New Parameterizations of Turbulence Statistics for the Atmospheric Surface Layer

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ABSTRACT: Recent work has shown that bulk-Richardson ($R_i$) parameterizations for friction velocity, sensible heat flux, and latent heat flux have similar, and in some instances better, performance than long-standing parameterizations from Monin–Obukhov similarity theory (MOST). In this work, we expanded upon new $R_i$ parameterizations and developed parameterizations of turbulence statistics, i.e., standard deviations in the 30-min $u$ (horizontal), $v$ (meridional), and $w$ (vertical) wind components (i.e., $\sigma_u$, $\sigma_v$, and $\sigma_w$, respectively), which allowed us to derive $R_i$-based parameterizations of turbulent kinetic energy ($\epsilon$), and standard deviations in the 30-min temperature and moisture measurements ($\sigma_T$ and $\sigma_q$, respectively). We used datasets from three 10-m micrometeorological towers installed during the Land Atmosphere Feedback Experiment (LAFE) conducted in Oklahoma from 1 to 31 August 2017 and evaluated the new parameterizations by comparing them against parameterizations from MOST. We used the LAFE datasets and fully independent datasets obtained from two micrometeorological towers installed in Alabama between February 2016 and April 2017 to evaluate the performance of the parameterizations. Based on the slope of the relationship between the observed and parameterized turbulence statistics ($\theta_i$) and the coefficient of correlation ($r$), we found that the $R_i$ relationships generally performed better than MOST at parameterizing $\sigma_u$, $\sigma_v$, $\sigma_w$, and $\sigma_T$ and the $R_i$ relationships performed better at low wind speeds than at high wind speeds. These results, coupled with recent developments of $R_i$ parameterizations for surface-layer momentum, heat, and moisture fluxes, provide further evidence to consider using $R_i$-based parameterizations in weather forecasting models.

SIGNIFICANCE STATEMENT: Deficiencies in Monin–Obukhov similarity theory (MOST) are well known, yet MOST forms the basis in weather forecasting models for describing heat, moisture, and momentum transfer between the land surface and atmosphere. We expanded upon previous work suggesting a MOST alternative called the bulk-Richardson approach. We used data collected from meteorological towers installed in Oklahoma and compared the bulk-Richardson approach with MOST. We evaluated these two approaches using data from meteorological towers installed in Oklahoma and Alabama and found that, overall, the bulk-Richardson approach performed better than MOST in determining the 30-min variability in temperature, moisture, and wind. This result provides additional motivation to use a bulk-Richardson approach in weather forecasting models because doing so will likely yield improved forecasts.

KEYWORDS: Surface layer; Turbulence; Parameterization

1. Background

For more than 50 years, Monin–Obukhov similarity theory (MOST) has been used to quantify near-surface exchanges of heat, moisture, and momentum in numerical weather prediction (NWP) models. MOST expresses gradients in surface-layer wind, temperature, and moisture fields as a function of a dimensionless stability length $\xi$, defined as

$$\xi = \frac{z - d}{L}. \quad (1)$$

In Eq. (1), $d$ is the displacement height of the vegetation, $z$ is the sampling height, and $L$ is the Monin–Obukhov length scale defined as

$$L = -\frac{\theta_i \mu_b^3}{g \kappa w \bar{u}^3}. \quad (2)$$

In Eq. (2), $\theta_i$ is the virtual potential temperature, $u_i$ is the friction velocity, $\kappa$ is the Von Kármán constant, $g$ is the gravitational acceleration, and $w^0 \bar{u}$ is the kinematic heat flux.

MOST is also used as the basis for scaling surface-layer turbulence quantities. Many different scaling relationships have been proposed that nondimensionalize the standard deviations in the wind components by dividing these quantities by $u_i$ and then relating the nondimensionalized forms to $\xi$. In general, under unstable conditions (i.e., when $\xi < 0$), these relationships are nonlinear and have the following form that has been suggested in many previous studies (e.g., Panofsky and McCormick 1960; Panofsky et al. 1977; Panofsky and Dutton 1984; Wilson 2008; de Franceschi et al. 2009; Srivastava et al. 2020):

$$\frac{\sigma_{u,w}}{u_i} = \alpha_{\sigma_{u,w}} (1 - \beta_{\sigma_{u,w}} \xi)^{1/3}. \quad (3)$$

In Eq. (3), $\sigma_u$, $\sigma_v$, and $\sigma_w$ are the standard deviations in the $u$ (i.e., horizontal), $v$ (i.e., meridional), and $w$ (i.e., vertical) wind components; and $\alpha_{\sigma_{u,w}}$ and $\beta_{\sigma_{u,w}}$ are empirically determined fitting coefficients. Under nearly neutral atmospheric regimes (i.e., when $\xi \approx 0$), values for $\sigma_u/u_i$ have been reported to range from 1.9 to 4.5, whereas values for $\sigma_v/u_i$ and $\sigma_w/u_i$ can range from 1.6 to 3.8 and from 1.1 to 1.5, respectively (e.g., de Franceschi et al. 2009).

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Under stable conditions (i.e., when $\zeta > 0$), there have been different functional forms that have been proposed for the relationship between $\sigma_{u_{x},w}/u_{x}$ and $\zeta$. For example, Al-Jiboori et al. (2002), de Franceschi et al. (2009), Quan and Hu (2009), and Srivastava et al. (2020) proposed that this relationship has the following form, again where $\sigma_{u_{x},w}$ and $\beta_{u_{x},w}$ are empirically determined fitting coefficients but only for stable atmospheric regimes:

$$\frac{\sigma_{u_{x},w}}{u_{x}} = \alpha_{u_{x},w} \left(1 + \beta_{u_{x},w} \zeta\right)^{1/3}. \quad (4)$$

In contrast, other studies (e.g., Pahlow et al. 2001) have reported the following relationship when $\zeta > 0$ (where $\gamma_{u_{x},w}$ is an empirically determined fitting coefficient):

$$\frac{\sigma_{u_{x},w}}{u_{x}} = \alpha_{u_{x},w} + \beta_{u_{x},w} \zeta^{\gamma_{u_{x},w}}. \quad (5)$$

There also remains no consensus on the forms of the relationship between the nondimensionalized forms of the standard deviation in temperature or moisture, i.e., $\sigma_{/q_{x}}$ and $\sigma_{/q_{x}}$, respectively; and $\zeta$. Andreas et al. (1998) reported that $\sigma_{/q_{x}} = 3.2(1 - 28.4\zeta^{-1/3})$ and $\sigma_{/q_{x}} = 3.2$ when $\zeta < 0$ and $\zeta > 0$, respectively; Quan and Hu (2009) reported that $\sigma_{/q_{x}} = -1.5(-\zeta)^{-1/3}$ and $\sigma_{/q_{x}} = 3.0(\zeta)^{-1/3}$ when $\zeta < 0$ and $\zeta > 0$, respectively. Ramana et al. (2004) found that $\sigma_{/q_{x}} = 6.56(1 - 9.5\zeta^{-1/3})$ and $\sigma_{/q_{x}} = 6.45(1 + 0.25\zeta)^{-1}$ when $\zeta < 0$ and $\zeta > 0$, respectively. In instances when $\zeta < 0$, Quan and Hu (2009) reported that $\sigma_{/q_{x}} = 1.07(1 - 2.71\zeta^{1/3})$ but did not find a similarity relationship for stable conditions, noting that MOST could not adequately represent water vapor fluctuations.

In addition to a lack of consensus on the functional forms of the relationships between $\sigma_{u_{x},w}/u_{x}$ and $\zeta$, as well as between $\sigma_{/q_{x}}$ and $\zeta$ and between $\sigma_{u_{x},w}$ and $\zeta$, there are other caveats to MOST that include, e.g., 1) the assumption of a homogenous flux layer near the surface (e.g., Businger et al. 1971; Foken 2006), 2) statistical self correlation (e.g., Hicks 1978, 1981, 1995), and 3) being poorly suited for stratified surface layers (e.g., Sun et al. 2020). Thus, there remains a strong need to revise MOST (e.g., Wilson 2008). To this end, researchers have investigated the utility of the Richardson number, $Ri$, instead of $\zeta$ as a scaling variable (e.g., Sorbjan 2006; Mauritzen et al. 2007; Sorbjan 2010, 2017; Greene et al. 2022). Furthermore, recent studies have found that the bulk-Richardson number ($Ri_{b}$), in which local gradients present in $Ri$ are approximated as bulk gradients, is sometimes better than MOST for parameterizing near-surface temperature, moisture, wind gradients (Lee and Buban 2020), as well as $u_{x}$, sensible heat flux ($H$), and latent heat flux ($E$) (Lee et al. 2021). For this reason, in the present study, we extended the studies by Lee and Buban (2020) and Lee et al. (2021) by developing $Ri_{b}$ parameterizations for $\sigma_{u_{x}}$, $\sigma_{e}$, $\sigma_{w}$, $\sigma_{u_{x}}$, and $\sigma_{q}$ and comparing these parameterizations against parameterizations that use a MOST-based scaling approach.

2. Datasets

To develop parameterizations for $\sigma_{u_{x},w,b,q}$, we used datasets obtained from three 10-m micrometeorological towers that were deployed during the Land Atmosphere Feedback Experiment (LAFE) in northern Oklahoma in August 2017. LAFE used a unique suite of surface and boundary layer profiling systems coupled with numerical simulations to study land atmosphere feedback processes and to improve the representation of these processes in NWP models. We refer the reader to e.g., Wulfmeyer et al. (2018) for more details about the experimental design and LAFE’s objectives.

The 10-m towers were installed along a 1.7-km line oriented southwest to northeast in an early-growth soybean field (tower 1), native grassland (tower 2), and mature soybean field (tower 3) and consists of the same array of instruments at all towers to sample bulk quantities and turbulence statistics. The surface roughness length ($z_{0}$) was approximately 0.10, 0.11, and 0.07 m for towers 1, 2, and 3, respectively, and the canopy displacement heights ($d$) at towers 1, 2, and 3 were around 0.91, 0.75, and 0.96 m, respectively. Sensitivity tests in which we vary $z_{0}$ indicate that minor differences in the values used for $z_{0}$ do not have a significant impact on the results in this study.

Datasets obtained between 1 and 31 August 2017 were used in all analyses. All turbulence statistics were sampled at 3 m above ground level (AGL) and 10 m AGL and at a 10-Hz sampling frequency. We applied standard postprocessing techniques to the 10-Hz datasets, including the coordinate rotations described in e.g., Meyers (2001) and Lee et al. (2019), as well corrections for angle-of-attack errors described in Kochendorfer et al. (2012). The mean diurnal cycles for $\sigma_{u_{x},w,b,q}$ over the August 2017 sampling period are shown in Fig. 1. These analyses indicated that $\sigma_{u}$, $\sigma_{e}$, and $\sigma_{w}$ had similar agreement among the three sites. $\sigma_{u}$ and $\sigma_{e}$ ranged from $\sim0.4$ m s$^{-1}$ during the nighttime to around 1.2 m s$^{-1}$ in the early-middle afternoon hours, whereas $\sigma_{w}$ ranged from $\sim0.2$ to 0.5 m s$^{-1}$. $\sigma_{q}$ and $\sigma_{q}$ showed slightly more variability among the sites than $\sigma_{u}$, $\sigma_{e}$, and $\sigma_{w}$; daytime values of $\sigma_{q}$ were lower over the soybean crop, and values of $\sigma_{q}$ were correspondingly larger. We also found a small $\sigma_{q}$ increase around sunset at all three sites. Not surprisingly, the $\sigma_{q}$ increase varied as a function of wind speed and was largest under weak wind conditions around sunset that are most conducive to the development of local near-surface temperature gradients.

To ensure that we had high-quality datasets with which to develop the parameterizations for $\sigma_{u_{x},w,b,q}$, we applied additional filtering procedures to the 30-min values of $\sigma_{u_{x},w,b,q}$, $\zeta$, and $Ri_{b}$. The application of additional filtering procedures on the datasets prior to developing the parameterizations in the present study was necessary to mitigate localized effects on the measurements. The filtering procedures for the LAFE datasets were originally described in Lee and Buban (2020) and Lee et al. (2021) and are briefly summarized here. We first removed observations made when the wind direction had a northerly component (i.e., $>270^\circ$ or $<90^\circ$) because Lee and Buban (2020) noted that the longest fetch at each of the
towers occurred when winds were from the south (i.e., Lee and Buban 2020). Doing so was particularly critical at tower 1 because the tower was installed near the boundary between an early-growth soybean field that extended approximately 210 m to the south of the tower and a mixed cropland north of the tower.

To further ensure high-quality, representative measurements we omitted 30-min averaging periods in which there was flux divergence occurring, as evident in the datasets by large differences in the fluxes at 3 and 10 m AGL (e.g., Lee et al. 2019, 2021). To this end, when developing the parameterizations for $u, y, w, u, q$, we removed all 30-min periods when the percent difference in $u^*$ between 3 and 10 m AGL exceeded 15%. Furthermore, we removed all 30-min periods when the percent difference in $H$ between 3 and 10 m AGL exceeded 15% when determining the parameterizations for $su$, and we removed all 30-min periods when the percent difference in $E$ between 3 and 10 m AGL exceeded 15% when determining the parameterizations for $sq$.

3. Derivation of parameterizations of turbulence statistics

a. $\zeta$ parameterizations

As there currently exists no widely accepted form for the relationship between $\sigma_{u,w,v,q}$ and $\zeta$, we developed our own $\zeta$-based parameterizations using the processed datasets from all three micrometeorological towers installed during LAFE. Doing so required that we first parameterize $u_*$. Following e.g., Lee et al. (2021), we compute $u_*$ as

$$u_* = \frac{\kappa U}{\ln\left(\frac{z - d}{z_0}\right) - \psi_m\left(\frac{z - d}{L}\right) + \psi_m\left(\frac{z_0}{L}\right)}.$$  \hspace{1cm} (6)

In Eq. (6), $U$ is the wind speed which was sampled 10 m AGL, $z$ is the sampling height, and the remaining variables have been defined previously. The integrated momentum similarity function $\psi_m$ varies as a function of stability (e.g., Jiménez et al. 2012):

$$\psi_m = \begin{cases} 2 \ln\left(\frac{1 + [\phi_m(\zeta)]^{-1}}{2}\right) + \ln\left(\frac{1 + [\phi_m(\zeta)]^{-2}}{2}\right) - 2\tan^{-1}\phi_m^{-1} + \pi \over 2, & \zeta < 0, \\ -\delta_m\phi_m(\zeta), & \zeta > 0 \end{cases}.$$  \hspace{1cm} (7)

---

FIG. 1. Mean diurnal cycle of (a) $\sigma_u$, (b) $\sigma_v$, (c) $\sigma_w$, (d) $\sigma_h$, and (e) $\sigma_q$ as a function of time of day (LST = UTC – 6 h) at 10 m AGL at tower 1 (red line), tower 2 (blue line), and tower 3 (green line) averaged over 1–31 Aug 2017.
In Eq. (7), we computed $\phi_m$ using Eq. (8), where the coefficients in Eq. (8) are those that Lee et al. (2021) determined using the LAFE datasets:

$$
\phi_m = \begin{cases} 
1.57(1 - 6.71 \xi^{0.25}) & , \ 
\xi < 0 \\
4.04 \xi + 1.50 & , \ 
\xi > 0 
\end{cases}
$$

(8)

To parameterize $\sigma_u$ and $\sigma_q$, we first used Eq. (6) to parameterize $u_*$ and used the parameterized values for $u_*$ in the equations below to derive $H$ and $E$:

$$
H = -\kappa \Delta \theta u_* c_p \rho \left[ \ln \left( \frac{z - d}{z_1 - d} \right) - \psi_h \left( \frac{z - d}{L} \right) + \psi_q \left( \frac{z_1 - d}{L} \right) \right],
$$

(9a)

$$
E = -\kappa \Delta q u_* \rho L_v \left[ \ln \left( \frac{z - d}{z_1 - d} \right) - \psi_h \left( \frac{z - d}{L} \right) + \psi_q \left( \frac{z_1 - d}{L} \right) \right],
$$

(9b)

In Eq. (9), $c_p$ is the specific heat capacity of air; $\rho$ is the air density; $\Delta \theta$ is the near-surface potential temperature gradient, which was sampled between 10 m AGL (i.e., $z_2$) and 2 m AGL (i.e., $z_1$); $\Delta q$ is the near-surface specific humidity gradient, which was sampled between 10 and 3 m AGL; $L_v$ is the latent heat of vaporization; and $\psi_h$ and $\psi_q$ are the integrated similarity functions for heat and moisture, respectively, that were obtained from Lee et al. (2021). As is the case for $\psi_h$, both $\psi_h$ and $\psi_q$ vary as a function of atmospheric stability and have the following relationship:

$$
\psi_h = \begin{cases} 
2 \ln \left( 1 + \left( \frac{\phi_h}{2} \xi \right)^2 \right) & , \ 
\xi < 0 \\
-\phi_h \xi, \ 
\xi > 0 
\end{cases}
$$

(10)

In the above equation, we computed $\phi_h$ using Eq. (11), where the coefficients are those that Lee et al. (2021) determined using the LAFE datasets:

$$
\phi_h = \begin{cases} 
1.06(1 - 1.10 \xi^{0.5}), \ 
\xi < 0 \\
10.90 \xi + 1.05, \ 
\xi > 0 
\end{cases}
$$

(11)

Similarly, we computed $\phi_q$ using Eq. (12), again where the coefficients in Eq. (12) were obtained by Lee et al. (2021):

$$
\phi_q = \begin{cases} 
1.15(1 - 5.15 \xi^{0.5}), \ 
\xi < 0 \\
5.51 \xi + 1.64, \ 
\xi > 0 
\end{cases}
$$

(12)

Once we parameterized $u_*$, $H$, and $E$, we performed nonlinear least squares regressions to relate the nondimensionalized forms of the turbulence statistics (i.e., by dividing $\sigma_n u_n$ by $u_*$ and dividing $\sigma_u$ and $\sigma_q$ by $\theta_*$ and $q_*$, respectively) to $\xi$, $\theta_*$, and $q_*$, are the temperature and moisture scales, respectively, and are computed following Eq. (13):

$$
\theta_* = -\frac{\overline{w^* \theta'}}{u_*},
$$

(13a)

$$
q_* = -\frac{\overline{w^* q'}}{u_*},
$$

(13b)

In the above equations, $\overline{w^* \theta'}$ and $\overline{w^* q'}$ are the kinematic forms of the heat and moisture flux, respectively.

For unstable conditions, we found that a $1/3$ power law best fit the data for $\sigma_{u,n}/u_*$, whereas a $-1/3$ power-law best fit the data for $\sigma_{q}/\theta_*$ and $\sigma_{q}/q_*$. Therefore, we used the following relations:

$$
\frac{\sigma_{u,n}}{u_*} = \alpha_{u,n}(1 - \beta_{u,n} \xi)^{1/3},
$$

(14a)

$$
\frac{\sigma_{q}}{\theta_*} = \alpha_{q}(1 - \beta_{q} \xi)^{-1/3},
$$

(14b)

$$
\frac{\sigma_{q}}{q_*} = \alpha_{q}(1 - \beta_{q} \xi)^{-1/3}.
$$

(14c)

For stable conditions, the relationships have the following form:

$$
\frac{\sigma_{u,n}}{u_*} = \mu_{u,n} \exp(v_{u,n} \xi),
$$

(15a)

$$
\frac{\sigma_{q}}{\theta_*} = \mu_{q} \exp(v_{q} \xi),
$$

(15b)

$$
\frac{\sigma_{q}}{q_*} = \mu_{q} \exp(v_{q} \xi).
$$

(15c)

In Eqs. (14) and (15), $\alpha_{u,n,A}, \beta_{u,n,A}, \mu_{u,n,A}$, and $v_{u,n,A,\xi}$ are empirically determined fitting coefficients. To determine these coefficients, we first computed the 1-sigma uncertainties in each observation. We then performed a Levenberg–Marquardt least squares fit which, using the observations and uncertainty within each observation, iteratively searches for the fitting coefficients until the chi-squared values are minimized. To determine the uncertainties in the observations, we used the approach described by e.g., Markowski et al. (2019) and Lee et al. (2021). To this end, the uncertainty in $\sigma_{u,n}/u_*$, i.e., $\delta(\sigma_{u,n}/u_*)$, is calculated as

$$
\delta(\frac{\sigma_{u,n}}{u_*}) = \left[ \frac{\delta \sigma_{u,n} u_*^{-1}}{\partial \sigma_{u,n}} \delta \sigma_{u,n} + \left( \frac{\partial \sigma_{u,n} u_*^{-1}}{\partial u_*} \delta u_* \right) \right]^{0.5}
$$

$$
= \left[ \left( \frac{\delta \sigma_{u,n} u_*^{-1}}{\partial u_*} \delta u_* \right)^{2} + \left( \frac{\delta \sigma_{u,n}}{u_*^2} \delta u_* \right) \right]^{0.5}
$$

(16)

The uncertainty in $\sigma_{q}/\theta_*$ and $\sigma_{q}/q_*$, i.e., $\delta(\sigma_{q}/\theta_*)$ and $\delta(\sigma_{q}/q_*)$, respectively, is computed the same way as we computed $\delta(\sigma_{u,n}/u_*)$.
Table 1. Best-fit parameters using nonlinear least squares for \( \sigma_{u,\text{raw}}/u_0 = \alpha_{\sigma_{u,\text{raw}}} (1 - \beta_{\sigma_{u,\text{raw}}} \xi)^{1/3} \), \( \sigma_{\theta}/\theta_0 = \alpha_{\theta} (1 - \beta_{\sigma_{\theta}} \xi)^{-1/3} \), and \( \sigma_{q}/q_0 = \alpha_{q} (1 - \beta_{\sigma_{q}} \xi)^{-1/3} \) over the range \(-2 < \xi < 0\). Also shown is 1-sigma uncertainty in each parameter, \( r, p \) value, and number of samples (\( N \)).

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( r )</th>
<th>( p )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{u}/u_0 )</td>
<td>2.419</td>
<td>0.019</td>
<td>1.127</td>
<td>0.137</td>
<td>0.514</td>
</tr>
<tr>
<td>( \sigma_{\theta}/\theta_0 )</td>
<td>2.100</td>
<td>0.022</td>
<td>4.067</td>
<td>0.313</td>
<td>0.584</td>
</tr>
<tr>
<td>( \sigma_{q}/q_0 )</td>
<td>1.196</td>
<td>0.014</td>
<td>1.492</td>
<td>0.222</td>
<td>0.778</td>
</tr>
<tr>
<td>( \sigma_{\theta}/\theta_0 )</td>
<td>4.354</td>
<td>0.551</td>
<td>39.524</td>
<td>18.755</td>
<td>0.626</td>
</tr>
<tr>
<td>( \sigma_{q}/q_0 )</td>
<td>6.303</td>
<td>0.132</td>
<td>40.906</td>
<td>3.933</td>
<td>0.614</td>
</tr>
</tbody>
</table>

\[
\delta \left( \frac{\sigma_{\theta}}{\theta_0} \right) = \left[ \left( \frac{\delta \sigma_{\theta}}{\delta \theta_0} \right)^2 + \left( \frac{\sigma_{\delta \theta}}{\theta_0} \right)^2 \right]^{1/2}, (17a)
\]

\[
\sigma_{\text{raw}} = \begin{cases} \frac{\kappa U}{\ln \left( \frac{z - d}{z_0} \right) - \psi_h \left( \frac{z - d}{L} \right) + \psi_h \left( \frac{z_0}{L} \right)} \alpha_{\sigma_{\text{raw}}} (1 - \beta_{\sigma_{\text{raw}}} \xi)^{1/3}, & \xi < 0 \\ \frac{\kappa U}{\ln \left( \frac{z - d}{z_0} \right) - \psi_h \left( \frac{z - d}{L} \right) + \psi_h \left( \frac{z_0}{L} \right)} \mu_{\sigma_{\text{raw}}} \exp(\nu_{\sigma_{\text{raw}}} \xi), & \xi > 0 \end{cases} (18)
\]

As we do for \( \sigma_{\text{raw}} \), we compute \( \sigma_{q} \) as

\[
\sigma_{q} = \begin{cases} \frac{\kappa \Delta \theta}{\ln \left( \frac{z_2 - d}{z_1 - d} \right) - \psi_h \left( \frac{z_2 - d}{L} \right) + \psi_h \left( \frac{z_1 - d}{L} \right)} \alpha_{\sigma_q} (1 - \beta_{\sigma_q} \xi)^{-1/3}, & \xi < 0 \\ \frac{\kappa \Delta \theta}{\ln \left( \frac{z_2 - d}{z_1 - d} \right) - \psi_h \left( \frac{z_2 - d}{L} \right) + \psi_h \left( \frac{z_1 - d}{L} \right)} \mu_{\sigma_q} \exp(\nu_{\sigma_q} \xi), & \xi > 0 \end{cases} (19)
\]

Similarly, we compute \( \sigma_{\theta} \) as

\[
\sigma_{\theta} = \begin{cases} \frac{\kappa \Delta \theta}{\ln \left( \frac{z_2 - d}{z_1 - d} \right) - \psi_h \left( \frac{z_2 - d}{L} \right) + \psi_h \left( \frac{z_1 - d}{L} \right)} \alpha_{\sigma_{\theta}} (1 - \beta_{\sigma_{\theta}} \xi)^{-1/3}, & \xi < 0 \\ \frac{\kappa \Delta \theta}{\ln \left( \frac{z_2 - d}{z_1 - d} \right) - \psi_h \left( \frac{z_2 - d}{L} \right) + \psi_h \left( \frac{z_1 - d}{L} \right)} \mu_{\sigma_{\theta}} \exp(\nu_{\sigma_{\theta}} \xi), & \xi > 0 \end{cases} (20)
\]

The values for \( \sigma_{\text{raw}}, \beta_{\text{raw}}, \mu_{\text{raw}}, \) and \( \nu_{\text{raw}} \) for \( \xi < 0 \) and \( \xi > 0 \) are shown in Tables 1 and 2, respectively, along with the 1-sigma errors in each fitting parameter.

b. \( R_{ib} \) parameterizations

We again used the processed datasets from all three micro-meteorological towers to compute the \( R_{ib} \). In the \( R_{ib} \) approach,
which was described in e.g., Lee and Buban (2020) and Lee et al. (2021), local gradients present in the equation for the Richardson number are approximated as bulk gradients (e.g., Stull 1988):

\[
\text{Ri} = \sqrt{\frac{g}{\overline{u}'(\overline{u}'^2 + \overline{v}'^2)}} \approx \frac{g\Delta \overline{u}' \Delta z}{\overline{u}'((\Delta \overline{u}')^2 + (\Delta \overline{v}')^2)} = \text{Ri}_b. \tag{21}
\]

Lee et al. (2021) then parameterized \( u_* \), \( \overline{w}' \overline{\theta}' \), and \( \overline{w}' \overline{q}' \) as functions of the \( \text{Ri}_b \):

\[
\begin{align*}
\overline{w}' \overline{\theta}' &= -\Delta \theta u_* C_t(R_i_b), \tag{22b} \\
\overline{w}' \overline{q}' &= -\Delta q u_* C_t(R_i_b). \tag{22c}
\end{align*}
\]

For unstable conditions and stable conditions, i.e., when \( \text{Ri}_b < 0 \) and \( \text{Ri}_b > 0 \), respectively, \( C_{u,t,r} \) has the following form:

\[
C_{u,t,r} = \begin{cases} 
\lambda_{u,t,r} (1 - \omega_{u,t,r} R_i_b)^{1/3}, & \text{Ri}_b < 0 \\
\chi_{u,t,r} \exp(\gamma_{u,t,r} R_i_b), & \text{Ri}_b > 0.
\end{cases} \tag{23}
\]

In the above equations, \( \lambda_{u,t,r}, \omega_{u,t,r}, \chi_{u,t,r}, \) and \( \gamma_{u,t,r} \) are empirically determined fitting coefficients. As we did for the MOST parameterizations, we determined \( \lambda_{u,t,r}, \omega_{u,t,r}, \chi_{u,t,r}, \) and \( \gamma_{u,t,r} \) using the Levenberg–Marquardt least squares fit described in section 3a. In this study, we used the same fitting coefficients as those found in Lee et al. (2021), which are reproduced in Table 3.

In Eq. (23), \( C_u, C_t, \) and \( C_r \) are the friction, heat-transfer, and moisture-transfer coefficients and are computed as

\[
\begin{align*}
C_u &= \frac{u_*}{U_t}, \tag{24a} \\
C_t &= \frac{\theta_u}{\theta_v - \theta_w}, \tag{24b} \\
C_r &= \frac{q}{q_v - q_w}. \tag{24c}
\end{align*}
\]

In Eqs. (24a)–(24c), the subscript \( s \) denotes the surface values which, in the absence of temperature and moisture measurements directly at the land surface and following e.g., Seidel et al. (2012), Lee and Buban (2020), and Lee et al. (2021), we take to be the lowest sampling height on the tower which is 2-m AGL for \( \theta_s \) and 3-m AGL for \( q_s \).

We found that a 1/3 power law best represented the relationship between \( \sigma_{u,t,r} u_* / \text{Ri}_b \) and \( \text{Ri}_b \). Thus, for unstable conditions:

\[
\sigma_{u,t,r} = \begin{cases} 
U \lambda_{u,t,r} (1 - \omega_{u,t,r} R_i_b)^{1/3}, & \text{Ri}_b < 0 \\
U \chi_{u,t,r} \exp(\gamma_{u,t,r} R_i_b) \chi_{u,t,r}, & \text{Ri}_b > 0
\end{cases}\tag{28}
\]

In contrast, we found that that a \(-1/3\) power law best represented the relationship between \( \sigma_{q}/q_\ast \) and \( \text{Ri}_b \), as well as between \( \sigma_{\theta}/\theta_\ast \) and \( \text{Ri}_b \), when \( \text{Ri}_b < 0 \). These relationships can be expressed following Eqs. (26a) and (26b), respectively:

\[
\begin{align*}
\frac{\sigma_\theta}{\theta_*} &= \lambda_\theta (1 - \omega_{\theta} R_i_b)^{-1/3}, \tag{26a} \\
\frac{\sigma_q}{q_*} &= \lambda_q (1 - \omega_q R_i_b)^{-1/3}. \tag{26b}
\end{align*}
\]

Under stable conditions, we found that the functional forms of \( \sigma_{\theta}, \sigma_{q}, \sigma_{\theta}/\theta_\ast, \) and \( \sigma_{q}/q_\ast \) have the same relationship as the relationships between \( C_{u,t,r} \) and \( \text{Ri}_b \); thus \( \sigma_{u,t,r} / u_* \), \( \sigma_{\theta}/\theta_\ast \), and \( \sigma_{q}/q_\ast \) are computed as

\[
\begin{align*}
\frac{\sigma_{u,t,r}}{u_*} &= \chi_{u,t,r} \exp(\gamma_{u,t,r} R_i_b), \tag{27a} \\
\frac{\sigma_{\theta}}{\theta_*} &= \chi_{\theta} \exp(\gamma_{\theta} R_i_b), \tag{27b} \\
\frac{\sigma_q}{q_*} &= \chi_{q} \exp(\gamma_{q} R_i_b). \tag{27c}
\end{align*}
\]

In the above equations, \( \lambda_{\theta}, \omega_{\theta}, \chi_{\theta}, \gamma_{\theta}, \) and \( \gamma_{u,t,r} \) are empirically determined from the LAFE datasets, again using the Levenberg–Marquardt least squares fit described in section 3a.

To compute \( \sigma_{u,t,r} \), we combine the equation for the parameterized \( u_* \), derived using the \( \text{Ri}_b \) approach with the above relationships to compute \( \sigma_{u,t,r} \) as

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
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<tr>
<td>( \lambda_\theta )</td>
<td>0.08</td>
</tr>
<tr>
<td>( \lambda_\theta )</td>
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</tr>
<tr>
<td>( \lambda_q )</td>
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<td>( \omega_\theta )</td>
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<td>( \chi_q )</td>
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<td>( \gamma_\theta )</td>
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<td>( \gamma_q )</td>
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<td>-9.25</td>
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<td>( \gamma_q )</td>
<td>-13.59</td>
</tr>
</tbody>
</table>

\[
\frac{\sigma_{u,t,r}}{u_*} = \lambda_{u,t,r} (1 - \omega_{u,t,r} R_i_b)^{1/3}. \tag{25}
\]
Table 4. Best-fit parameters using nonlinear least squares for $\sigma_{u,s}/u_* = \lambda_{u,s}(1 - \omega_{u,s} R_i b)^{1/3}$, $\sigma_{\theta,s}/\theta_s = \lambda_{\theta,s}(1 - \omega_{\theta,s} R_i b)^{-1/3}$, and $\sigma_{q,s}/q_s = \lambda_{q,s}(1 - \omega_{q,s} R_i b)^{-1/3}$ over the range $-2 < R_i b < 0$. Also shown is the 1-sigma uncertainty in each parameter, r, p value, and N.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\lambda$ uncertainty</th>
<th>$\omega$ uncertainty</th>
<th>r</th>
<th>p</th>
<th>N</th>
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<tbody>
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<td>$\sigma_{u,s}/u_*$</td>
<td>2.449 0.018</td>
<td>2.206 0.277</td>
<td>0.539 $&lt; 0.01$</td>
<td>696</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{u,s}/u_*$</td>
<td>2.204 0.019</td>
<td>6.717 0.532</td>
<td>0.576 $&lt; 0.01$</td>
<td>696</td>
<td></td>
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<tr>
<td>$\sigma_{u,s}/u_*$</td>
<td>1.217 0.013</td>
<td>2.743 0.423</td>
<td>0.723 $&lt; 0.01$</td>
<td>696</td>
<td></td>
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<tr>
<td>$\sigma_{\theta,s}/\theta_s$</td>
<td>2.743 0.120</td>
<td>15.003 3.709</td>
<td>0.566 $&lt; 0.01$</td>
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<tr>
<td>$\sigma_{q,s}/q_s$</td>
<td>3.493 0.057</td>
<td>8.075 0.869</td>
<td>0.453 $&lt; 0.01$</td>
<td>362</td>
<td></td>
</tr>
</tbody>
</table>

As the parameters for $\sigma_q$ are a function of $\theta$, which itself is a function of the parameterized $H$, we can compute $\sigma_q$ as

$$
\sigma_q = \left\{ \begin{array}{ll} 
\Delta \theta \lambda_s (1 - \omega R_i b)^{1/3} \lambda_{\theta,s} (1 - \omega_{\theta,s} R_i b)^{-1/3}, & R_i b < 0 \\
\Delta \theta \chi \exp(\gamma R_i b) \chi_{\theta,s} \exp(\gamma_{\theta,s} R_i b), & R_i b > 0 
\end{array} \right.
$$

Similarly, the parameters for $\sigma_q$ are a function of $q$, which is a function of $E$; thus, we compute $\sigma_q$ as

$$
\sigma_q = \left\{ \begin{array}{ll} 
\Delta q \lambda_q (1 - \omega R_i b)^{1/3} \lambda_{q,s} (1 - \omega_{q,s} R_i b)^{-1/3}, & R_i b < 0 \\
\Delta q \chi \exp(\gamma R_i b) \chi_{q,s} \exp(\gamma_{q,s} R_i b), & R_i b > 0 
\end{array} \right.
$$

The values for $\lambda_{\lambda,s,\omega,s,\chi,s}$, $\omega_{\lambda,s,\omega,s,\chi,s}$, $\chi_{\lambda,s,\omega,s,\chi,s}$, and $\gamma_{\lambda,s,\omega,s,\chi,s}$ for $R_i b < 0$ and $R_i b > 0$ are shown in Tables 4 and 5, respectively, along with the uncertainties in each fitting parameter. The values for r for the fits for $\sigma_{u,s}/u_*$ and $\sigma_{\theta,s}/\theta_s$ under unstable conditions are comparable between the MOST and Ri b parameterizations; however, the values of r for the fits for the remaining variables are lower for the Ri b parameterizations than for the MOST parameterizations. For stable regimes, both the MOST and Ri b parameterizations have smaller values of r than for the unstable regimes. The fits for all variables except for $\sigma_{\theta,s}/\theta_s$ have larger values of r for the Ri b parameterizations. We suspect that the lower values of r for the fits arises due to the nonlinearity of the formulations themselves and to the small sample size.

4. Evaluation of $\zeta$ and $R_i b$ parameterizations for turbulence statistics

a. Datasets

We evaluated the $\zeta$ and $R_i b$ parameterizations of $\sigma_{u,s,\omega,s,\chi,s}$ using the observed 30-min $\sigma_{u,s,\omega,s,\chi,s}$ sampled 10 m AGL at all three LAFE micrometeorological towers. Unlike when developing the parameterizations, however, we did not filter the datasets by wind direction, nor did we remove periods when there were large differences in $u_*$, $H$, or $E$ between 3 and 10 m AGL.

Additionally, we used a separate, fully independent dataset to evaluate the parameterizations. These datasets were obtained from two 10-m micrometeorological towers that were installed to support the Verification of the Origins of Rotation in Tornadoes Experiment-Southeast (VORTEX-SE) campaign in northern Alabama. VORTEX-SE was a campaign focused on better understanding the surface and boundary layer processes that lead to severe weather genesis over the Southeast United States. We used datasets from towers installed near Belle Mina, Alabama, and Cullman, Alabama. The land surface surrounding the tower near Belle Mina consisted of grazed pasture, whereas the land surface surrounding the tower at Cullman was ungrazed grassland. Belle Mina and Cullman had values of $z_0$ of around 0.15 and 0.25 m, respectively, and d of about 0.4 and 1.1 m, respectively. More details about the VORTEX-SE experimental design and the sites themselves appear in, e.g., Lee et al. (2019) and Wagner et al. (2019).

We used the entire period of record from both towers (February 2016–April 2017), and we used 10-m observations of $\sigma_{u,s,\omega,s,\chi,s}$, again computed over 30 min and based on measurements sampled at 10 Hz, to evaluate the $\zeta$ and $R_i b$ parameterizations that we developed in the present study. The measurement suite at the towers at Belle Mina and Cullman was identical to the measurement suite at each of the LAFE towers, and we used the same data processing techniques for the high-frequency measurements as we did for the LAFE measurements. Following our approach for using the LAFE datasets to evaluate the $\zeta$ and $R_i b$ parameterizations, we did not apply any filtering criteria to either the Belle Mina or Cullman observations of $\sigma_{u,s,\omega,s,\chi,s}$ when evaluating the $\zeta$ and $R_i b$ parameterizations.

b. Weighting technique to evaluate $\zeta$ and $R_i b$ parameterizations

We followed the procedures described in Lee et al. (2021) to weight the errors from the observed $\sigma_{u,s,\omega,s,\chi,s}$ and parameterized $\sigma_{u,s,\omega,s,\chi,s}$. We then used the uncertainties in the observations and the uncertainties in the parameterized values as inputs into the following equations (Neri et al. 1989; Cantrell 2008; Lee et al. 2021):

$$
W_i = \frac{w_s w_{\omega}}{w_{\omega} + w_{\omega} + w_{\omega}},
$$

$$
W_i = \frac{w_s w_{\omega}}{w_{\omega} + w_{\omega} + w_{\omega}},
$$

$$
b = \sum W_i (\gamma - m_i x_i) \sum W_i.
$$
In Eqs. (31a)–(31c), $x_i$ and $y_i$ correspond with the observed $\sigma_{u,\text{w},q}$ and parameterized $\sigma_{u,\text{w},q}$, respectively; $m_0$ is the slope of the fit; and $w_x$ and $w_y$ are the weights in the observations of $\sigma_{u,\text{w},q}$ and parameterized values of $\sigma_{u,\text{w},q}$ respectively. The equations were then solved iteratively until values for $m_0$ and $b$ were found that satisfied Eq. (31c).

5. Results and discussion

a. Relationship between $\sigma_{u,\text{w},q}$, $\zeta$, and $R_i_b$

Once the uncertainties in the each of the variables were computed using the approach described in section 3b, we were able to develop the relationships between $\sigma_{u,\text{w},q}$ and $\zeta$, as well as between $\sigma_{u,\text{w},q}$ and $R_i_b$ that weighted the observations as a function of the uncertainty in each measurement (Figs. 2 and 3). As noted in Tables 1 and 4 and briefly summarized here, when developing the MOST and $R_i_b$ parameterizations, we used the same range for $\zeta$ for $R_i_b$ for unstable conditions, i.e., $-2 < \zeta < 0$, and $-2 < R_i_b < 0$, respectively. As noted in Tables 2 and 5, though, we used a different range of stabilities for $\zeta$ for $R_i_b$, for stable conditions, i.e., $0 < \zeta < 1$ and $0 < R_i_b < 0.25$, respectively, because of the relatively small number of cases when $R_i_b > 0.25$ compared to cases when $\zeta > 0.25$ and to remove the effects of outliers on the fits.

Consistent with previous work (e.g., Lee and Buban 2020; Lee et al. 2021), the scatter in these relationships was larger for stable conditions than for unstable conditions, which resulted in lower coefficients of correlation ($r$) under stable conditions, both for the $\zeta$-based parameterizations and $R_i_b$-based parameterizations (cf. Tables 1, 2, 4, and 5). Furthermore, the nonlinear least squares fits for $\sigma_{u,\text{w},q}$ had a larger $r$ than the fits for $\sigma_{u,\text{w},q}$. We also note the comparatively small number of data points used to generate the fits for stable conditions because of the filtering criteria applied to the LAFE datasets which removed many periods during the nighttime in which the difference in the sensible heat fluxes and latent heat fluxes between the two sampling heights was nontrivial.

Furthermore, the relationships that we obtained between $\sigma_{u,\text{w},q}/u_*$ and $\zeta$ were consistent with previous work. For example, when $\zeta = 0$, $\sigma_{u,\text{w},q}/u_*$ and $\sigma_{u,\text{w},q}/u_*$ were around 2.4, 2, and 1.2, respectively, which are within the range of values reported from the literature (e.g., Panofsky and Dutton 1984; de Franceschi et al. 2009). In contrast, we found a discontinuity near 0 in the relationship between $\sigma_{u,\text{w},q}/u_*$ and $\zeta$, as well as in the relationship between $\sigma_{u,\text{w},q}/q_*$ and $\zeta$, that arose due to the scatter present in these fits, particularly for stable conditions when $r$ was 0.31 and 0.10, respectively. Consequently, $\sigma_{u,\text{w},q}/u_*$ was around 4.3 for slightly unstable conditions, but was around 7 for slightly stable conditions, which are values that are near the maximum values reported in the literature (e.g., de Franceschi et al. 2009). $\sigma_{u,\text{w},q}/q_*$ was ~6.3 and 8.0 for slightly stable conditions.
unstable conditions and slightly stable conditions, respectively. We acknowledge, though, that there are relatively few studies with which to compare our relationships between $s_{q}/q^{*}$ and $z$. However, we note that Quan and Hu (2009) found values of $s_{q}/q^{*}$ around 1.1 for near-neutral conditions, which were lower than the values we found using the LAFE datasets.

b. $z$ and $R_i$ parameterizations of $u, v, w$

When we used the similarity relationships developed in the previous section to calculate $s_{u}, s_{v}, s_{w}$, we found that the parameterized values of $s_{u,v,w}$ were lower than the observed $s_{u,v,w}$ when using the $z$ parameterizations, not only the LAFE micrometeorological towers, but also at the VORTEX-SE micrometeorological towers. For example, $m_{b}$, which is the slope of the best-fit line between the observed and parameterized values and calculated after including the uncertainties in the observed and parameterized values following Eq. (31), was 0.80, 0.77, and 0.81 for $s_{u}, s_{v},$ and $s_{w}$, respectively, at the LAFE towers (Figs. 4a–c). At Belle Mina, $m_{b}$ was $-1$ for $s_{u}$, but was $-0.6$ and $-0.8$ for $s_{v}$ and $s_{w}$, respectively (Figs. 4d–f). At Cullman, the correlation coefficients between the parameterized and observed $s_{u,v,w}$ were comparable with Belle Mina (Figs. 4g–i), but $m_{b}$ was lower at Cullman than at Belle Mina.

We found that the $R_i$ parameterizations generally performed better than the $z$ parameterizations, though this was not always the case. Whereas the values of $r$ were comparable among the LAFE sites for the $R_i$ and $z$ parameterizations, $m_{b}$ increased from 0.80 to 0.86, 0.77 to 0.82, and 0.81 to 0.99 for $s_{u}, s_{v},$ and $s_{w}$, respectively (Figs. 5a–c). At Belle Mina, $m_{b}$ increased for $s_{u}, s_{v},$ and $s_{w}$; as a result the parameterizations overestimated these values (Figs. 5d–f). However, $r$ increased from 0.62 to 0.68 and from 0.69 to 0.76 for $s_{u}, s_{v},$ and $s_{w}$, respectively. For $s_{u}, r$ decreased from 0.84 to 0.72 when using the $R_i$ parameterizations instead of the $z$ parameterizations at Belle Mina. At Cullman, the $R_i$ parameterizations for $s_{v}$ performed better than the $z$ parameterizations for $s_{v}$, as $m_{b}$ was closer to 1 and $r$ was higher, but the $R_i$ parameterizations did not perform as well as the $z$ parameterizations for $s_{u}$ and $s_{w}$ based on these metrics (Figs. 5g–i).

c. $z$ and $R_i$ parameterizations of $e$

We used the parameterized $s_{u}, s_{v},$ and $s_{w},$ computed using the MOST approach and the $R_i$ approach, to parameterize turbulent kinetic energy, $e$, in the surface layer, and compared the parameterized $e$ against the observed $e$ which was computed as

$$
e = 0.5(s_{u}^{2} + s_{v}^{2} + s_{w}^{2}).$$

(32)
As expected based on the relationships between the parameterized and observed s_u, both the z and Ri_b parameterizations underpredicted s_u, in which the parameterizations underpredicted s_u, both the z and Ri_b parameterizations also underpredicted e at the LAFE and VORTEX-SE micrometeorological towers (Fig. 6). At the LAFE towers, the z and Ri_b parameterizations performed similarly in their predictions of e. However, at Belle Mina, the Ri_b parameterizations performed better than the z parameterization based on the increase in m_b, as m_b was 0.87 and 0.77, for the Ri_b and z parameterizations, respectively. In contrast, the opposite occurred at Cullman, as m_b was 0.67 for the z parameterization of e but was 0.53 when we used the Ri_b parameterization.

FIG. 4. (a) Density plot showing the relationship between the ζ-parameterized s_u and observed s_u for all three LAFE towers combined. (b),(c) As in (a), but for s_y and s_w, respectively. (d)–(f),(g)–(i) The relationships between the parameterized and observed s_u, s_y, and s_w, respectively, at Belle Mina and Cullman, respectively. The dotted line shows the 1:1 line, and the solid line shows the line of best fit. N, r, and formula for the best-fit line (here, \( X = m_b \)) are shown at the bottom right of each panel.
d. ζ and Riₚ parameterizations of σ₀,q

When evaluating the relationships between the parameterized and observed values of σ₀,q, we found that the ζ parameterizations overpredicted σ₀ and σ₂ at the LAFE towers; m₁ was 1.04 and 1.17, respectively (Figs. 7a,b). At the VORTEX-SE towers, the opposite was true, and both m₁ and r were considerably lower than at the LAFE towers. m₂ for the relationship between the parameterized and observed σ₀ at Bella Mina and Cullman was 0.55 and 0.42, respectively; for σ₂, m₂ was 0.24 and 0.91, respectively (Figs. 7c–f).

The Riₚ parameterizations generally yielded improved predictions for σ₀ and σ₂. At the LAFE towers, m₁ was 0.85 and 0.92 for σ₀ and σ₂, respectively (Figs. 8a,b). At Belle Mina, there was marked improvement in m₁ and r. m₁ increased from 0.55 to 0.78 and 0.24 to 0.67 for σ₀ and σ₂, respectively, when using the Riₚ parameterizations instead of the ζ parameterizations;

---

**Fig. 5.** As in Fig. 4, but for the Riₚ parameterizations.
increased from 0.43 to 0.64 and 0.07 to 0.45 for $\sigma_\theta$ and $\sigma_{q_v}$, respectively, when using the Ri_b parameterizations instead of the $z$ parameterizations (Figs. 8c,d). We found similar increases in $m_b$ and $r$ when applying the Ri_b parameterizations to the Cullman datasets. $m_b$ increased from 0.42 to 0.76 for $\sigma_\theta$ when using the Ri_b parameterizations instead of the $z$ parameterizations, but the Ri_b parameterizations overestimated $\sigma_{q_v}$. However, $r$ increased from 0.46 to 0.70 and 0.23 to 0.55 for $\sigma_\theta$ and $\sigma_{q_v}$, respectively (Figs. 8e,f).

Regardless, there is a nontrivial amount of scatter present in the relationship between the parameterized $\sigma_{q_v}$ and observed $\sigma_{q_v}$, both in the $z$ and Ri_b parameterizations, as $r$ is typically smaller than the values found in the relationships for $\sigma_\theta$. There are multiple reasons for the scatter between the parameterized and observed values. One of the reasons is that we did not filter either the LAFE or VORTEX-SE datasets by wind direction, nor did we remove periods when there were large differences in the fluxes at the two sampling heights, when evaluating the parameterizations not only for $\sigma_\theta$ and $\sigma_{q_v}$, but also when we evaluated the parameterizations of $\sigma_m$, $\sigma_{w_m}$, $\sigma_{q_v}$, and $e$. As expected, applying the same filtering criteria that we had used when developing the original parameterizations does reduce the scatter between the parameterized and observed turbulence statistics.

Overall, though, the $\sigma_{q_v}$ parameterizations show the largest amount of scatter. Difficulties parameterizing near-surface moisture are well known. Lee and Buban (2020) and Lee et al. (2021) reported that parameterizations for $\Delta q$ and $E$, respectively, in the surface layer yielded values that were at times significantly different from the observed values. As noted by Lee et al. (2021) and summarized here, the comparatively poor performance of moisture parameterizations in the surface layer, compared to parameterizations for surface-layer heat or momentum, can be attributed to advection and variability in landuse type, which are not represented in either the $z$ or Ri_b parameterizations.

We also note much lower values of $r$ at both Belle Mina and Cullman as compared with the values of $r$ at the LAFE towers when evaluating the $\zeta$ and Ri_b parameterizations. This is true not only for $\sigma_{q_v}$ but also for $\sigma_\theta$. The poorer agreement...
at Belle Mina and Cullman than at the LAFE sites arises because the parameterizations were developed over a semiarid region and then applied to a region in which $E$ is a larger term in the surface energy budget.

**Fig. 7.** (a) Density plot showing the relationship between the $\zeta$-parameterized $\sigma_\theta$ and observed $\sigma_\theta$ at the LAFE towers. (b) As in (a), but for $\sigma_q$. (c),(d) The relationships between the parameterized and observed $\sigma_\theta$ and $\sigma_q$, respectively, at Belle Mina. (e),(f) As in (c) and (d), but for Cullman. The dotted line shows the 1:1 line, and the solid line shows the line of best fit. $N$, $r$, and formula for the best-fit line ($X = m_0$) are shown at the bottom right of each panel.

e. Performance of $\zeta$ and $\text{Ri}_b$ parameterizations of $\sigma_{u,\text{w},\text{v},\theta,q}$ for different wind regimes

Discussion so far has focused on the performance of the $\zeta$ and $\text{Ri}_b$ parameterizations independent of wind speed.
However, recent work has shown that the Ri_b parameterizations generally perform better than the MOST parameterizations for low wind speeds (Lee et al. 2021). To this end, we distinguished between low wind speeds and high wind speeds by computing the median wind speed at the LAFE towers and at the Belle Mina and Cullman towers for the entire period of record at each site, which was 2.7, 2.0, and 2.5 m s\(^{-1}\), respectively. We classified 30-min time periods with wind speeds below these thresholds as cases with low wind speeds, and we classified the 30-min periods with wind speeds above these thresholds as cases with high wind speeds.

The Ri_b parameterizations of \(\sigma_u\), \(\sigma_v\), and \(\sigma_w\) for the subset of cases with low wind speeds have larger \(r\) and \(m_b\) than the
Table 6. $N$, $r$, and formula for the best-fit line for the MOST and $R_i_b$ parameterizations of $\sigma_u$, $\sigma_v$, $\sigma_w$, $\epsilon$, $\sigma_q$, and $\sigma_q$ for low wind speeds at the LAFE towers and for the towers installed at Belle Mina and Cullman.

<table>
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<tr>
<th></th>
<th>$\sigma_u$</th>
<th>$\sigma_v$</th>
<th>$\sigma_w$</th>
<th>$\epsilon$</th>
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<tr>
<td>$N$</td>
<td>MOST 2055</td>
<td>2053</td>
<td>2053</td>
<td>2053</td>
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<td>2074</td>
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<td>2025</td>
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<td>$r$</td>
<td>MOST 0.556</td>
<td>0.628</td>
<td>0.726</td>
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<td>0.593</td>
<td>0.498</td>
</tr>
<tr>
<td></td>
<td>$R_i_b$ 0.651</td>
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<td>0.866</td>
<td>0.655</td>
<td>0.402</td>
<td>0.570</td>
</tr>
<tr>
<td>Best fit</td>
<td>MOST 0.313$m_b + 0.248$</td>
<td>0.406$m_b + 0.177$</td>
<td>0.42$m_b + 0.131$</td>
<td>0.30$m_b + 0.103$</td>
<td>0.654$m_b - 0.127$</td>
<td>0.839$m_b - 0.124$</td>
</tr>
<tr>
<td></td>
<td>$R_i_b$ 0.562$m_b + 0.0674$</td>
<td>0.603$m_b + 0.0635$</td>
<td>0.821$m_b + 0.0183$</td>
<td>0.408$m_b + 0.0437$</td>
<td>0.524$m_b + 0.0031$</td>
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Belle Mina

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<th>$\sigma_u$</th>
<th>$\sigma_v$</th>
<th>$\sigma_w$</th>
<th>$\epsilon$</th>
<th>$\sigma_q$</th>
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<td>5799</td>
<td>2838</td>
<td>3359</td>
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<td>$R_i_b$ 5854</td>
<td>6215</td>
<td>5114</td>
<td>4245</td>
<td>3644</td>
<td>3998</td>
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<tr>
<td>$r$</td>
<td>MOST 0.524</td>
<td>0.0886</td>
<td>0.0491</td>
<td>0.132</td>
<td>0.511</td>
<td>0.0959</td>
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<td></td>
<td>$R_i_b$ 0.511</td>
<td>0.552</td>
<td>0.728</td>
<td>0.459</td>
<td>0.596</td>
<td>0.480</td>
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<td>Best fit</td>
<td>MOST 0.462$m_b$ $- 0.0202$</td>
<td>0.152$m_b$ $+ 0.319$</td>
<td>0.078$m_b$ $+ 0.187$</td>
<td>0.144$m_b$ $+ 0.134$</td>
<td>0.30$m_b$ $- 0.0033$</td>
<td>0.31$m_b$ $+ 0.786$</td>
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<td>$R_i_b$ 1.018$m_b$ $- 0.146$</td>
<td>1.257$m_b$ $- 0.181$</td>
<td>1.379$m_b$ $- 0.0941$</td>
<td>0.901$m_b$ $+ 0.0626$</td>
<td>0.481$m_b$ $- 0.124$</td>
<td>0.610$m_b$ $- 0.0367$</td>
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Cullman

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<tr>
<td>Best fit</td>
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<td>0.379$m_b$ $+ 0.130$</td>
<td>0.289$m_b$ $+ 0.111$</td>
<td>0.286$m_b$ $+ 0.0327$</td>
<td>0.660$m_b$ $+ 0.931$</td>
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<td>1.020$m_b$ $- 0.155$</td>
<td>1.017$m_b$ $- 0.0833$</td>
<td>0.799$m_b$ $- 0.009$</td>
<td>0.631$m_b$ $- 0.177$</td>
<td>1.247$m_b$ $- 0.078$</td>
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MOST parameterizations when evaluated using the LAFE datasets, whereas $m_b$ is lower for the $R_i_b$ parameterizations of $\sigma_q$ and $\sigma_q$ (Table 6). At both Belle Mina and Cullman, $m_b$ is better for the $R_i_b$ parameterizations than for the MOST parameterizations for all turbulence statistics. Similar results are found for $r$; the only exception is that $r$ is lower for the $R_i_b$ parameterizations of $\sigma_q$ than the MOST parameterizations at both sites.

Results are more mixed for the subset of cases with high wind speeds. The $R_i_b$ parameterizations of $\sigma_q$ and $\sigma_q$ do not perform as well as the MOST parameterizations at the LAFE sites, whereas the performance is comparable for $\sigma_u$ (Table 7).

In the case of $\sigma_u$, $r$ is lower for the $R_i_b$ parameterizations, but $m_b$ is nearer to 1 than for the MOST parameterizations. The opposite is true for $\sigma_q$ at the LAFE sites. At Belle Mina, $m_b$ is nearer to 1 for all turbulence statistics for the $R_i_b$ parameterizations than for the MOST parameterizations, yet $r$ is lower when comparing the parameterized and observed values of $\sigma_u$, $\sigma_v$, $\sigma_w$, and $\epsilon$. At Cullman, $r$ and $m_b$ are better for the $R_i_b$ parameterizations of $\sigma_q$ and $\sigma_q$ as compared with the MOST parameterizations.

6. Summary and outlook

Previous work found that using a $R_i_b$-based scaling approach works is comparable, if not better, than using a $\zeta$-based scaling approach derived from MOST to compute near-surface gradients in wind, temperature, and moisture (Lee and Buban 2020) and fluxes of these respective quantities (Lee et al. 2021). Thus, in this work we developed parameterizations for $\sigma_u$, $\sigma_v$, $\sigma_w$, $\sigma_q$, and $\sigma_q$ that were a function of $R_i_b$ and compared these parameterizations against techniques that use $\zeta$ as a scaling variable. To develop the $\zeta$ and $R_i_b$ parameterizations, we used datasets from three 10-m micrometeorological towers installed during LAFE in August 2017. We evaluated the new parameterizations of turbulence statistics using not only the LAFE datasets but also by evaluating these parameterizations over a different region of the United States using fully independent datasets obtained from two 10-m micrometeorological towers installed during VORTEX-SE. Based on $r$ and $m_b$, both of which are summarized in Fig. 9, between the $R_i_b$-parameterized quantities and observed turbulence statistics, we found that the $R_i_b$ parameterizations better represented $\sigma_u$, $\sigma_v$, $\sigma_w$, and $\sigma_q$ than...
the MOST parameterizations, and the most significant improvements were for the Ri_b parameterizations of σ_u and σ_v. Furthermore, the Ri_b parameterizations performed better than the MOST parameterizations under low wind speeds.

One important caveat about the Ri_b parameterizations that was described in Lee et al. (2021) and that also applies in the present study is that the Ri_b parameterizations use a stability term (i.e., Ri_b) which is closely related to the stability term used in MOST [i.e., L; cf. Eqs. (1), (2), and (21)], to represent surface-layer turbulence quantities. Thus, the Ri_b parameterizations are simply a different approach to represent surface layer physics through the use of a different term for near-surface stability; there is nothing inherently new about the Ri_b parameterizations of the underlying physics. Additionally, both the MOST and Ri_b parameterizations have self-correlation present. Self-correlation arises in the friction velocity term and the wind gradients in the MOST parameterizations and Ri_b parameterizations, respectively. However, the advantage of the Ri_b parameterizations is that bulk gradients are easier to measure than friction velocities (e.g., Lee et al. 2021). Furthermore, the Monin–Obukhov length scale in the MOST equations [cf. Eq. (2)] is a function of u*. Thus, errors in the measured u* can have a nontrivial impact on the Monin–Obukhov length scale (e.g., Markowski et al. 2019).

Overall, the results in this study, coupled with earlier studies arguing the need to revise MOST (e.g., Wilson 2008) and work by Lee and Buban (2020) and Lee et al. (2021), provide additional evidence that the functional forms of the similarity equations used in surface layer parameterization schemes within NWP models should be revisited and may need to be revised to use an Ri_b-based approach. Moving toward implementing an Ri_b-based approach in NWP models, however, requires that the newly suggested Ri_b parameterizations are evaluated over different landuse types and across a range of climatic conditions and atmospheric stability regimes (i.e., very unstable and very stable regimes) than has been so far been done (e.g., Olson et al. 2021). Furthermore, the newly suggested Ri_b parameterizations must be tested in large-eddy simulations, whereby not only are the parameterizations that we suggest in this study are tested but also the parameterizations for u*, H, and E from Lee et al. (2021) are evaluated within these modeling frameworks prior to the parameterizations being implemented into operational NWP models.
Fig. 9. Summary of (a) $r$ and (b) $m_0$ for the MOST (red bars) and Rb (blue bars) parameterizations of $\sigma_u$, $\sigma_v$, $\sigma_w$, $e$, $\sigma_b$, and $\sigma_q$ at the LAPE and VORTEX-SE towers. Horizontal black lines separate different variable–site combinations, and the vertical black line in (b) indicates a 1:1 relationship between the parameterized and observed turbulence statistics.
REFERENCES


