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AN EMPIRICAL STUDY OF VERTICAL VELOCITIES IN THE LOWER STRATOSPHERE*

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ABSTRACT

Vertical velocities have been computed for the lower stratosphere for two independent winter periods, by employing a form of the adiabatic method. The regions studied were in both cases outside the polar vortex. The flow pattern was divided into stationary (long-wave) and transient (short-wave) components. The vertical velocity pattern associated with the stationary long wave is precisely that described by Kochanski [3]; i.e., the air rises in moving from warm troughs to cold ridges. The pattern associated with the short waves is more complicated. There is a maximum of warm advection in the vicinity of short-wave ridges, and cold advection near troughs. Local temperature changes, however, very nearly compensate the advection, with the net result that in the mean the vertical velocities associated with short-wave patterns are small, but tend to be positive near ridges and negative near troughs. Superimposition of the short and long wave, however, can lead to any conceivable combination of signs of advection, local temperature change, vertical velocity, and position with respect to ridge or trough.

The single parameter which is most useful in specifying the vertical velocity is the temperature advection.

1. INTRODUCTION

In the middle 1940's, spurred by recent developments in atmospheric sounding techniques, meteorologists began to investigate in considerable detail the nature of the vertical motions in the lower and middle troposphere and their relation to the horizontal flow. Now, in the late 1950's, as further improvements have increased the density and reliability of data in the lower stratosphere, it is feasible to extend our investigations to those levels. The present study is offered as an early step in that direction. A hypothesis will be formulated concerning the manner in which the vertical velocities are related to the troughs and ridges in the lower stratosphere, in the hope that this may serve as a basis for further studies.

The earliest attempt at an empirical evaluation of the vertical velocities in the lower stratosphere was that of Kochanski in 1951 (cf. [3]). Mean soundings at troughs and ridges were compared, and vertical velocities were computed on the assumption of adiabatic motion by computing the time it would take for parcels of air to move along a contour from ridge to trough. Average vertical velocities of the order of 3 cm. sec.$^{-1}$ were found near the tropopause, diminishing to almost negligible amounts just above 100 mb. Brockmeyer [1] studied vertical velocities in the lower stratosphere for the period January 1–10, 1956 by means of the usual adiabatic method. His results were quite similar to those of Kochanski; i.e., values of vertical velocity ranging up to 3 to 4 cm. sec.$^{-1}$ at 200 mb. with smaller values at higher levels, and, in general, subsidence from ridge to trough and rising motion from trough to ridge.

Craig and Hering [2], in a study of the sudden warming in the polar stratosphere in January 1957, estimated that to account for changes in temperature along a trajectory, assuming adiabatic processes, vertical velocities of about 5 cm. sec.$^{-1}$ were required at 25 mb. This, of

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course, was a somewhat anomalous situation. Teweles [4], in discussing the same situation, estimated average rates of subsidence of 1 to 3 cm. sec.$^{-1}$ over somewhat longer paths.

2. METHOD

The technique chosen for computing vertical velocities is a modified version of the adiabatic method. This method is particularly well suited to the stratosphere because of the large stability in that region. The basis of the adiabatic method is the equation:

$$w = -(\mathbf{V}\cdot\nabla T + \partial T/\partial t)/(\gamma_{ad}-\gamma). \qquad (1)$$

The assumption is made that the horizontal advection of temperature ($\mathbf{V}\cdot\nabla T$) is very nearly equal to constant pressure advection.

To simplify the computations it is also assumed that the lapse rate (γ) may be neglected in the stratosphere, in comparison with the adiabatic lapse rate (γ_{ad}). This is not likely to be in error, in the regions studied, by more than 15 or 20 percent. Finally, then, if the vertical velocity (w) is measured in units of cm. sec.$^{-1}$, and if the units of advection and local temperature change are °K.(12 hr.)$^{-1}$, equation (1) may be rewritten

$$w = -0.24(\mathbf{V}\cdot\nabla T + \partial T/\partial t). \qquad (2)$$

In most previous applications the right hand side of equation (2) has been evaluated by estimating temperature changes along trajectories. This is unsatisfactory for two reasons. First, there is the difficulty of constructing trajectories from synoptic maps; second, the vertical velocities computed are then averages both in time and space over the length of the trajectory. Since a parcel of air may travel hundreds or even thousands of kilometers in 12 hours, this leads to considerable ambiguity in interpreting the results with respect to the synoptic pattern.

By evaluating the two terms on the right hand side of equation (2) separately, a method is developed by which adiabatic vertical velocities, although averages over 12-hour time intervals, refer to specific areas, which, in the present case, are 10° latitude by 10° longitude. There is no longer any question as to where, along a trajectory, the vertical velocities occurred.

The basic grid is indicated in figure 1. The input information was the temperature at every point in the 5° x 5° grid (numbers in fig. 1) and mean winds within every 5° box (indicated by letters). The temperatures were estimated to the nearest ½° K. on the basis of an isothermal analysis made for every 2° K. The vector winds were estimated subjectively to the nearest 5 knots and 10°, from observed winds wherever possible, but supplemented by geostrophic winds when necessary.

The mean temperature at a point, such as point 5 in figure 1, is defined as:

$$\overline{T}_5 = \frac{1}{16}(T_1 + 2T_2 + T_3 + 2T_4 + 4T_5 + 2T_6 + T_7 + 2T_8 + T_9).$$

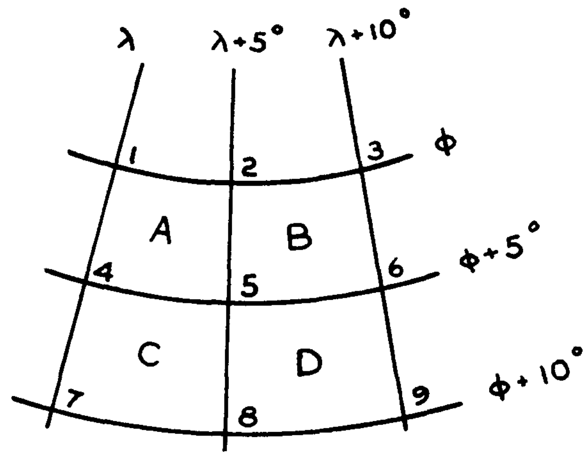

FIGURE 1.—Grid employed in vertical velocity computations.

The mean advection at the same point is defined by:

$$(\overline{\mathbf{V}\cdot\nabla T})_5 = \frac{1}{4}[(\mathbf{V}\cdot\nabla T)_A + (\mathbf{V}\cdot\nabla T)_B + (\mathbf{V}\cdot\nabla T)_C + (\mathbf{V}\cdot\nabla T)_D]$$

where

$$(\mathbf{V}\cdot\nabla T)_A = \frac{Cu_A}{\cos(\varphi+2\tfrac{1}{2}°)}[(T_2-T_1)+(T_5-T_4)] + Cv_A[(T_1-T_4)+(T_2-T_5)].$$

Here u_A and v_A are, respectively, the west-east and south-north wind components corresponding to the mean vector wind in box A. C is a proportionality factor which involves the grid size and the conversion of units.

Averages over time (12 hours) of advection and temperature change were accomplished by averaging the advection fields on two successive maps and subtracting the temperature fields. In other words, if the times of successive maps are t_o and t_o+12,

$$\left(\frac{\partial \overline{T}}{\partial t}\right)_5^{(t_0+6)} = \overline{T}_5^{(t_0+12)} - \overline{T}_5^{(t_0)}$$

and

$$(\overline{\overline{\mathbf{V}\cdot\nabla T}})_5^{(t_0+6)} = \frac{1}{2}[(\mathbf{V}\cdot\nabla T)_5^{(t_0)} + (\mathbf{V}\cdot\nabla T)_5^{(t_0+12)}].$$

3. SOME EXAMPLES OF THE RESULTS

Computations of advection and local temperature change in the lower stratosphere (200, 150, and 100 mb.) were made for two winter periods, December 17–23, 1953, and January 1–10, 1956. The latter period had been studied by Brockmeyer [1] by means of the trajectory technique.

Figures 2–5 are examples of some of the results of the study. The contour patterns at 150 mb. for December 17, 1953 and for January 4, 1956 are shown in figures 2 and 3 along with the corresponding patterns of vertical velocity. Figures 4 and 5 contain latitudinal cross sec-

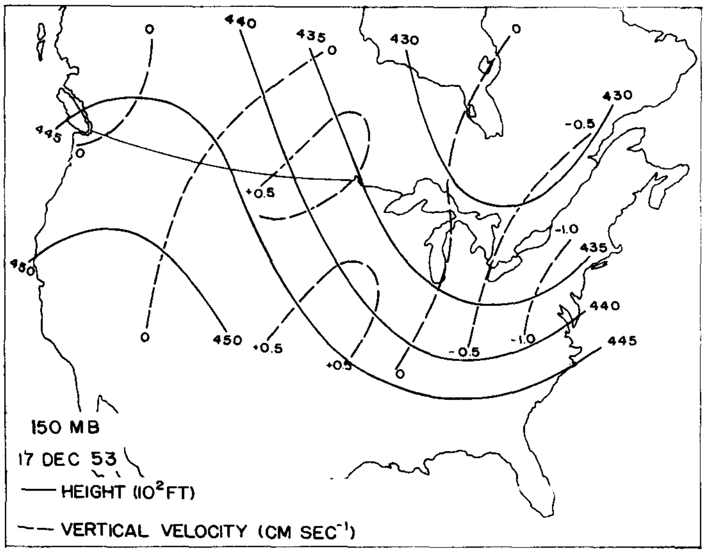

FIGURE 2.—Contours and vertical velocities at 150 mb. on December 17, 1953.

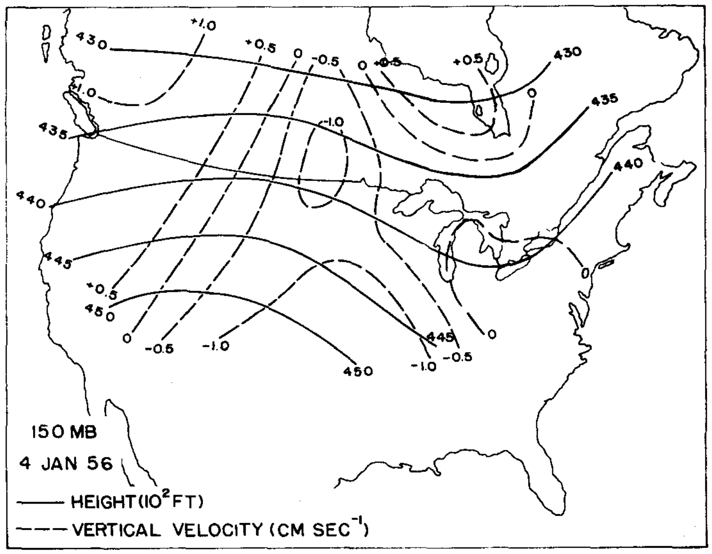

FIGURE 3.—Contours and vertical velocities at 150 mb. on January 4, 1956.

tions on the same dates. In all four figures, the vertical velocities are averages of two successive sets of computations; i.e., they are 24-hour averages.

On December 17 (fig. 2) there was a pronounced ridge near 115° W. longitude and a trough near 75° W. Between these two meridians was a zone of positive vertical velocities above 200 mb. The magnitude of the vertical velocities was not large but a smooth pattern is indicated and it persisted for the next several periods. The evidence from the cross section (fig. 4) is that there was slight horizontal convergence in this region. Although the vertical velocities were still positive (or zero) even at 200 mb., the pattern suggests that within the troposphere the air was probably subsiding as it moved from ridge to trough. Thus, it appears there was a reversal of the sign of the vertical velocity between the troposphere and the lower stratosphere.

The case of January 4 (fig. 3) tells a different story. Here there was a ridge near 105° W. longitude and a trough near 80° W. In this case, however, the region between these meridians was one of pronounced subsidence. The cross section (fig. 5) shows that the subsidence increased downward. There was no change of sign at the tropopause.

Meteorologists have learned to expect subsidence and convergence in the upper troposphere in association with flow of air from ridge to trough. Neither of the cases presented here, in spite of their differences, conflicts with this concept. In both cases, the indicated sign of the convergence in the lower stratosphere is in agreement with that which we would expect in the upper troposphere. In one case this convergence resulted in decreasing subsidence with increasing height; in the other the vertical gradient of vertical velocity ($\partial w/\partial z$) appears to have been of such a magnitude that there was a reversal of the sign of the vertical velocity between the upper troposphere and the lower stratosphere, and the rising motion in the lower stratosphere increased with height.

We are confronted, then, with one case in which the stratospheric pattern of vertical velocities was that with which we are familiar in the troposphere; i.e., subsidence in air moving with a northerly component; and another in which the sign of the vertical velocity was reversed. It should be pointed out here that the two examples presented here are not extreme cases, although they were chosen to illustrate the wide range of vertical velocity patterns. It should also be pointed out that all possible combinations of vertical velocity and meridional flow were encountered, particularly during the period of December 1953.

4. THE LONG-WAVE PATTERN

In an attempt to create some understanding out of the seeming disorder apparent on the daily maps, each sequence was separated into a time-mean component and deviations from the mean. The means were computed by averaging the values at each grid point, and here are identified with the long-wave systems which are indeed very nearly stationary. The deviations from the means are identified with the short-wave systems. In this section we shall discuss only the long-wave (mean) components.

The map in figure 6 illustrates the mean contour, isotherm, and advection fields at 150 mb. for the period December 17–23, 1953. (The mean advection is determined not by $\overline{\mathbf{V}}\cdot\overline{\nabla T}$, but rather as the average of all the fields of advection ($\overline{\mathbf{V}\cdot\nabla T}$). Any correlation between the deviations of $\mathbf{V}$ and ∇T from their means will cause differences between these two types of averages. Thus, the field of advection in figure 6 (and also in fig. 7) need not agree with that which would be given by the contours and isotherms on the maps). Over this period there was a slight

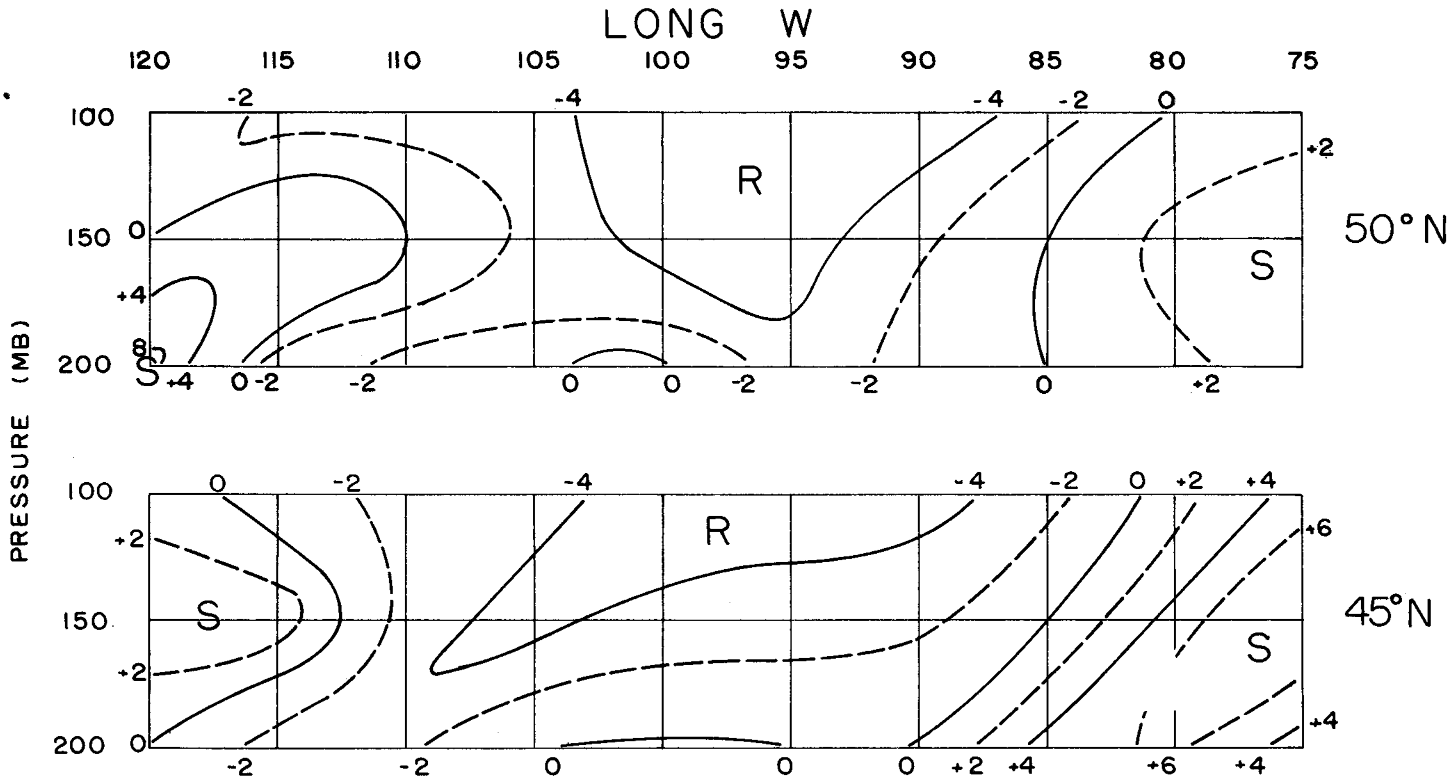

FIGURE 4.—Vertical velocity cross sections in the lower stratosphere along latitudes 45° N. and 50° N. on December 17, 1953. R indicates rising motion, S subsidence. Isopleths are labeled in units of mb. (12 hr.)$^{-1}$. This choice of units, plus a linear pressure scale as the abscissa, makes it possible to interpret the vertical velocity directly in terms of convergence or divergence.

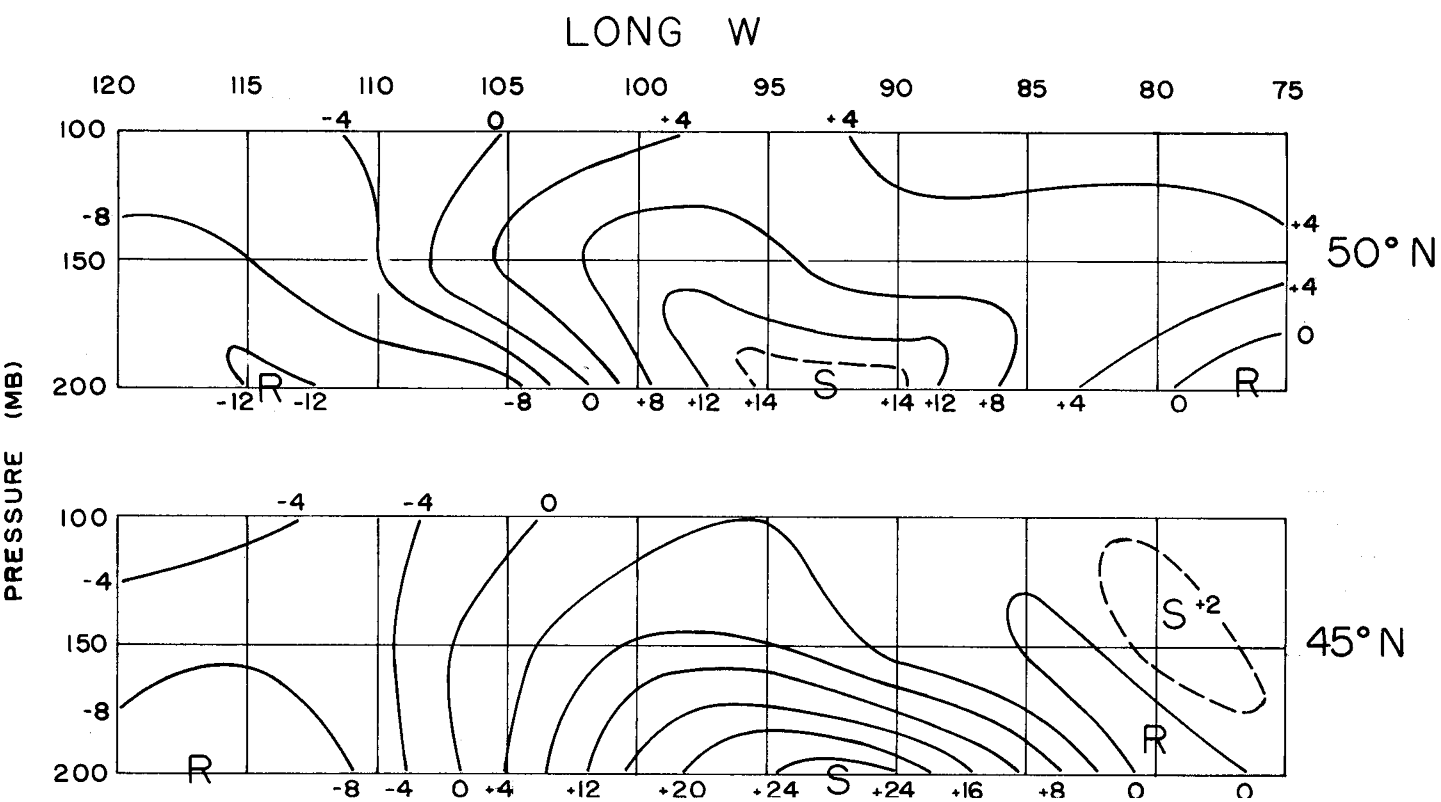

FIGURE 5.—Vertical velocity cross sections in the lower stratosphere along latitudes 45° N. and 50° N. on January 4, 1956.

decrease in temperature which occurred almost uniformly over the entire area. This decrease in temperature is taken to be a diabatic radiational heat loss and is therefore not included in the determination of the mean vertical velocity. One then writes:

$$\overline{w} = -0.24\, \overline{\mathbf{V}\cdot\nabla T} \qquad (3)$$

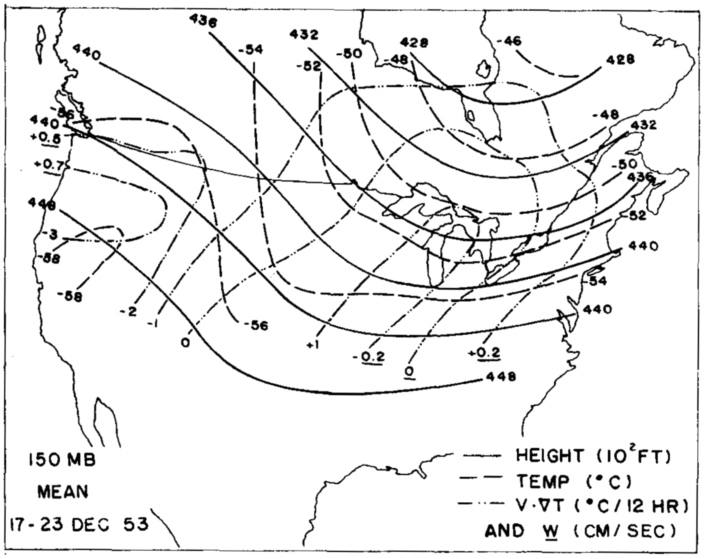

FIGURE 6.—Mean 150-mb. map for the period December 17–23, 1953. The isopleths of mean temperature advection are labeled in °K. (12 hr.)$^{-1}$ and also as mean vertical velocities in cm. sec.$^{-1}$ according to equation (3). The underlined values are the vertical velocities.

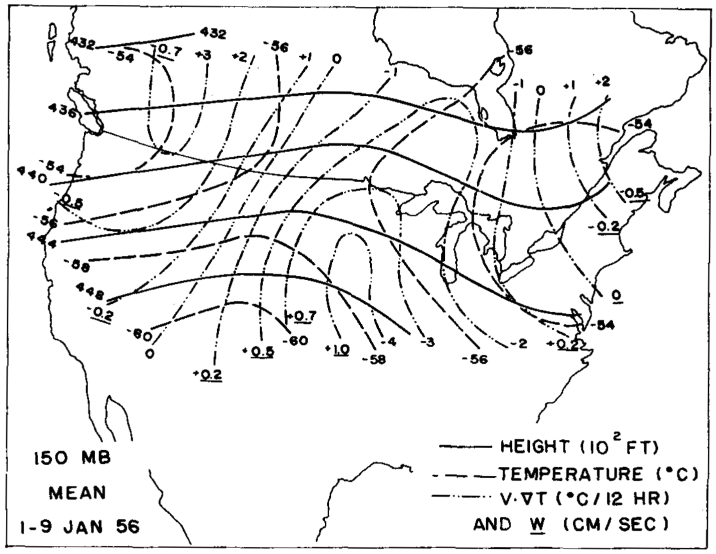

FIGURE 7.—Mean 150-mb. map for the period January 1–10, 1956. Isopleths are labeled as in figure 6.

Certainly, if the long waves are indeed stationary and in a steady state, there should be no local temperature changes which could be associated with them. Thus, the isopleths of advection in figure 6 are labeled at one end in units of °K. (12 hr.)$^{-1}$ and at the other in units of cm.sec.$^{-1}$.

The mean map for the period January 1–9, 1956 is given in figure 7. In this case, it should be pointed out, the averages of the local temperature changes, although everywhere small (mostly less than ¼° K. per 12 hours), were not uniformly negative and did exhibit some semblance of a pattern. This is due to a combination of effects: the fact that an integral number of short waves need not have passed a given location during the period; non-uniform radiational effects caused by differences in the temperature and nature of the earth's surface and of the air itself; also, a slight change of the long-wave position may have occurred. Nevertheless, the local temperature changes were sufficiently small that they are neglected in figure 7, and equation (3) was employed directly as in figure 6.

The fact that the average local temperature changes were so small and uniform offers a considerable degree of "a posteriori" justification for identifying the mean fields with the atmospheric long waves. If the long waves had moved significantly during the periods studied they would have carried with them their temperature fields, and this movement would have shown up in the average temperature derivatives. Examination of the daily maps of the deviations of the contour fields from their means (not shown here) also supports the contention that the long waves were very nearly stationary during these periods.

Both situations exhibit certain fundamental features which we have come to recognize as characteristic of the lower stratosphere outside the polar vortex, namely, warm troughs and cold ridges, with the isotherms showing greater amplitude than the streamlines. This corresponds, as Kochanski pointed out, to air descending from ridge to trough and ascending from trough to ridge. Kochanski's results indicated an average vertical velocity of 1.4 cm. sec.$^{-1}$ near the point of inflection at 140 mb., a value which is in reasonable agreement with those shown in figures 6 and 7.

5. SHORT-WAVE COMPONENTS

One may obtain an array of deviations from their means of $\mathbf{V}\cdot\nabla T, \partial T/\partial t, w$, and h (the height of the 150-mb. surface) by subtracting from the daily mean values the appropriate means as shown in figures 6 and 7 (and also the mean values of $\partial T/\partial t$). These deviations are taken as representing the effects on each of the parameters of the migratory short-wave disturbances. To study the interrelations of the various terms, correlation coefficients were computed. The results are summarized in tables 1 and 2. The variables correlated are shown by the subscripts to r according to the following scheme: $1=\mathbf{V}\cdot\nabla T$, $2=\partial T/\partial t$, $3=w, 4=h$.

The largest coefficients are clearly those between advection and vertical velocity. The positive value means here that cold air sinks and warm air rises; i.e., a direct circulation. The principal significance of these coefficients is that the local temperature change, which is also strongly correlated with the advection, is not closely related to the vertical motion.

On the basis of the correlation coefficients it becomes possible to suggest a pattern of the relationships between the short-wave troughs and ridges and the corresponding components of the vertical velocity. In the vicinity of the short-wave troughs there tends to be a maximum of cold air advection and local cooling. The advection and

TABLE 1.—*Correlation coefficients computed for the period December 17–22, 1953. Subscripts on r refer to deviations from their mean values of the variables as follows:* $1=\mathbf{V}\cdot\nabla T$, $2=\partial T/\partial t$, $3=w$, *and* $4=h$. *n is the number of points on which the respective coefficients are based.*

	n	r_{12}	r_{13}	r_{14}	r_{23}	r_{24}	r_{34}
All days:							
40°	54	−0.442	−0.816	−0.416	−0.158	0.158	0.170
45°	60	−.549	−.852	−.599	.030	.556	.367
50°	66	−.761	−.767	−.676	.167	.749	.463
55°	66	−.767	−.446	−.739	−.233	.730	.101
All latitudes:							
Dec. 17	41	−.502	−.929	−.916	.146	.486	.840
Dec. 18	41	−.040	−.733	−.020	−.651	.645	−.424
Dec. 19	41	−.576	−.852	−.112	.062	.244	−.020
Dec. 20	41	−.387	−.794	−.422	−.254	.365	.201
Dec. 21	41	−.763	−.569	−.823	−.097	.655	.434
Dec. 22	41	−.696	−.848	−.527	.209	.755	.159
All days and all latitudes	246	−.616	−.765	−.592	−.036	.592	.267

the local temperature change, being of opposite sign, tend to cancel, and the vertical velocity associated with the short wave is consequently of small magnitude. However, since the magnitude of the advection in general exceeds that of the local temperature change, there is a tendency for the air to sink as it passes through troughs. Whether the vertical velocity which is here associated with the migratory disturbances is sufficiently persistent, or of great enough magnitude, to be of synoptic significance is doubtful. However, in spite of the small magnitude of the correlation coefficients, r_{34}, the agreement between the two independent periods lends credence to the pattern presented above.

In principle, variables which are poorly correlated may still be highly dependent upon one another, if the variables are 90° out of phase. Additional study of the data, however, indicates that this is not the case with regard to the relationship between vertical velocity and contour height.

6. SUMMARY

The vertical velocities in the lower stratosphere do not present the consistent pattern which is normally found in the upper troposphere. Flow from ridge to trough sometimes has a positive and at other times a negative component of vertical velocity. Nevertheless, the patterns of horizontal convergence in the lower stratosphere appear to be continuous with those in the upper troposphere.

The stationary and transient waves have been treated separately. The present evidence supports the findings of Kochanski [3] that, in association with the long-wavelength, quasi-stationary waves, the air subsides in flowing from cold ridges to warm troughs. Study of the shorter-wavelength, transient waves indicates a considerable degree of correlation between temperature advection and the height anomaly (−0.592 and −0.394 in the two independent cases), between local temperature change and height anomaly (+0.592 and +0.353), and between advection and local temperature change (−0.616 and −0.684). There is also a lesser correlation between vertical velocity and height anomaly (+0.267 and +0.293). These results suggest a model in which

TABLE 2.—*Correlation coefficients computed for the period January 1–9, 1956. Subscripts on r refer to deviations from their mean values of the variables as follows:* $1=\mathbf{V}\nabla\cdot T$, $2=\partial T/\partial t$, $3=w$, *and* $4=h$. *n is the number of points on which the respective coefficients are based*

	n	r_{12}	r_{13}	r_{14}	r_{23}	r_{24}	r_{34}
All days:							
40°	81	−0.727	−0.906	−0.537	0.368	−0.509	0.430
45°	99	−.682	−.908	−.324	.314	−.380	.153
50°	90	−.639	−.859	−.285	.156	−.276	.024
55°	72	−.715	−.859	−.549	.256	−.399	.449
All latitudes:							
Jan. 1	38	−.662	−.957	−.331	.414	.722	.122
Jan. 2	38	−.763	−.943	−.435	.504	.483	.404
Jan. 3	38	−.839	−.833	−.609	.397	.673	.255
Jan. 4	38	−.684	−.583	−.199	−.194	.271	.000
Jan. 5	38	−.158	−.905	−.377	−.277	.153	.301
Jan. 6	38	−.642	−.954	−.276	.383	.684	.066
Jan. 7	38	−.758	−.920	+.133	.441	−.358	.010
Jan. 8	38	−.796	−.678	−.303	.094	.080	.402
Jan. 9	38	−.165	−.926	−.550	−.218	.254	.447
All days and all latitudes	342	−.684	−.882	−.394	.259	.353	.293

there is considerable cold advection and local cooling, and a lesser degree of subsidence, in association with the short-wave troughs.

By superimposing the models suggested above, one arrives at a picture of the flow in the lower stratosphere which is in reasonable accord with observations. In particular, it is found that all possible combinations of positive and negative advection, local temperature change, vertical velocity, and meridional flow are to be expected, depending on the relative positions (and presumably also the intensities) of the long- and short-wave disturbances. Nevertheless the vertical velocity will always be well correlated with the advection; that is to say, the effect of local temperature change is small, by comparison, in the short waves, and nonexistent (or almost so) in the long waves. For the short waves the advection and vertical velocity were correlated by −0.765 and −0.882. For many purposes, then, the temperature advection, which can be determined from a single map, can be used as an adequate measure of vertical velocity.

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