CORRIGENDUM

JOHANNES M. L. DAHL[a] AND CHRISTIAN BOYER[b]

[a] *Department of Geosciences, Texas Tech University, Lubbock, Texas*
[b] *Department of Earth and Atmospheric Sciences, Central Michigan University, Mount Pleasant, Michigan*

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1. Introduction

This corrigendum concerns Figs. 3e,f, 5c, and 7e,f of Boyer and Dahl (2020), which show the integrals of the different terms contributing to the evolution of the horizontal vorticity magnitude. However, instead of showing the time integrals of Eq. (2) in the original paper as advertised, these figures show the magnitudes of the individual integrated terms of the horizontal vorticity vector equation [Eqs. (3) and (4) in the original paper]. This mistake does not affect the conclusions as discussed below, but we wish to clarify what was advertised, and what is actually shown in the figures. The difference is as follows.

2. Horizontal vorticity magnitude as advertised

Equation (2) in the original paper, which describes the time evolution of the horizontal vorticity magnitude along a parcel trajectory, may be written as

$$\frac{d|\boldsymbol{\omega}_h|}{dt} = \frac{\xi}{|\boldsymbol{\omega}_h|}\frac{d\xi}{dt} + \frac{\eta}{|\boldsymbol{\omega}_h|}\frac{d\eta}{dt}. \tag{1}$$

Defining the unit vector parallel to the horizontal vorticity vector as

$$\mathbf{s} \equiv \frac{\boldsymbol{\omega}_h}{|\boldsymbol{\omega}_h|}, \tag{2}$$

Eq. (1) may be written as

$$\frac{d|\boldsymbol{\omega}_h|}{dt} = \mathbf{s}\cdot\frac{d\boldsymbol{\omega}_h}{dt} = \mathbf{s}\cdot\sum_N \left.\frac{d\boldsymbol{\omega}_h}{dt}\right|_N, \tag{3}$$

where N reflects the different terms in the vorticity equation (stretching and tilting terms, baroclinic vorticity production, etc.). Another way of writing Eq. (3) is

$$\frac{d|\boldsymbol{\omega}_h|}{dt} = \sum_N \cos\alpha_N \left|\frac{d\boldsymbol{\omega}_h}{dt}\right|_N, \tag{4}$$

where α_N is the angle between $\boldsymbol{\omega}_h$ and $d\boldsymbol{\omega}_h/dt|_N$. In other words, these equations describe the projection of the horizontal vorticity vector evolution onto the horizontal vorticity vector, which equals the sum of the projection of each contribution onto the horizontal vorticity vector. So, only the component of the horizontal vorticity evolution in the direction of the horizontal vorticity vector is considered. This projection may attain positive or negative values, and upon integration, all the terms on the right-hand side of Eq. (4) add up to the total horizontal vorticity. The upside of this formulation is that only those terms that actually change the horizontal vorticity magnitude are considered. The downside is that the absence of, e.g., the contribution from baroclinic production or differential subgrid-scale (SGS) mixing does not necessarily mean that the parcel did not experience baroclinic production or differential SGS mixing. Rather, in some

Corresponding author: Johannes Dahl, johannes.dahl@ttu.edu

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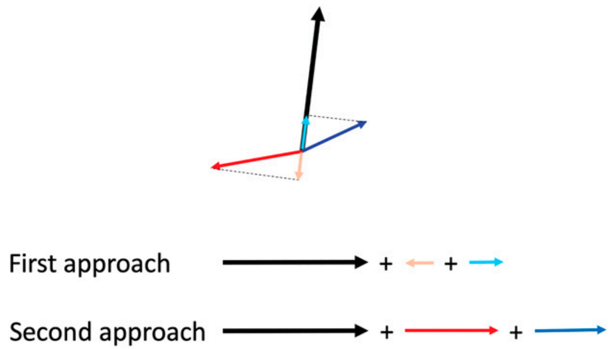

FIG. 1. The black arrow represents the horizontal vorticity vector, and the colored arrows represent two contributions leading to changes of the vorticity vector (due to, e.g., differential SGS mixing and baroclinic production) during a small time increment. The projections onto the horizontal vorticity vector are shown in light colors. In the first approach, based on Eq. (4), only the projections of these contributions are integrated. In the second approach, based on Eq. (5) the integral of each contribution itself is considered. The sum of the incremental changes in each approach is shown symbolically at the bottom. Should each of the two contributions have the same magnitude and point in opposite directions while also being normal to the horizontal vorticity vector, these contributions would be ignored in the first approach.

cases these contributions could have the same magnitude but point in opposite directions, normal to the horizontal vorticity vector. In that case the projections onto the horizontal vorticity vector remain zero and are not considered in the budget.

3. Horizontal vorticity magnitude as shown in the figures

Instead of implementing Eq. (1), however, the horizontal vorticity contributions were calculated by using the integrals of the governing equations for the horizontal vorticity components, (ξ, η), such that

$$|\boldsymbol{\omega}_h|_N = \sqrt{\xi_N^2 + \eta_N^2}, \tag{5}$$

with (ξ_N, η_N) being obtained by integrating the respective terms of the vorticity equations [Eqs. (3) and (4) in the original paper]. The total horizontal vorticity magnitude is given by

$$|\boldsymbol{\omega}_h| = \sqrt{\left(\sum_N \xi_N\right)^2 + \left(\sum_N \eta_N\right)^2}, \tag{6}$$

but now $\sum_N |\boldsymbol{\omega}_h|_N \geq |\boldsymbol{\omega}_h|$ i.e., the individual integrals $|\boldsymbol{\omega}_h|_N$, which are positive definite, do not in general add up to the total vorticity magnitude. Because this treatment does not consider whether or not a given contribution has a component parallel to the horizontal vorticity vector, one cannot infer whether a given integral contributes positively or negatively to the horizontal vorticity magnitude. However, one advantage of this implementation is that the entirety of the horizontal vorticity evolution is captured, rather than just the components parallel to the horizontal vorticity vector. Both formulations discussed in this corrigendum, the integral of Eq. (4) as well as Eqs. (5) and (6) are valid, but each highlight different aspects of the vorticity evolution. The difference between these two implementations is shown in Fig. 1. Note that if one were interested in the 3D vorticity vector magnitude, analogous arguments would apply.

4. Ramifications

The focus of the paper was on the evolution of the vertical vorticity along parcel trajectories, especially on the observation that the vertical vorticity does not increase before the parcels enter the base of the vortex. Rather, vertical vorticity only increases after horizontal vorticity is abruptly tilted into the vertical in the corner-like flow. Although the origin of that horizontal vorticity is mentioned, the conclusions do not depend on whether, e.g., that vorticity originates from differential SGS mixing or from baroclinic production. The one claim regarding horizontal vorticity that was made based on the integrated budgets is that there is a significant positive contribution from horizontal stretching. This claim was supported by the fact that the individual combined tilting and stretching terms were large in magnitude and that the wind speed increased along the trajectories. Further support is provided by the recent analysis by Fischer and Dahl (2022), who also analyzed the evolution of the horizontal vorticity magnitude along trajectories entering tornado-like vorticies (TLVs), but they used Eq. (1). In their Figs. 4b and 11a, pertaining to mature TLVs, the stretching term indeed acts to increase the horizontal vorticity magnitude before the parcels enter the vortex.

We thus argue that our conclusions in the paper are unaffected by using a different equation than advertised to calculate the different terms contributing to the horizontal vorticity magnitude, but we wished to clarify the meaning of Eq. (2) in the original paper and to present the correct formulas that Figs. 3e,f, 5c, and 7e,f are based on.

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