Electric Field Reversal in Sprite Electric Field Signature

Richard G. Sonnenfeld

Langmuir Laboratory and Physics Department, New Mexico Tech, Socorro, New Mexico

William W. Hager

Department of Mathematics, University of Florida, Gainesville, Florida

(Manuscript received 3 August 2012, in final form 18 December 2012)

ABSTRACT

In measurements of the electric field associated with the current of a sprite 450 km from ground-based field sensors, it was observed that the sign of the electric field was positive when positive charge was lowered from the ionosphere. A recent model for the electric field associated with the sprite current also predicts positive field changes at 450 km from the sprite. A well-known analysis of a vertical dipole in a thundercloud shows that the electric field on the ground reverses its sign at an easily computed distance from the dipole. A similar simplified electrostatic analysis of a sprite predicts a field reversal distance around 130 km. A more accurate electrodynamic analysis based on Maxwell's equations indicates that the field reversal distance should be between 70 and 80 km.

1. Introduction

In a recent paper (Hager-Sonnenfeld et al. 2012), we studied a series of sprites, luminous glows in the mesosphere above thunderclouds extending roughly from 50 to 90 km in altitude. Figure 1 shows a picture of a carrot sprite taken from Langmuir Laboratory, about 467 km west of the sprite, at 0527:09.69 UTC 15 July 2010. Three instruments (slow antennae) in the Langmuir Electric Field Array (LEFA) measured the electric field during the storm. Figure 2 shows the vertical electric field that was measured at LEFA station 2 during the carrot sprite. Observe that the electric field is predominantly positive while the sprite is descending from the ionosphere. A positive hump in the electric field from a sprite was also reported by Stanley et al. (2000) where it is referred to as the sprite's signature. In this paper, we study the dependence of the electric field on the distance from the sprite to the observer. We show that for the carrot sprite of Fig. 1, the sign of the electric field should change from positive to negative as the distance to the sprite decreases. For this particular sprite, the field reversal distance is between 70 and 80 km.

We make our argument in section 2 with a simplified model of a sprite to provide intuition about its electric field signature. In sections 3 and 4 a more accurate model is used to confirm our intuition. Beneath the sprite lies a thunderstorm with a complex charge structure that may be safely ignored; thunderstorm charging occurs on minute times scales (e.g., Golde 1977, 64–98), while the sprite we present here lasts only three milliseconds. Thus, thunderstorm charging processes should not be expected to appear in the sprite data.

2. Simplified electrostatic model

Before launching into a more complete model for the electric field from a sprite, let us first develop our intuition by reviewing a well-known analysis of a thunderstorm electric field (recently republished by Rakov and Uman 2003, 69–72). Figure 3 shows a vertical dipole charge over a perfectly conducting plane with a positive charge at height h_p underneath an equal negative charge at height h_n. (In analyzing a storm, one usually puts the positive charge over the negative charge, but we have a reason for switching the charges in this discussion.) When the observer on the ground is at a location P_1 close to the dipole, the lower positive charge results in

---

Corresponding author address: Richard G. Sonnenfeld, Langmuir Laboratory and Physics Department, New Mexico Institute of Mining and Technology, 801 Leroy Pl., Socorro, NM 87801-4681.
E-mail: rsonnenfeld@gmail.com

DOI: 10.1175/MWR-D-12-00220.1

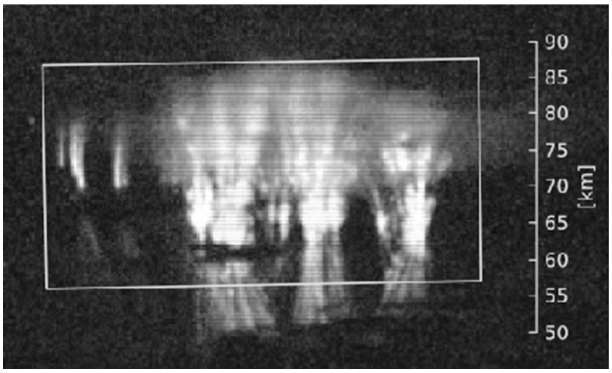

FIG. 1. The sprite of 0527:09 UTC as seen in a still frame from a 30 frames per second (FPS) Watec camera. The sprite extends between 50 and 90 km in altitude. The added rectangle shows the field of view of a telescopic video camera that also recorded this sprite at several thousand FPS.

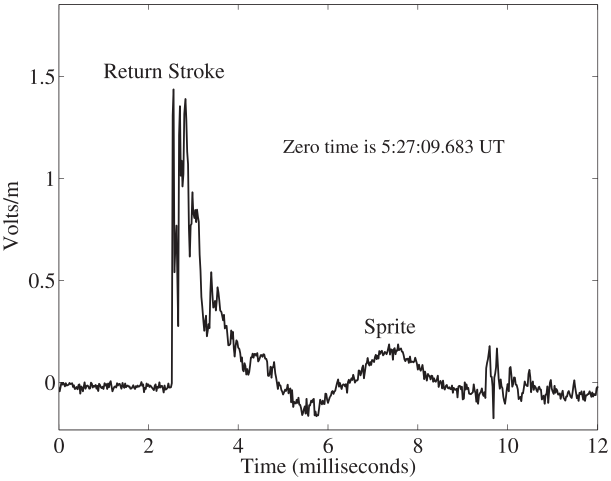

FIG. 2. Electric field seen at a point 450 km west of a large positive cloud-to-ground flash. The hump labeled "sprite" coincides to within a fraction of a millisecond with the peak in a light curve of the sprite produced by that flash.

a negative electric field (pointing downward). When the observer is at a location P_2 far from the dipole, both charges are about the same distance from the observer and produce fields of roughly the same magnitude. However, the negative charge at higher altitude has a larger vertical component (by simple trigonometry), resulting in a net positive electric field at the observer. Given that there is sign change between P_1 and P_2, there must be a distance (called the "field-reversal distance" and designated D_0) where the net electric field from the dipole is zero. A little algebra gives the formula:

$$D_0 = \sqrt{(h_p h_n)^{\alpha}(h_p^{\alpha} + h_n^{\alpha})}, \quad \alpha = 2/3. \tag{1}$$

It is thought that a sprite is a manifestation of classical breakdown caused by the increased fields above a storm that has just experienced a large positive cloud-to-ground flash; for example, see the theory of Pasko et al. (1997). In a simplified model of a sprite, we can consider it as inserting a positive charge in the atmosphere descending from the ionosphere. On time scales at which the ionosphere can be modeled as a perfect conductor, the positive charge, which is the sprite leader tip, should be mirrored by an equal and opposite negative charge that ascends above the ionosphere. Thus, Fig. 3, which at first glance appears to be a model of a thundercloud (with polarity reversed from the typical case), can be considered, with the addition of a conducting ionosphere midway between the two charges, to be an electrostatic model of a sprite.

If the simplified figure is the same, then the simplified math is also the same, and we can apply Eq. (1) to sprites. If the ionosphere is located at 100 km and the height of the positive charge is $h_p = 50$ km, then the height of the negative image charge is $h_n = 150$ km and the field reversal distance D_0 is about 127 km.

This calculation oversimplified the true physics. Both the conductive properties of the ionosphere and the conductivity of the surface of the earth must be accounted for. Moreover, for distant electromagnetic disturbances, the electrostatic contribution to the electric field to which Eq. (1) applies is often much smaller than the inductive and radiation contributions to the field. The next section of this paper provides a more accurate model for the electric field associated with the sprite current.

3. Modeled electric field for sprite

In Hager-Sonnenfeld et al. (2012) a model for the electric field from a sprite is developed and is based on

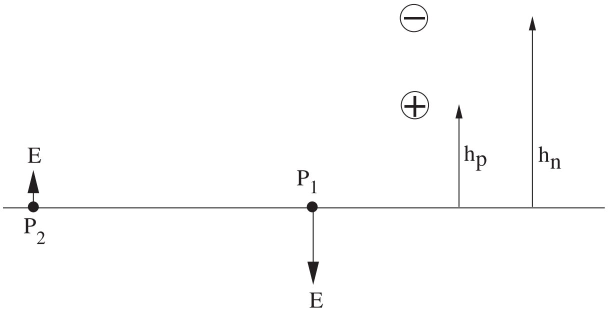

FIG. 3. Electrostatic field reversal associated with a vertical dipole. Observer P_1 on the conducting ground experiences a downward-directed (negative) electric field, while the distant observer at P_2 measures a positive electric field.

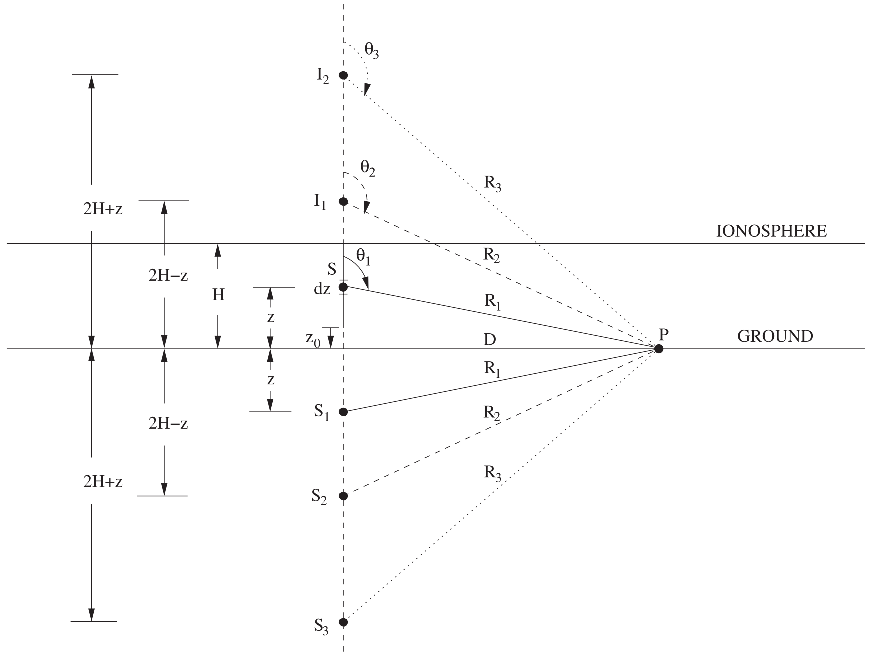

FIG. 4. The image dipoles generated by the source dipole current at altitude z; S_1 is the image of S reflected in the ground plane and I_1 is the image of S reflected in the ionospheric plane. For $k > 1$, S_k is the subterranean image associated with I_{k-1} above the ionosphere.

the following approximations: the earth and ionosphere are treated as perfectly conducting horizontal planes, and the sprite current is assumed to travel along an infinitely thin wire connecting the altitude z_0 and the ionosphere at altitude H. The formula for a special exact solution to Maxwell's equations given by Uman et al. (1975), leads to the following relation for the vertical electric field at an observation point P on the ground:

$$E(t) = \sum_{k=1}^{\infty} E_k(t), \tag{2}$$

where

$$\begin{aligned} E_k(t) = \frac{(-1)^{k+1}}{2\pi\varepsilon_0}\Bigg\{&\int_{z_0}^{H}\int_{0}^{t}\left[\frac{2-3\sin^2\theta_k(z)}{R_k(z)^3}\right] i\left[z,\tau-\frac{R(z)}{c}\right] d\tau\, dz \\ &+ \int_{z_0}^{H}\left[\frac{2-3\sin^2\theta_k(z)}{cR_k(z)^2}\right] i\left[z,t-\frac{R(z)}{c}\right] dz \\ &- \int_{z_0}^{H}\frac{\sin^2\theta_k(z)}{c^2R_k(z)}\frac{\partial i[z,t-R(z)/c]}{\partial t}\, dz\Bigg\}. \end{aligned} \tag{3}$$

Here i is the current in the sprite, and if D denotes the distance from P to the base of the sprite, then $R(z) = \sqrt{D^2 + z^2}$. Thus, $R(z)$ is the distance between a point on the sprite at altitude z and the observer. We also define $R_k(z) = \sqrt{D^2 + z_k^2}$ where

$$z_k = \begin{cases} kH - z & \text{if} \quad k \text{ is even}, \\ kH + z - H & \text{if} \quad k \text{ is odd}, \end{cases}$$

and $\sin\theta_k(z) = D/R_k(z)$. The three terms on the right-hand side of Eq. (3) are often called the electrostatic term, the induction term, and the radiation term.

Equations (2) and (3) were derived using image charge techniques for a *dipole current generator*. The parameters $R_k(z)$ give the location relative to the observation point P of the image dipole current generators associated with the source generator at altitude z (see Fig. 4). The current i was modeled as in a transmission line (e.g., Uman and McLain 1969):

$$i(z,t) = i(t + z/v),$$

where z is altitude and v is the velocity of the downward descending current pulse. If we take $v = 0.4c$, which approximates the mean velocity of about $0.37c$ for a lighting return stroke reported by Idone and Orville (1982), then the sprite current that best fits the measured electric field at LEFA station 2 is shown in Fig. 5. The current is negative, which indicates that a positive charge is transported down from the ionosphere by the sprite tip.

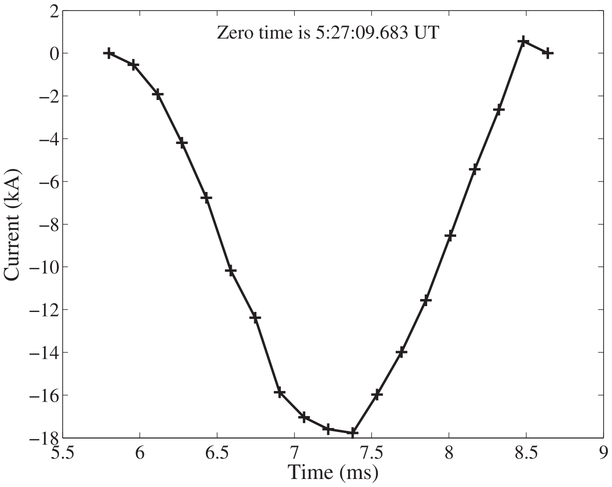

FIG. 5. The sprite current at the top of the sprite channel (the ionosphere) as a function of time. The current model decimates the measured electric field down to 19 samples, each of which is shown by a plus sign. The line joining the points is provided only for readability.

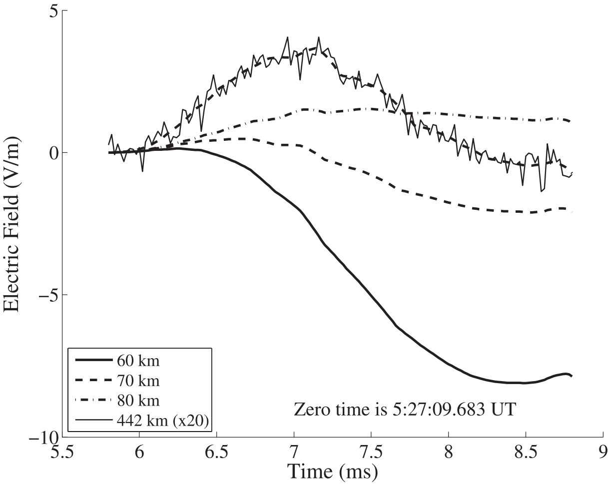

FIG. 6. The modeled sprite electric field at distances between 60 and 90 km from the sprite. Also shown is the measured electric field times the factor 20 for the sprite of 15 Jul 2010. The time axis is offset to have the same zero as the time axis used in Figs. 2 and 5. (The measured data are the thin ''noisy'' line, while the modeled electric field based on the current of Fig. 5 is the thicker line visible inside the measured data.)

4. Field reversal distance and discussion

The estimated sprite current, determined by fitting our model to the experimental data and shown in Fig. 5, can now be inserted in Eqs. (2)–(3) to obtain the electric field at various distances from the sprite. In Fig. 6, the electric field at distances from 60 to 90 km is shown. For comparison, the modeled and measured field at 442 km is also shown. Note that the modeled *E* field is much quieter than the actual *E* field. Our model decimates the measurements down to 19 points (shown by plus signs in the figure) so it is inherently less noisy. Observe that between 70 and 80 km, the modeled *E* field changes from negative, to almost zero, to positive. The simplified electrostatic model from section 2 was useful for giving intuition, and it gives a field reversal distance within a factor of 2 of the more accurate model based on an exact solution of Maxwell's equations and an infinite number of image dipoles.

Note that the *E*-field curve (in Fig. 6) at 442 km returns to zero at the end of the plot while the nearer models do not return to zero. This is to be expected. The electrostatic term of the exact solution depends on the cube of distance from the sprite, while the inductive and radiation terms have a $1/R^2$ and $1/R$ dependence, respectively. Thus, at the larger distance the field returns to zero because the current has gone to zero. However, at closer distances, the electrostatic term keeps the field away from zero because at nearer distance one can see that a net charge has been moved to lower altitude as the result of the sprite. The field reversal distance for a sprite is of course much larger than field reversal distances often observed for lightning in thunderstorms. For example, if $h_p = 10$ km and $h_n = 5$ km, typical values for a thunderstorm, then the field reversal distance is about 10 km, which can be compared to an estimated field reversal distance between 70 and 80 km for the sprite of 15 July 2010. To our knowledge, no one has reported observing sprite-electric fields from within the reversal distance, but we hope this happens soon.

Our conclusions are significant because they open up a possible way to automatically detect sprites. Electric-field measurements are one of the oldest ways to characterize effects in atmospheric electricity. However, it was roughly 50 years between the first lightning electric-field measurements by Schonland (Schonland 1932) and the creation of the U.S. National Lightning Detection Network (NLDN; Idone et al. 1998), which interpolates electric and magnetic field measurements to precisely locate every lightning strike that occurs in the continental United States. Sprites are considerably harder to detect than lightning, and usually require a camera and a fortuitously aimed telescope. As upper-atmospheric phenomena are increasingly important in our satellite-dependent world, it could be useful to locate and report every sprite that occurs over the planet. The technology to do so could be similar (or the same) as the NLDN; only new recognition algorithms would be required. Seeing a reversal of the field change between nearby and distant stations would be extremely helpful in identifying sprites and separating them from the many other electromagnetic effects detected by these sensors.

Acknowledgments. We thank our collaborators G. McHarg, T. Kammae, and H. Stenbaek-Neilsen for their permission to reprint their video data shown in Fig. 1. This work was funded by Grant 0724750 from the National Science Foundation. Additional funding was provided by the Irving and Marion Langmuir bequest to Langmuir Laboratory. We thank Dr. Tsutomu Takahashi for asking a basic question about what sign is to be expected for the sprite hump.

REFERENCES

Golde, R. H., 1977: *Lightning.* Vol. 1, *Physics of Lightning*, Academic Press, 496 pp.

Hager-Sonnenfeld, and Coauthors, 2012: Charge rearrangement by sprites over a north Texas mesoscale convective system. *J. Geophys. Res.,* **117,** D22101, doi:10.1029/2012JD018309.

Idone, V. P., and R. E. Orville, 1982: Lightning return stroke velocities in the Thunderstorm Research International Program (TRIP). *J. Geophys. Res.,* **87** (C7), 4903–4916.

——, D. A. Davis, P. K. Moore, Y. Wang, R. W. Henderson, M. Ries, and P. F. Jamason, 1998: Performance evaluation of the U.S. National Lightning Detection Network in eastern New York: 1. Detection efficiency. *J. Geophys. Res.,* **103** (D8), 9045–9055.

Pasko, V. P., U. S. Inan, T. F. Bell, and Y. N. Taranenko, 1997: Sprites produced by quasi-electrostatic heating and ionization in the lower ionosphere. *J. Geophys. Res.,* **102** (A3), 4529–4561.

Rakov, V. A., and M. A. Uman, 2003: *Lightning Physics and Effects*. Cambridge University Press, 698 pp.

Schonland, B. F. J., 1932: *Atmospheric Electricity*. Methuen and Co., Ltd., 95 pp.

Stanley, M., M. Brook, P. Krehbiel, and S. A. Cummer, 2000: Detection of daytime sprites via a unique sprite ELF signature. *Geophys. Res. Lett.,* **27** (6), 871–874.

Uman, M. A., and D. K. McLain, 1969: Magnetic field of the lightning return stroke. *J. Geophys. Res.,* **74** (28), 6899–6910.

——, ——, and E. P. Krider, 1975: The electromagnetic radiation from a finite antenna. *Amer. J. Phys.,* **43,** 33–38.