TRUE MEAN TEMPERATURE.

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SYNOPSIS.

The complexity of the question of obtaining the true mean temperature led the late A. Buchan to look favorably on the mean daily extremes (maximum plus minimum)/2 as an expression of the mean temperature. This view has become traditional among English-speaking races, but it has two very grave objections, and it is not adopted by continental or South American meteorologists. These objections are briefly: (1) Maximum and minimum thermometers, especially the latter, are far more likely to develop systematic errors than is the ordinary dry bulb. This remark applies with great force to hot countries, where in addition the observers are frequently untrained. In fact, it was this very difficulty which led to the adoption of a column of "True mean temperature" in *Reseau Mondial*. (2) The relation (maximum plus minimum)/2 to the true mean is not even approximately fixed, but varies with the location, with the cloudiness, and with the time of year.

The means of observations taken at 7 a. m., 1 or 2 p. m., and 9 p. m., giving the evening observation double weight, is recommended for general use.—*H. L.*

The ideal standard of mean temperature is the mean height of the trace given by a thermograph corrected for any sources of instrumental error. In practice the mean of 24 observations each day at intervals of one hour gives the mean temperature with amply sufficient accuracy, so that the terms "true mean" (here written T.) and 24-hourly mean are synonymous, and means of observations every two hours, three hours or even four hours may be accepted as sufficiently close to the true or 24-hourly mean to need no correction. But observations at such frequent intervals are the exception in most networks of stations, the majority of which are of the second order. Here, as a rule, three observations are taken each day, with maximum and minimum temperatures, and we are faced with the problem of reconstructing from these the true mean. Such a reconstruction is highly important as a necessary preliminary to the comparative study of temperatures in different countries, but it must be remembered that it can only be made satisfactorily from monthly or annual means, and in temperate countries at least should never be employed for individual days. The methods adopted in various publications fall under four heads:

(A) The combination of the means of the observations at the three fixed hours, or of the maximum and minimum in certain proportions which have been found to give approximately the true mean at "standard" stations where hourly observations are available.

(B) The calculation of appropriate additive corrections for various combinations of hours or for the mean of the maximum and minimum at standard stations, and their transference without modification to other stations in the vicinity. A development of this method is to plot the corrections at standard stations on maps, and read off the corrections at intervening stations from these maps.

(C) The corrections at the standard station may be multiplied by a factor proportional to the daily range (or to the difference, midday-evening observations) at the station to be corrected.

(D) All direct use of standard stations may be avoided and the reduction to true mean based on considerations connected with the general phenomena of the diurnal variation of temperature.

A and B may be described as "mechanical" methods, since they take no direct account of the diurnal variation, while C and D, which are based on the existence of a normal diurnal curve, may be described as "logical." But it can not be stated dogmatically that any one of them is better than the others. Each has advantages in special cases; for example, methods A and D are of value where the stations are isolated or the hours of observation irregular, B is the obvious method over the wide plains of Siberia or America, and C has advantages in the complicated relief of Norway. Every case must be considered on its merits.

A. The most general combination of hours is 7 h., 13 or 14 h., and 21 h; call the observations at these hours I, II, and III, respectively. The direct mean $(I+II+III)/3$ is too high. The mean

$$T=(I+II+2\times III)/4 \qquad \ldots \quad (1)$$

gives the best results of any of the combinations in general use, the differences from the mean being small and fairly constant. At stations in Greenland, where I refers to 8 h., the combination in use is

$$T=\{2(I+II)+5\times III\}/9 \qquad \ldots \quad (2)$$

A brief investigation of this type of formula by the method of least squares gave interesting results. The method employed was to take for a number of standard stations the deviations from true mean of I (7 h.), II (13 h.), and III (21 h.) and deduce from these figures the best-fitting equation of the form:

$$p\times I+q\times II+r\times III=o \qquad \ldots \quad (3)$$

Three groups of stations were tested:

(1) Western Europe, 22 stations ranging from Upsala to Vienna. Figures from *Angot, A, Etudes sur le climat de France. Température. Paris, Ann. Bur. centr. météor*, 1902, pt. 1, p. 41. The best combination (annual mean) is approximately the usual one $(I+II+2\times III)/4$.

Examples, Paris (Parc St. Maur) and Kew, T. (calc.) —T. (obs.):

	Jan.	Feb.	Mar.	Ap.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Paris....	−0.12	−0.08	+0.01	−0.09	−0.15	−0.15	−0.11	−0.03	+0.11	+0.10	−0.07	−0.12
Kew.....	− .08	− .07	− .01	− .01	− .18	− .23	− .18	− .05	+ .06	+ .04	− .05	− .09

(2) Subtropics, 22 stations. Figures from *Hann, J von, Die tägliche Gang der Temperatur in der aüsseren Tropenzone. Wien, Denkschr, Ak. Wiss.*, 80, 1906, p. 317; 81, 1907, p. 21.

The best combination is

$$(I+0.9\times II+1.1\times III)/3$$

which can also be written

$$\{I+II+III-\tfrac{1}{10}(II-III)\}/3 \qquad \ldots \quad (4)$$

This combination gave remarkably accurate results, the probable error of the true mean being only a fraction of a degree absolute.

Example, Mauritius, T. (calc.)—T. (obs.):

Error.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
$\{I+II+III-\frac{1}{10}(II-III)\}/3$......	0.1	0.1	−0.1	0.0	−0.2	−0.2	−0.2	−0.2	0.0	0.4	0.6	0.1

(3) Tropics, 30 stations. Figures from *Hann, J. von, Die tägliche Gang der Temperatur in der inneren Tropenzone. Wien, Denkschr. Ak. Wiss.*, 78, 1905, p. 249.

The best combination is No. (4), but is not quite so exact as for subtropical stations. An alternative is

$$\{2(I+II)+3\times III\}/7 \qquad \ldots \quad (5)$$

which is remarkably accurate at Batavia.

Example, Batavia, T. (calc.) – T. (obs.):

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
No. (4)..	–0.04	–0.04	–0.08	–0.09	–0.10	–0.11	–0.13	–0.13	–0.09	–0.06	–0.01	+0.01
No. (5)..	– .02	– .02	– .05	– .07	– .06	– .07	– .07	– .06	– .05	– .06	– .03	+ .01

For other hours the combinations would of course be different.

With regard to Sweden an interesting calculation has been carried out by Nils Ekholm.[1] Here observations are taken at all stations at 8 h., 14 h., and 21 h., *civil* time. Consequently, when this is converted to *local* time the hours of observation become increasingly earlier from east to west. The mean of five stations was taken as a standard for the whole country and the combination appropriate for any meridian was calculated in the form (3). For the hours of 8, 14, and 21 *local* time the calculated formula for the month of May is

$$T=0{\cdot}27\times I+0{\cdot}20\times II+0{\cdot}53\times III \quad \ldots \quad (6)$$

For other months the coefficients p, q, and r are somewhat different. The results when applied to Upsala are exceedingly accurate, viz:

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
49 years mean..	–0.01	–0.01	+0.03	+0.01	–0.02	–0.05	–0.05	0.00	+0.03	–0.02	–0.01	0.00

Method A is not necessarily limited to observations at fixed hours, but may be applied also to the mean daily maximum and minimum. For British stations the formula in use in the Weekly Weather Report is:

$$T=k\times M+l\times m$$ [2]

where $k+l=1$. This becomes in practice:

$$T=m+k\ (M-m) \qquad \ldots \quad (7)$$

k was determined empirically by reference to standard stations, and is a constant which varies only with the month.

At many stations the observations at fixed hours are combined with M and m in empirical formulæ. A few examples are:

Hamburg: May to August $(8+20h+M+m)/4$ · (8)
September to April $\frac{1}{2}\{(8+20h)/2+(8+14+20h)/3\}=(5\times 8h+2\times 14h+5\times 20h)/12$ . . . (9)

Tunis: $\{7+13+19h+\frac{1}{2}\ (19h+m)\}/4$ . . . (10)

Egyptian stations: $(8+14+20h+m)/4$ . . . (11)

This combination gives values which are too low; better results would be obtained by substituting M for 14 h.

For Cairo (Abbassia) we have the following values of T. (calc.) – T (obs.):

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
$(8+14+20h+m)/4$.	–1.1	–1.0	–1.0	–1.0	–0.8	–0.8	–0.5	–0.6	–0.5	–0.8	–0.9	–0.9
$(8+20h+M+m)/4$.	– .8	– .7	– .6	– .5	– .4	– .4	– .2	– .3	– .3	– .5	– .6	– .6

Ponta Delgada: $(9+21h+M+m)/4$ . . . (12)

This is a good combination for stations under a great variety of conditions, since $\frac{1}{2}\ (M+m)$ is too high and $\frac{1}{2}\ (9+21h)$ is too low, and the errors partially balance each other. For example, at Naha, Liu Kiu Islands:

Correction for—	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
$\frac{1}{2}\ (9+21h)$	+0.28	+0.19	+0.18	+0.04	0.00	+0.08	–0.01	–0.01	–0.06	–0.05	–0.01	+0.20
$\frac{1}{2}\ (M+m)$	– .22	– .17	– .20	– .31	– .26	– .48	– .43	– .38	– .52	– .24	– .16	– .18
Mean....	+ .03	+ .01	– .01	– .13	– .13	– .20	– .22	– .19	– .29	– .14	– .09	+ .01

Combination (12) is thus suitable for the winter and spring months, while for summer and autumn the simple mean $\frac{1}{2}\ (9+21h)$ is better.

B. The calculation of additive corrections is only suitable where conditions are essentially similar at the standard station and at the station to be corrected. Where available it is the simplest and best method. In Wild's hands in Russia and Siberia it reached its fullest development, since there conditions change only gradually over a wide area. Wild expresses his final results for $(7+13+21h)/3$ as a table giving figures for every intersection of a 5°-coordinate, a form very handy to use and requiring less space than a series of charts. In the United States it is suitable in the eastern half of the continent, but is unsafe in the western half, especially when applied to $\frac{1}{2}\ (M+m)$. Similar remarks apply to India,[3] where the corrections are sound over most of the country but are probably doubtful for the mountain stations.

Method B can be combined with A further to correct the slight errors of the best combinations of hours available. An example is the Chilian network, where the following additive corrections have been found for the combination $(I+II+2\times III)/4$, the hours of observation being 7, 14, and 21 h.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Mollendo (Peru)..	+0.2	+0.2	+0.2	+0.3	+0.2	+0.1	+0.1	+0.1	+0.1	0.0	0.0	+0.1
Santiago..........	– .4	– .2	.0	+ .2	+ .2	+ .1	+ .1	+ .2	+ .1	– .1	– .3	– .4
Punta Arenas.....	.0	.0	+ .1	+ .1	.0	.0	.0	+ .1	+ .1	.0	.0	– .1

It would have been better if Wild had employed this compound method for use with the Russian stations, as the mean $(7+13+21h)/3$ involves considerable corrections in summer, e. g. Nertchinsk.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
$(7+13+21h)/3$..	–0.28	+0.01	+0.07	–0.26	–0.37	–0.50	–0.42	–0.24	–0.05	–0.02	–0.23	–0.33
$(7+13+2\times 21h)/4$	– .09	+ .16	+ .27	+ .06	+ .01	– .11	– .03	+ .14	+ .25	+ .21	– .05	– .11

C. In countries of varied relief it is not safe to assume that the diurnal variation is similar at two neighboring stations. The amplitude especially may show considerable fluctuations between, say, hill stations and valley

[1] Stockholm, Institute Central de Météorologie. Vol. 56, 1914, App: Calcul de la température moyenne mensuelle de l'air aut stations météorologiques suédoises. Uppsala 1916.

[2] M is employed throughout as the symbol for mean daily maximum, and m for mean daily minimum.

[3] Calcutta, Meteor. Office. Indian Meteor. Memoirs, vol. 17, Calcutta, 1904.

stations. To meet this difficulty Kämtz suggested the following method [4] for use at stations where observations are taken three times daily: Find the difference $II-III$ at the standard station and at the station to be reduced, call these values "s" and "a," respectively. Let Cs be the additive correction to $(I+II+III)/3$ at the standard station. Then the additive correction at the station to be corrected is given by

$$Ca = Cs \times (a/s) \qquad \ldots (13)$$

This should eliminate the influence of the amplitude. But it appears that at stations with similar climates the amplitude is similar, and if the climates are dissimilar, the method is hazardous. For example, let us take the case of Los Andes, in a valley at the foot of the Andes,

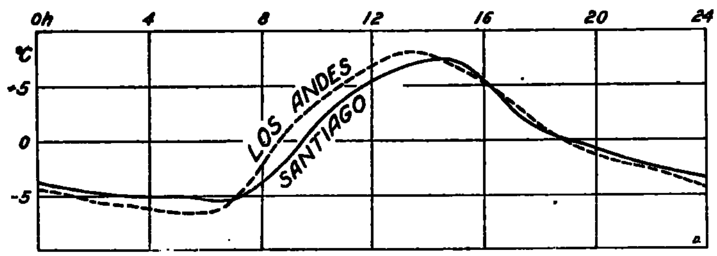

FIG. 1.—Diurnal variation at two neighboring stations. The range is similar but the shape of the curves differ.

and the neighboring station of Santiago, on the coastal slope. In July 1912 we have for the hours of 7, 13, and 21 h.:

	I	II	III	$II-III$	a/s	$(I+II+III)/3$	Correction.		T. (Calc.).	T. (Obs.).
							Obs.	Calc.		
Los Andes, °C	4.7	18.0	8.3	9.7	0.87	10.3	−0.6	......	......	9.7
Santiago, °C	2.3	13.9	5.5	8.4		7.2	......	−0.5	6.7	7.1

Here the required corrections for Santiago was -0.1, and the correction found was -0.5. Drawing the curves of hourly temperatures (fig. 1) it is seen at once that the source of the error is the sharp right minimum at Los Andes.

The method has been developed by Mohn for use at Norwegian stations into the formula:

$$T = n - k(n-m) \qquad \ldots (14)$$

where n is the mean of the three fixed daily observations and k is a factor varying with the station and month. The genesis of the formula is as follows: Write $n = (I+II+III)/3$ and give m the variable coefficient β. Then we have

$$T = (3n + \beta m)/(3+\beta) = n - \beta(n-m)/(3+\beta)$$

whence $k = \beta/(3+\beta)$; β can readily be found for standard stations. Maximum thermometers are not in use in Norway.

A somewhat similar method was developed independently in the case of Horta, where observations are taken at 9, 12, 15, and 21 h. The mean of these four observations is obviously too high and the formula adopted was

$$T = (9+12+15+21h)/4 - k(M-m) \quad . \; . \; (15)$$

By comparison with Ponta Delgada k was given the value 0.13, which is valid throughout the year.

[4] Lehrbuch der Meteorologie, Leipzig, 1831, Bd. 1, p. 89.

D. In many tropical regions no standard station can be found near enough to be of value in a direct reduction. This is peculiarly the case in Africa, where the difficulty was further complicated by the great diversity in the hours of observation. After much consideration a method was adopted which did not involve the direct use of standard stations, the corrections being based on a consideration of the normal diurnal variation of temperature as expressed by the Fourier series

$$T_h = T + a_1 \sin(t_h + A_1) + a_2 \sin(2t_h + A_2) \, . \, . (16)$$

A detailed description of the method has been reported to the Royal Meteorological Society,[5] but the more important formulæ are reproduced here. Using absolute degrees, a_1 and a_2, the coefficients of the first and second harmonics, are calculated from the equations

$$\left.\begin{aligned} a_1 &= -0.72 + 0.44(M-m) \\ a_2 &= +0.54 + 0.08(M-m) \end{aligned}\right\} \quad \ldots (17)$$

(1) Given three observations a day at intervals of six, six, and twelve hours (e. g., 6, 12, 18 h) we have

$$T = (I + 2 \times II + III)/4 - a_1 \times k \quad \ldots (18)$$

where $k = \frac{1}{2}(1 - (I-III)^2/4a_1^2)^{\frac{1}{2}}$.

This formula is not difficult to use and is generally accurate to less than $0.2a$. The chief uncertainty lies in the calculation of a_1 by (17).

(2) Given three observations, the first and last not separated by 12 hours,

$$T = p \times I + q \times II + r \times III - a_2 \times h \; . \; . \; . \; (19)$$

where p, q, r, and h depend solely on the hours of observation. For details the original paper must be consulted. Where the hours are 7, 13, or 14 and 21 h. it is better to use $(I + II + 2 \times III)/4$ or one of its variants.

(3) Given two observations a day separated by 12 hours (I and III), we have

$$T = (I + III)/2 + a_2 \times l \qquad \ldots (20)$$

where l depends chiefly upon the hours of observation. For observations at 9 and 21 h. at stations within the Tropics we may take $l = 0.4$, for 8 and 20 h. $l = 0.8$: Thus we have, for 9 and 21 h.

$$T = (I + III)/2 + 0.03(M-m) + 0.2 \quad . \; . \; (21)$$

This is quite a useful formula.

In this paper some attempt was made to deduce additive corrections to $\frac{1}{2}(M+m)$ which should be generally applicable in the Tropics, and a table was compiled giving the correction in terms of $(M-m)$ and ϕ. The latter symbol stands for the term $2A_1 - A_2$, where A_1 and A_2 are the phase angles of the first and second terms of the Fourier series. ϕ is calculated from an empirical formula involving the latitude, the height, and the cloudiness. Some more direct method is obviously desirable, and for the tropical and subtropical regions between 30° N. and 30° S. a simple empirical formula has been calculated involving only the height and the mean daily range. The stations were divided into two groups, and for each station 36 monthly means were taken from the *Reseau Mondial*, 1911–1913. From the figures the following parabolic equations were obtained. R is the daily range, $M-m$.

[5] Brooks, C. E. P. The reduction of temperature observations to mean of 24 hours and the elucidation of the diurnal variation in the continent of Africa. London, Q. J. R. Meteor. Soc., 43, 1917, p. 375. (8)

(1) Stations below 1,000 meters; mean height 68 m.:

$$T=(M+m)/2-0.30-0.0025R-0.0030R^2$$

(2) Stations above 1,000 meters; mean height 1,922 m.:

$$T=(M+m)/2-0.23-0.14R+0.0085R^2$$

From these two equations we may deduce the general one,

$$T=(M+m)/2+(a+bR+cR^2) \quad . \quad . \quad . \quad (22)$$

where

$a=-0.30\ \ +0.04h$
$b=\ \ \ 0.00\ \ -0.07h$
$c=-0.0034+0.006h$

h being the height of the station in kilometers. The figures refer to absolute degrees.

A preliminary trial of this formula gave encouraging results, and it was tested for 23 stations for each month of 1914, giving 276 cases. The results showed that T (calc.) has a probable error of 0.18a, so that the application of the method is distinctly worth while when no other means of correction is available.

The average difference, $\frac{1}{2}(M+m)-T$, is 0.5a, and if this is applied as a constant correction, the probable error of the result is 0.23a.

For the convenient application of formula (22) Table 1 has been constructed, showing the correction at various heights and with various ranges.

TABLE 1.—*Corrections to* $\frac{1}{2}(M+m.)$

Daily range.	Height in meters.							
	0	500	1,000	1,500	2,000	2,500	3,000	3,500
a.								
4	−0.35	−0.42	−0.49	−0.56	−0.63	−0.70	−0.77	−0.84
6	− .42	− .50	− .58	− .66	− .74	− .82	− .90	− .98
8	− .52	− .59	− .65	− .72	− .78	− .85	− .91	− .98
10	− .61	− .67	− .70	− .73	− .76	− .79	− .82	− .85
12	− .79	− .76	− .73	− .70	− .67	− .64	− .61	− .58
14	− .97	− .85	− .73	− .61	− .49	− .37	− .25	− .13
16	−1.17	− .94	− .71	− .48	− .25	− .02	+ .21	+ .44
18	−1.40	−1.04	− .68	− .32	+ .04	+ .40	+ .76	+1.13
20	−1.66	−1.14	− .62	− .10	+ .42	+ .94	+1.46	+1.98

THE MEAN OF THE DAILY EXTREMES.

The complexity of the question of obtaining the true mean temperature led the late A. Buchan in his strivings after uniformity to look favorably on the mean of the daily extremes $(M+m)/2$ as an expression of the mean temperature. This view has become traditional among the English-speaking races, but it has two very grave objections, and is not adopted by continental or South American meteorologists. These objections are briefly:

(1) Maximum and minimum thermometers, especially the latter, are far more likely to develop systematic errors than is the ordinary dry bulb. This remark applies with great force to hot countries, where in addition the observers are frequently untrained and adequate inspection or comparison of readings is impossible. This has been brought out repeatedly in preparing normals for use in the *Reseau Nondial,* and it was in fact this very difficulty which led to the introduction of a column for "True mean temperature."

(2) The relation of $(M+m)/2$ to true mean is not even approximately fixed but varies with the location, with the cloudiness and with the time of year. The absolute range of the necessary correction, taking the monthly mean as the unit, is from about 1a to −2a, three degrees absolute, which is a very considerable degree of uncertainty. In cases of doubt Hann frequently assumes a correction of −1a, but this is too great, as we have seen. Assuming a correction of −0.5a for the Tropics, we are faced with a probable error of 0.23a, whereas in the case of most of the reductions based on observations at fixed hours the probable error is less than 0.1a.

It is at subtropical desert stations that the anomalous values of $T-(M+m)/2$ are chiefly found. At these

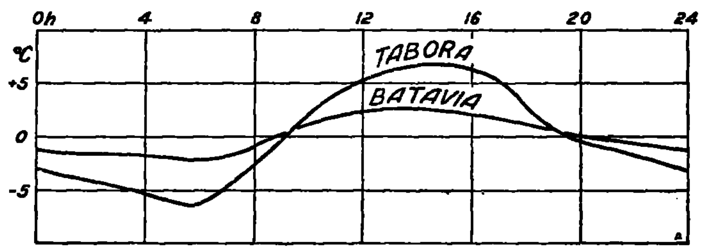

FIG. 2.—Diurnal variation at Batavia (humid) and Tabora (Tanganyika Terr.) (dry). Note the sharp minimum at the latter station

stations, where the air is very dry, nocturnal radiation is strong, and temperature continues to fall at an almost uniform rate throughout the night, giving a sharp minimum about sunrise (fig. 2, Tabora). On the other hand at stations where the air is humid the rate of fall slows down after a few hours, and the minimum is not sharp (fig. 2, Batavia). Hence at desert stations the minimum is much farther below the mean temperature of the night than at humid stations. The maximum temperature of the day is also affected, but not to the same extent, so that the sharp minimum at desert stations lowers the mean of the daily extremes relatively to the true mean, and the correction to reduce the former to the latter may be positive instead of negative. The irregularity of the correction at desert stations is due to the fact that a small difference in the slope of the curve before dawn, such as may easily be introduced owing to the great variability of the humidity in such situations, makes a great difference in the minimum temperature but has little influence on the 24 hourly mean.

CONCLUSION.

The conclusion to which all this tends is that meteorologists in the English-speaking countries would be well advised to make a less exclusive use of the mean of the daily extremes and to place more reliance on observations at fixed hours, the best combination being $(7+13 \text{ or} 14+2\times21)/4$.