radial velocities, respectively.

Figure 15a shows the error from the single assimilation window case. The comparison between the 0-h forecast (solid line, with data assimilation) at the initial time at 1810 UTC and the model background field (dotted line, without assimilating radar data) indicates that the analysis from the assimilation has smaller errors at all vertical levels. The results in the forecasted radial velocity show smaller errors in the lower levels compared to the higher levels. Three reasons may explain the large errors in the upper levels: first, the increment (resolution) in elevation angle of the McGill S-band radar increases geometrically with elevation, and this results in fewer radar measurements being available for assimilation at higher altitudes. Second, for regions with high reflectivity gradients (in the storm top), beam broadening could result in a systematic bias of the velocity information. Third, ice phase processes are absent in the assimilation system and may produce larger errors at higher levels. We note that the vertical distribution of the errors in the radial wind in our study is comparable with the results in Sun (2005b).

The errors for the cycling process are presented in Fig. 15b. The behavior patterns of the errors are consistent with those in Fig. 15a. Comparison of the forecasting errors at 1910 and 1940 UTC reveals larger errors at the upper levels with the cycling process, which may explain the greater eastward drift in time of the forecast storm system relative to the radar observations. However, this interpretation should be considered with caution as the error in the tangential component of the wind is unknown.

b. Wavelet analysis of predictability at different scales

The observed (Fig. 4d) and simulated (Fig. 12b) reflectivities at 1940 UTC display similar patterns in cell structure, but discrepancies are also evident. To determine at what scale the model exhibits strong predictability in the forecast lead time, we have performed an analysis using the wavelet transform technique described in Turner et al. (2004).

Wavelet analysis decomposes data into different scales or resolutions. Compared with Fourier analysis, the localized pulse functions in the wavelet transform can provide a better representation of localized, intermittent fields. In this study, the 2D Haar wavelet and its associated smoothing function are used. The total spectrum represents the sum of the contributions from the observations (or forecasts) from each length-scale interval, and the cospectrum presents the product of the wavelet coefficients of observations and forecasts at each scale. To compute the wavelet spectrum $S$ and cospectrum $(Co)$, coefficients from the two-dimensional wavelet transform are used:

$$
S(m) = 4^{-m} \sum_{i=1}^{3} [W_i(m, x, y)]^2 \quad \text{and}
$$

$$
Co_{fi}(m) = 4^{-m} \sum_{i=1}^{3} W_i(m, x, y) W_i^*(m, x, y),
$$

where $W_i(m, x, y)$ are the wavelet coefficients of the $i$th wavelet component for a scale $m$. The subscripts $f$ and $o$ stand for the forecast and the observation, respectively. The overbar denotes a spatial average. Interested readers are referred to Turner et al. (2004) for further details.

The wavelet spectra are computed after applying the cycling process. Figure 16 depicts the spectra from the observed and the computed radar reflectivities (dBZ) every 20 min from 1840 to 1940 UTC on 12 July 2004. The spectral values are normalized by the mean square of the radar reflectivity (dBZ). In the observed spectra, the maximum occurs near the 10–20-km interval for all four times, which is indicative of the scale of the convective storms under consideration. In contrast, the peak values in the computed spectra vary with forecasting time. At 1840 UTC, the observed and computed peak values agree due to the assimilation of the radar observations. After 20 min, at 1900 UTC, the computed peak values are found at smaller scales relative to the observations. After 1920 UTC, the computed spectral peaks shift back to the larger scales, in better agreement with the observations. These results reveal that, after the initialization at 1840 UTC, the model undergoes some adjustment at 1900 UTC. Following the adjustment, the predicted spectra improved.

The cospectra depicted in Fig. 17 are normalized by the product of the root-mean-square of the forecast and observations. A cospectra value of 1 means that the observed and simulated reflectivities are identical. The cospectra peak at a scale between 10 and 20 km in the first 20 min, in good agreement with the observations. After 20 min, the peak values of the forecast cospectra decrease and their length scales shift to 32 km, although there are secondary peaks at scales of 10–20 km. The results of the cospectra analysis indicate that, even with a model resolution of 1 km, the highest predictability occurs at scales of less than 30 km for a 30-min forecast when storms are rapidly evolving.
6. Summary and future work

The McGill data assimilation system deals with the very short-term forecasts at convective scales. To take into account the model errors, and to reduce the large number of computations, a variational algorithm with a weak constraint is applied in the assimilation system. A recursive filter is implemented to improve the background term and the optimal analysis is then used to initialize the MC2 cloud-resolving model. Using single–Doppler

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Fig. 15. Verification of the radial velocity: (a) from a single assimilation window forecast and (b) from the cycling process.
radar observations, and with one assimilation window of the radar reflectivity and radial velocity, the McGill radar assimilation system successfully initiated the convective storms near the observed locations. In contrast with previous studies that used a linear wind as background, we have instead taken the output of a short-range forecast. This new strategy has the advantage that the minimization problem has a unique solution and the background field from the short-range forecast is more consistent with the model dynamics. Furthermore, with the CAPE parameter provided by the background short-range forecast, the initial thermodynamic environment is better prescribed than in previous studies (e.g., Montmerle et al. 2001, 2002) that were based on a single sounding far from the location of the convection and that needed an adjusted CAPE value to initiate convection.

With only one assimilating window, the MC2 model simulates the selected convective storms, but the forecast precipitation deviates substantially from the observations after 1 h of simulation. The underlying reason is that the initial fields from the assimilation system contain substantial errors. The observed storms evolved rapidly and the forecast error from the numerical model grew quickly over time. The cycling process improves the predicted intensity and location of the storm beyond 1 h, but the 1.5-h simulation indicates the obvious positional errors. An analysis of the evolution of the errors in the radial component of the wind reveals large errors are at the higher levels and may explain why the predicted storm system is displaced farther downwind from its true location. The cospectra of the wavelet analysis indicate that the predictable scale in the first 20 min of the forecasts is quite comparable with the scale of the convective storms (10–20 km), but the predictability degrades to the larger scales of ∼30 km beyond 30 min.
More case studies need to be tested in order to confirm the conclusions of the current study. In addition, we are attempting to estimate the forecast (background) errors at the convective scale via ensemble forecasts. Other issues, such as the nature and structure of the error covariance of the radar reflectivity and radial velocity, and the problem of model errors in data assimilation, still need to be addressed in data assimilation at the convective scale. The improvement of numerical weather prediction at storm-scale depends on their successful resolution.

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