BEHAVIOR OF THE PRINCIPAL HARMONICS OF SELECTED 5-DAY MEAN 500-MB. CHARTS

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ABSTRACT

The coefficients A_n and B_n, and the percentage of the total height variance accounted for by each harmonic were computed for the first three longitudinal harmonics of 5-day mean 500-mb. height-contour charts at each 10° of latitude from 20° N. to 80° N. from December 1, 1959, to May 31, 1960. The phase angles and amplitudes of the first three harmonics were computed at 30° N., 50° N., and 70° N. and plotted as a function of time. Retrogression of the waves was found at high and low latitudes, while relatively stationary conditions prevailed at middle latitudes. The correlation coefficient between the contribution of the sum of the first three harmonics and the entire wave train was found to be almost +0.9.

1. INTRODUCTION

This is a report of a study of the behavior of the long waves of the atmosphere at the 500-mb. level and is part of a larger project designed to predict their movement and intensity.

The first step in this project, and the one with which this paper is largely concerned, is simply the observation of the movements and intensities of the three longest waves, in order to become familiar with their individual characteristics and climatological variations. Comparisons may then be made between their actual movements and those predicted by present methods. A careful investigation in terms of synoptic events may reveal methods by which the present forecasting techniques can be improved.

In this part of the project the Extended Forecast Branch's harmonic analysis program for the IBM 704 computer was used to compute the phase angles and amplitudes of each harmonic on 5-day mean 500-mb. height-contour charts using height profiles along selected latitudes during the 6-month period, December 1, 1959, to May 31, 1960. By an automatic data-processing method, height values were interpolated from twice-daily charts and 5-day means obtained thrice weekly at every 10° of longitude around the globe in the Northern Hemisphere. It is estimated that the resulting 5-day mean heights should be accurate to the nearest 30 feet.

2. HARMONIC ANALYSIS PROCEDURE

A good discussion of harmonic analysis may be found in Panofsky and Brier [1]. Some aspects of this technique will be repeated here in order to make clear what was done.

A finite sum of sine and cosine terms is found which together add up to the height variations at 36 grid points along a given latitude circle. The first harmonic (wave 1) has a wavelength of 360° longitude or the entire distance around the earth. This is called the "fundamental wavelength." The second harmonic (wave 2) has a wavelength equal to one-half the fundamental wavelength, or 180° longitude. The third harmonic has a wavelength equal to one-third the fundamental wavelength, or 120°, and so forth. The phase angle and amplitude of each harmonic were computed for every 10° of latitude from 20° N. to 80° N.

If N is the number of height values around a given latitude circle (36 in this case), there can be only $(N/2)-1$ sine harmonics and $N/2$ cosine harmonics. The sum of the entire series is given by

$$z=\bar{z}+\sum_{n=1}^{n=(N/2)-1}[A_n \sin n\lambda]+\sum_{n=1}^{n=N/2}[B_n \cos n\lambda] \qquad (1)$$

where z is the height value at any longitude (λ); $\bar{z}$ is the zonal average height; n is the wave number; and A_n and B_n are amplitude factors. This equation states that a local height value z is equal to the mean zonal height plus the sum of all harmonics.

For each given 500-mb. 5-day mean chart the coefficients A_n and B_n were obtained for each wave number n, until their sum explained 99 percent of the total height-variance (not necessarily from $n=1$ to $n=N/2$). This was done for each 10° of latitude from 20° N. to 80° N.

Once the coefficients A_n and B_n have been found for wave number n, its amplitude (C_n) is given by

$$C_n=\sqrt{A_n^2+B_n^2} \qquad (2)$$

and its phase angle by

$$\lambda_n=\frac{1}{n}\arctan\,(A_n/B_n) \qquad (3)$$

The phase angle is measured in degrees longitude west of Greenwich and indicates the first position at which the wave-number n is at a maximum deviation; i.e., it marks the first ridge position. Since arctan (A_n/B_n) has two values between 0° and 360°, the correct choice of angle must be decided from the additional expression:

$$\lambda_n=\frac{1}{n}\arcsin\,(A_n/C_n) \tag{4}$$

It is important to know what percent of the total variance explained by *all* the waves is explained by each *individual* wave. Since the harmonics are all uncorrelated, no two harmonics can explain the same part of the variance of the sample. Therefore, the variances explained by the different harmonics can be added. For example, if wave 1 accounts for 40 percent and wave 2 for 20 percent, then waves 1 and 2 combined account for 60 percent of the total variance. The equation for the variance of a single harmonic n is $C_n^2/2$, except for the last harmonic in the series for which it is C_n^2. If S_z^2 is the variance of the entire sample, then the percentage of the total variance which is explained by wave n is $C_n^2/2S_z^2$ when $n<N/2$.

3. CHARACTERISTICS OF THE VARIOUS WAVES

In the following discussion, an attempt will be made to describe the characteristics of the long waves as found on 5-day mean 500-mb. maps using the three previously discussed parameters, i.e., the position and amplitude of the wave, and the percent of the total height variance explained by each wave. Graphs showing the first two of these features plotted as a function of time for waves 1, 2, and 3 at latitudes 70° N., 50° N., and 30° N. are shown in figures 1 through 9.

Before discussing each of these graphs, a few general remarks should be made. Continuity in time for the phase angle of each harmonic is established from thrice-weekly charts separated by intervals of two or three days. This presents no special problem when, during this time interval, the longitudinal displacement of the wave is small. However, occasionally the phase angle appears to "jump" over a large longitudinal range, as illustrated in figure 1 in December when the first harmonic moved 180° of longitude in 2 days (pentad 11 to pentad 12). In such cases it cannot be definitely established whether the phase angle moved westward or eastward. Only 5-day mean charts constructed at shorter time intervals could resolve this difficulty. In fact, it is not even certain that continuity can be established at all in these cases, for the jumps frequently occur when the amplitude is quite small, often (as in the case mentioned above) approaching the limiting accuracy of 30 feet. It is possible in these cases that the original wave component completely died out and a new one subsequently formed in a different location. A few of the interesting examples of such jumps will be included in the discussion of the individual graphs.

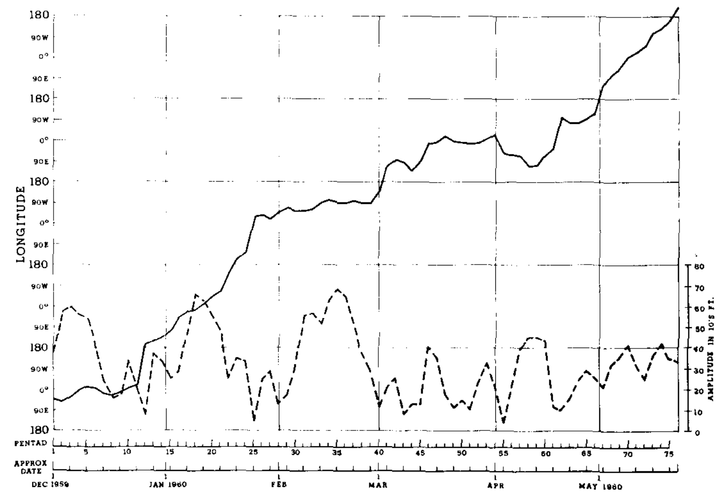

FIGURE 1.—Phase angle and amplitude of wave-number 1 at 70° N. as a function of time. Time scale (abscissa) is in consecutively numbered "pentads" rather than days. A pentad number is assigned in sequence to each 5-day mean 500-mb. chart used in computing the amplitude and phase angles. The time interval between consecutive midtimes of the overlapping 5-day means is either 2 or 3 days. Solid curve: phase angle in degrees longitude (scale at left). Dashed curve: amplitude in 10's of feet (scale at right).

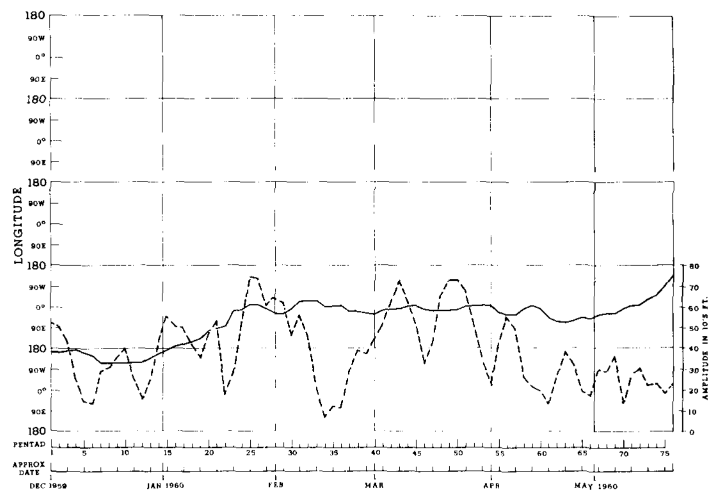

FIGURE 2. Same as figure 1, except wave 2 at 70° N.

Figure 1 shows the phase angle and amplitude of wave 1 at 70° N. plotted as a function of time. This fundamental harmonic moved generally westward around the North Pole during the 6-month period beginning December 1, 1959 and ending May 30, 1960. During this period wave 1 made five complete revolutions around the Pole, averaging one every 5 weeks. One revolution in January, however, took only 3 weeks, and another in the spring, 8 weeks. Twice wave 1 remained stationary for at least 2 weeks, and on two occasions it progressed eastward for at least 1 week. In most cases the periods

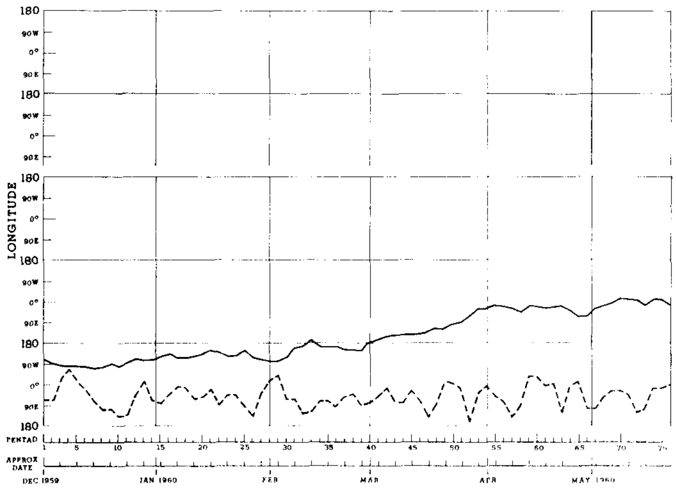

Figure 3.—Same as figure 1, except wave 3 at 70° N.

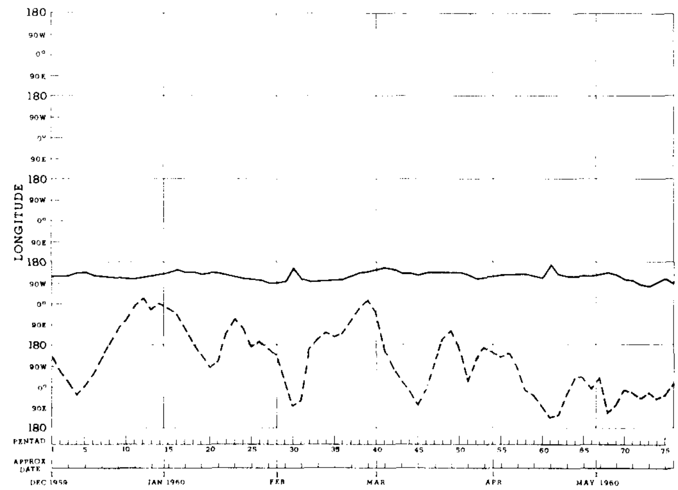

Figure 5.—Same as figure 1, except wave 2 at 50° N.

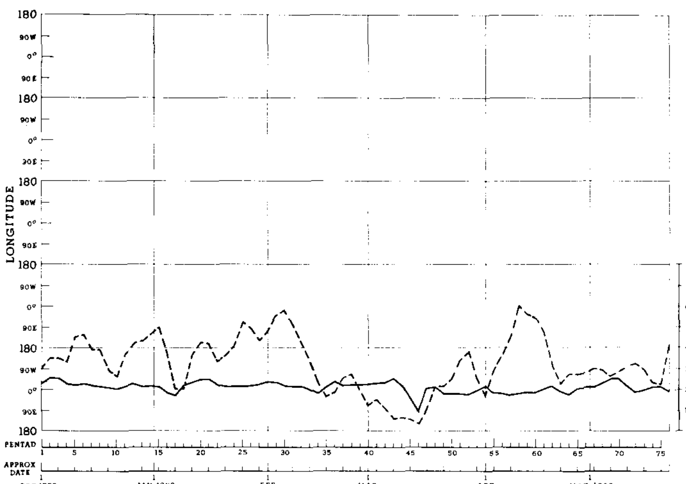

Figure 4.—Same as figure 1, except wave 1 at 50° N.

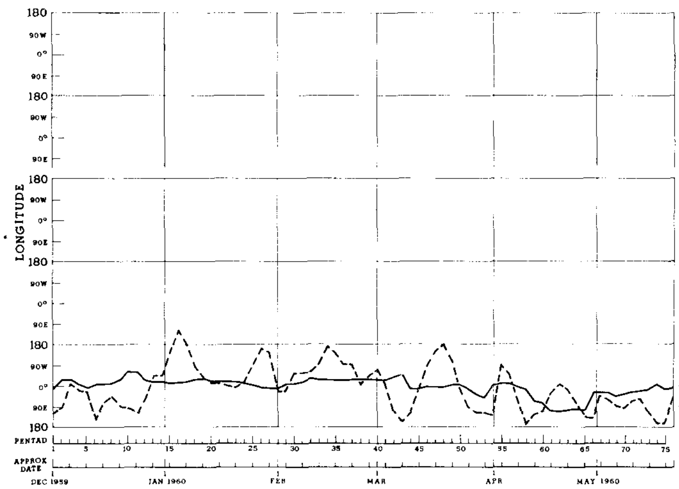

Figure 6.—Same as figure 1, except wave 3 at 50° N.

of rapid movement were associated with relatively low amplitudes, and slow movements were associated with relatively large amplitudes. In general, the amplitude was larger during the three winter months than during the three spring months. During the 6-month period, wave 1 at 70° N. explained, on the average, 39 percent of the total variance.

The steady westward movement which characterized wave 1 was clearly lacking in wave 2 at 70° N. (fig. 2). Two periods of slow steady retrogression (90° longitude per month) were observed—one during the first 3 weeks of January and the other from the end of April through the first 3 weeks of May. For 9 weeks beginning on January 21, wave 2 could be found within 15° east or west of Greenwich, and during the entire 6-month period it made only one revolution around the Pole. As with wave 1, the slower the movement, the larger the amplitude. During the few cases of progression the amplitude seemed to reach a minimum. At 70° N., wave 2 explained, on the average, 46 percent of the total variance, and waves 1 and 2 combined accounted for 85 percent of the total variance.

Wave 3 at 70° N. (fig. 3) showed less movement than either wave 1 or wave 2. During December and January it was relatively stationary near 90° or 100° W. During February and especially March, it retrogressed to 20° E. where it remained practically motionless for the next 2 months. During the three spring months, the amplitude curve tended to reach a small maximum every 9 to 14 days. On the average, wave 3 accounted for 10 percent of the total variance at 70° N. latitude. When combined, waves 1, 2, and 3 accounted for 95 percent of the total variance at 70° N. We should not expect this dominance of the lower wave numbers to persist as we move southward, because the distance around the globe increases at lower latitudes.

The persistent and rapid retrogression of the large-scale wave components at high latitudes during May has been

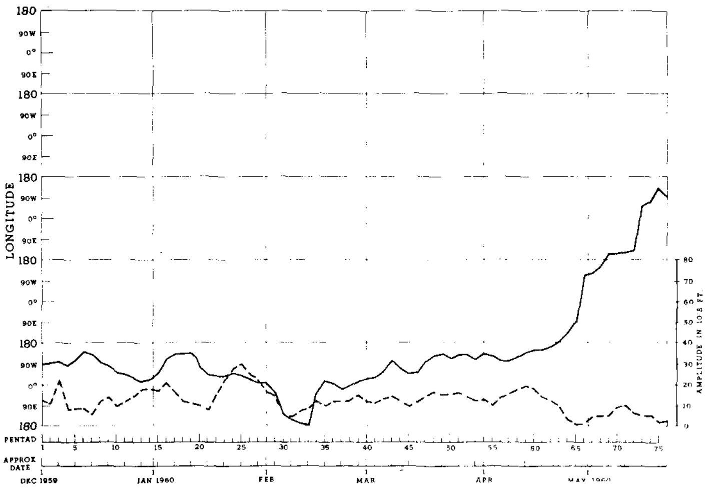

FIGURE 7.—Same as figure 1, except wave 1 at 30° N.

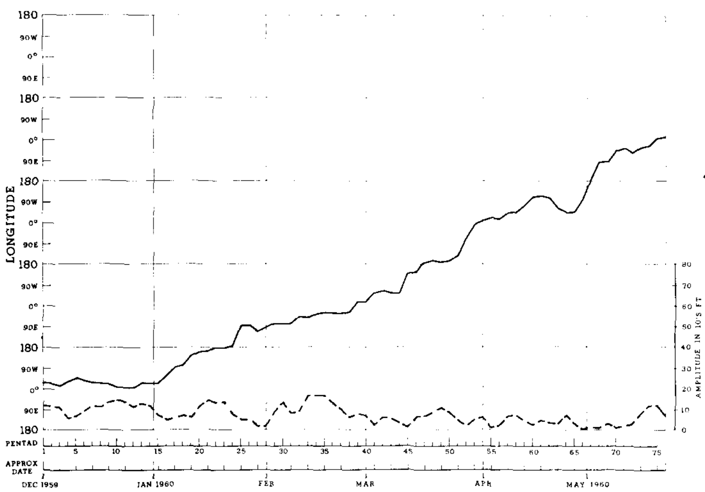

FIGURE 8.—Same as figure 1, except wave 2 at 30° N.

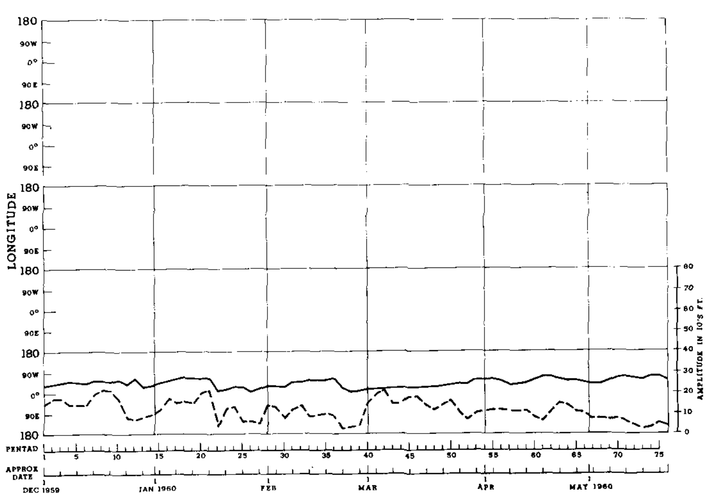

FIGURE 9.—Same as figure 1, except wave 3 at 30° N.

discussed by Andrews [2]. Evidently a period of equally pronounced retrogression of these wave components was present in January, as shown by figures 1 and 2.

Farther south at 50° N. there were no periods of persistent retrogression in any of the waves, and on the whole, the amplitudes, whether high or low, tended to persist longer. At 50° N. wave 1 (fig. 4) held a rather stationary position, passing back and forth across 0° Greenwich 17 times during the 6-month period, and only once did it move more than 20° east or west of it. On March 8, wave 1 started moving eastward and on the 5-day mean map near mid-month it had reached its most easterly position 100° E. More detailed studies may show a correlation between this marked eastward motion and the severe snow storms that lashed the east coast of the United States during the same period. Wave 1 at 50° N. accounted for, on the average, 33 percent of the total variance.

Compared to wave 1 at the same latitude, wave 2 (fig. 5) showed longer periods of slow persistent movement of the order of 25° longitude per month. This persistent movement generally lasted 3 to 4 weeks after which time the wave would reverse its direction for another 3 or 4 weeks. Twice during the period wave 2 seemed to jump rapidly westward, once in early February and again in mid-April. However, in both cases it returned just as quickly to near its original position. These two apparent "jumps" occurred at a time when the wave's amplitude had reached its two lowest values, and it is questionable whether or not they can be regarded as real. On the average, wave 2 accounted for 29 percent of the total variance at that latitude.

Wave 3 (fig. 6) showed more irregularities and rapid movements than either wave 1 or 2 at 50° N., although it was generally confined between 45° W. and 100° E. During the winter months (December, January, and February) it remained almost stationary near 10° W. From early March through the end of the period it was characterized by irregular eastward and westward movements, progressing as far east as 100° E. during April. Consistent with the other waves, little or no movement took place during the periods of relatively high amplitude, and the most rapid movements were associated with low amplitude. At 50° N., wave 3 accounted for, on the average, 12 percent of the total variance, and waves 1, 2, and 3 together accounted for 74 percent.

At 30° N., wave 1 (fig. 7) was characterized by alternating periods of slow and rapid movements. During the first 2½ months, wave 1 moved generally eastward from 90° W. to near 180°. During the 2 months that followed, wave 1 moved slowly westward back to near 100° W., after which the amplitude was so low that the wave's apparent movement no longer retained continuity or meaning. During this last month it accounted for only 11 percent of the total variance, whereas during the entire 6-month period wave 1, on the average, accounted for 24 percent of the total variance at 30° N.

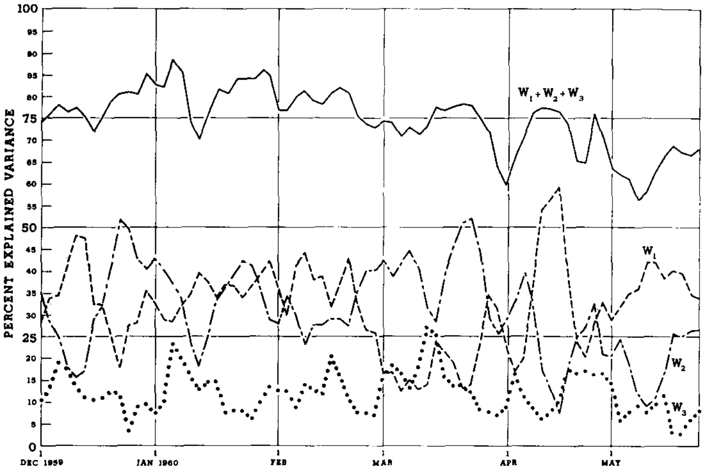

Figure 10.—Percentage of the total variance (ordinate) explained by wave 1 (dashed line), wave 2 (dash-dotted line), wave 3 (dotted line), and waves 1, 2, and 3 combined (solid line) at all seven latitudes (20° N., 30° N., . . . , 80° N.) as a function of time.

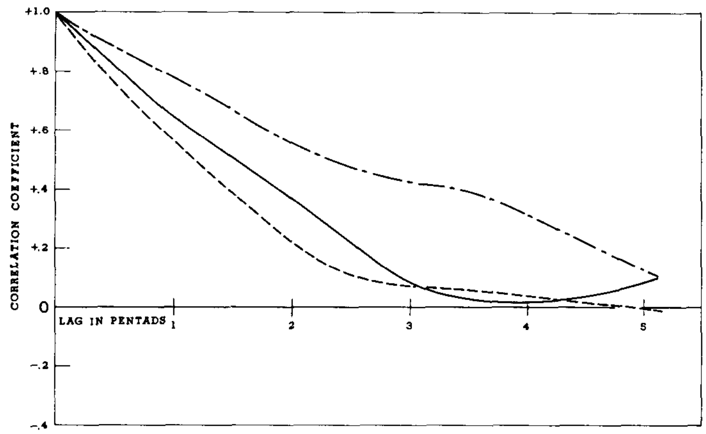

Figure 11.—Autocorrelation coefficient (ordinate) between phase angles of waves 1, 2, and 3 at 30° N., at time lags from 0 to 5 pentad numbers. A pentad number is assigned in sequence to each 5-day mean 500-mb. chart; the time interval between the midtimes of consecutive overlapping pentads is equal to 2 or 3 days. Solid line, wave 1; dashed line, wave 2; dash-dot line, wave 3.

The outstanding feature of wave 2 at 30° N. (fig. 8) was its continued westward movement around the globe. During the 6-month period, wave 2 completely circled the earth three times despite the fact that during the first month (December) it remained nearly stationary. Only twice did it move eastward for longer than a week. Since for the same angular velocity, the linear velocity at 30° N. is two and one-half times as great as it is at 70° N., wave 2 at 30° N. moved at about one and one-half times the speed of wave 1 at 70° N. On the average, wave 2 accounted for 11 percent of the total variance at 30° N.

Wave 3 (fig. 9) at 30° N. showed none of the irregularities of wave 1 nor the continued revolution around the earth which characterized wave 2. During the 6-month period it seemed to follow a pattern of very slow retrogression (35° longitude per month) followed by a rapid eastward motion (30° longitude per week) after which this apparent cycle was repeated again. During the entire period, wave 3 was located between 0° Greenwich and 80° W., and on the average it accounted for 22 percent of the variance. Waves 1, 2, and 3 together accounted for 57 percent of the total variance at 30° N.

Figure 10 shows the percentage of the total variance which was explained by wave 1 (dashed line); wave 2 (dash-dotted); wave 3 (dotted line); and waves 1, 2, and 3 combined (solid line) at all seven latitudes (20° N., 30° N., . . . , 80° N.) as a function of time. One can easily tell by looking at the solid curve that the three waves together accounted for a greater percentage of the total variance during the first three months than during the last three. Specifically, they accounted for 79.4 percent during the three winter months, and, on the average, only 70.2 percent during the three spring months.

It should also be mentioned that during periods of low

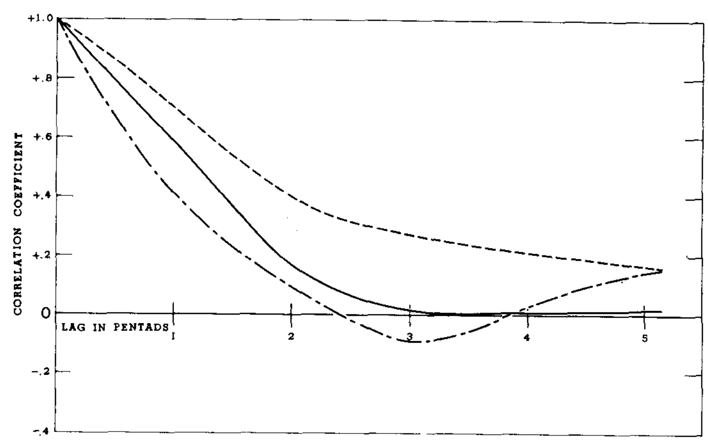

Figure 12.—Same as figure 11, except at 50° N.

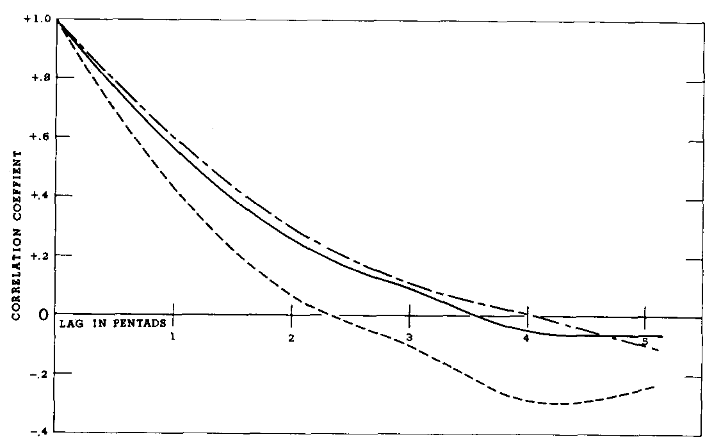

Figure 13.—Same as figure 11, except at 70° N.

zonal index, waves 1, 2, and 3 explained a relatively small percent of the total variance (end of March and first part of May) and during periods of high zonal index they explained a large percent of the total variance (mid-April).

Autocorrelations were computed for the phase angles of waves 1, 2, and 3 at 30° N., 50° N., and 70° N., and the correlograms are shown in figures 11, 12, and 13. It was hoped that a power spectrum analysis could be carried out on those waves which showed some possibility of periodicity (i.e., those whose correlograms do not fall away exponentially). However, time restrictions made this impossible.

4. CONCLUSIONS AND SUGGESTIONS FOR FUTURE STUDY

Eliasen's [3] study, using daily maps over a shorter time period, showed the semipermanence of the longest waves ($n=1$, 2, 3, and 4) at 50° N. An inspection of figures 1–9 reveals this semipermanence at mid-latitudes (50° N.) and also shows a tendency for retrogression of these long waves at high (70° N.) and low (30° N.) latitudes. The stationary character at middle latitudes may be, as Eliasen maintains, a consequence of the large mountain ranges, or it may be that baroclinic action counteracts the retrogression tendencies of the long waves at middle latitudes.

Inspection of the graphs reveals that there is little similarity between the movement of the different long waves at the same latitude. Eliasen [3], however, showed that during two fall months, there was great similarity between the motion of two short waves ($n=6$ and 7) at 50° N. The graphs also show that there is little correspondence of phase and amplitude of the long waves between adjacent latitudes. This indicates frequent sheering of full-latitude troughs and ridges rather than a concurrent movement at all latitudes.

The movements of the waves and the fluctuations of their amplitudes would be of greater significance if the corresponding climatological values were removed from these two parameters. A laborious method of doing this would be to perform an harmonic analysis on about 10 years of data and find the mean position of the waves for each particular 5-day mean chart. Similar results may be obtained by performing an harmonic analysis on the normal monthly and mid-monthly maps, interpolating for values near the beginning and end of the month. Finally, a power analysis of the correlograms after more lags have been computed might reveal further statistical characteristics of the movements of these long waves.

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