SEMIDIURNAL TIDAL MOTIONS IN THE FRICTION LAYER

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ABSTRACT

The observed semidiurnal wind in the troposphere at five mid-latitude stations shows a phase lead over the wind deduced from frictionless theory, and a smaller amplitude, especially near the earth's surface, than the theoretical value. By making use of the observed fact that the tidal pressure-dependent (frictionless) wind changes much less rapidly with height than does the observed wind in the friction layer, it is possible to extend the Ekman theory to explain approximately the observed solar semidiurnal wind distribution. It is concluded that the existence of surface friction causes the time of maximum of the semidiurnal pressure wave to be advanced relative to the time that frictionless theory should predict; or, in agreement with Gold's conclusion that friction produces a phase lag of the tide with increase in elevation. The theoretical argument would appear to be valid for the lunar atmospheric tide also, but direct evidence that similar conclusions apply to the lunar tide is lacking.

1. INTRODUCTION

Perhaps because observations of the semidiurnal wind have been comparatively rare, both at the earth's surface and in the lower troposphere [5, 13, 6, 7, 8], the question of a frictional effect on the twice-daily air tide has been little discussed in the literature. Gold [5] computed the amplitude and phase of the diurnal and semidiurnal winds on the assumption that the frictional force is proportional and opposite to the motion, but he had only surface data and few points at which to verify his conclusions. Recently, however, Finger, Townshend, and Teweles [4] have obtained harmonic analyses of wind, pressure, and temperature from rawinsonde data at three mid-latitude stations in North America. The analyses for these stations (Fort Worth, Tex., St. George, Bermuda, and Valparaiso, Fla.) confirm in a general way a suggestion of a systematic effect on the semidiurnal motions, near the earth's surface, noted in earlier studies for Washington, D.C. [6] and the Azores [7]. All five stations are located between latitudes 30° and 40° N.

The purpose of this article is to present evidence for attributing the apparent systematic effect to surface friction and to assess qualitatively the probable influence of friction on the semidiurnal tide. The origin of the solar atmospheric tide has not yet been satisfactorily explained, and it therefore seems especially appropriate to investigate the possible influence of a force like friction which has not received wide consideration. Gold [5] concluded that the phase retardation of the semidiurnal pressure wave with increasing height above the earth's surface is the result of friction, and one objective of the current work was to reexamine this inference in the light of new data and a slightly different theoretical approach.

2. THE OBSERVATIONS

The data to be explained are presented in table 1, which is based on earlier published material [6, 7] and on the more recent unpublished results [4]. The amplitude and phase of the semidiurnal wind variation were obtained, in the manner described in [6] and [7], from pairs of months, one before and one after the date of the change, in 1957, of the scheduled times of upper air observations. Thus, in table 1, the values for Washington are based on 6 months' records; Fort Worth and Valpariaso, 20 months'; and the Azores and Bermuda, 24 months'. In spite of the undoubted existence of considerable "noise" in the determinations of the semidiurnal wind from a relatively short record (as suggested in table 2), the different stations represented in table 1 show some fairly consistent features which may be presumed to be real.

Table 1 shows the departure of the observed tidal wind from a theoretical value. Stolov [12] has given an expression for the latitudinal distribution of the semidiurnal wind, based on a solution of the linearized equations of motion for frictionless flow and on the observed pressure oscillation at the earth's surface. The theoretical phase of the tidal wind in table 1 is identical with that assumed by Stolov, but the amplitudes of both eastward and northward components of the wind are roughly 4 cm. sec.$^{-1}$ less than Stolov's values. This difference results from two restrictions in addition to those commonly used: namely, that the velocity vanish at the poles and that the mass divergence throughout the depth of the atmosphere satisfy the pressure variation adopted by Stolov and other authors [11]. Thus the theoretical amplitudes used here may be said to represent a mean value, with respect to mass, through the depth of the atmosphere.

TABLE 1.—*Departure from theoretical value of the semidiurnal wind at (1) Washington, D.C., June through August; (2) Fort Worth, Tex., June through March; (3) Valparaiso, Fla., June through March; (4) Terceira, Azores, entire year; and (5) St. George, Bermuda, entire year. Theoretical amplitudes at latitude 35°: eastward component, 28.9 cm. sec.$^{-1}$; northward component, 30.8 cm. sec.$^{-1}$. Theoretical phase (time of maximum): eastward component, 3.86 (15.86) hours: northward component, 0.86 (12.86) hours*

Mean Pressure, mb.	Amplitude Departure, cm. sec.$^{-1}$												Phase Departure, hours											
	Eastward Component						Northward Component						Eastward Component						Northward Component					
	(1)	(2)	(3)	(4)	(5)	Average	(1)	(2)	(3)	(4)	(5)	Average	(1)	(2)	(3)	(4)	(5)	Average	(1)	(2)	(3)	(4)	(5)	Average
SFC	7	−9	5	−21	−12	−9.5	−28	−19	10	−10	−17	−15.1	−3.3	−3.2	−1.4	−1.5	−0.9	−2.22	−6.0	0.1	0.3	−1.4	−0.9	−0.30
1000	5	-----	−2	−17	−10	−9.4	−16	-----	10	−14	−8	−15.4	−3.2	-----	−1.6	−0.6	−0.7	−1.83	0.6	-----	0.3	−1.4	−1.3	−0.73
950	2	−1	−4	−15	2	−5.9	20	−17	9	−9	−1	−5.1	−2.7	−0.8	−0.8	−1.3	−0.5	−1.16	−2.8	−0.6	0.0	−0.9	−0.9	−1.16
900	−14	−19	−4	−10	−6	−11.0	−9	−7	4	−8	0	−6.7	−1.4	−0.5	0.0	−1.4	−0.4	−0.43	−2.5	−0.4	0.2	−0.7	−1.0	−0.73
850	2	−20	−10	−15	−2	−11.0	−5	−8	6	−8	3	−2.4	0.6	−2.5	−0.1	0.4	−0.7	−0.13	−0.2	−0.7	−0.5	−0.2	−0.6	−0.40
800	4	−11	−7	−7	−11	−8.6	−3	3	11	0	−3	−1.2	0.8	−1.4	0.1	−1.2	−0.2	−0.16	0.7	−0.8	−1.6	0.3	−1.4	−0.53
750	14	0	0	−7	−11	−2.3	5	6	5	−2	5	−7.2	0.9	−0.9	0.0	0.0	0.0	0.14	−0.2	−0.9	−1.1	0.0	−0.9	−0.63
700	14	4	−8	−11	−12	−3.6	13	8	15	−9	15	8.1	0.4	−0.9	0.1	−0.8	−0.4	−0.23	−1.0	−0.3	−1.0	−0.8	−1.1	−0.86
650	11	8	11	−9	−12	0.9	16	12	8	−8	22	9.3	0.6	−0.1	−0.5	−0.4	−1.0	−0.13	−0.8	−0.3	0.5	−1.4	−2.2	−0.76
600	9	7	9	−9	−4	1.0	12	15	12	−15	12	6.1	0.7	−0.4	−0.8	0.2	−0.6	−0.16	−0.7	−0.3	−1.4	0.3	−1.0	−0.70
550	13	−5	11	2	6	−0.2	−8	15	−3	−15	13	−0.6	1.0	−0.4	−1.6	−1.6	−0.8	−0.60	−0.5	0.2	−0.8	0.1	−1.1	−0.43
500	9	6	3	−4	−7	−2.5	−8	13	−3	−16	14	−1.7	0.9	−0.1	−1.9	0.1	−1.0	−0.30	−0.4	0.4	0.1	−1.8	−0.7	−0.26
450	6	−2	3	−3	1	−5.6	−7	17	−19	−11	20	−0.9	0.4	0.2	−2.9	−0.5	−0.2	−0.46	−0.6	0.1	−0.7	0.4	−0.9	−0.30
400	−1	−3	7	−1	−19	−6.4	1	5	−2	−21	24	−2.0	−0.2	0.3	−2.2	−0.4	−0.1	−0.63	−1.0	0.7	0.0	1.7	−1.2	−0.33
350	−17	−8	11	13	0	−3.3	15	1	−11	−15	13	−1.8	−1.1	1.3	−1.6	−0.4	−0.8	−0.63	−0.8	0.7	−1.0	1.7	−0.4	−0.20
300	−8	10	−12	22	−3	−2.4	1	15	−19	−17	6	−4.2	2.4	0.4	−0.4	−1.4	−1.2	0.00	−1.1	0.2	−0.9	0.2	−1.3	−0.53
Average	3.7	−2.9	0.5	−4.7	−5.9	−4.69	1.8	3.9	2.2	−11.9	9.0	−1.71	0.01	−0.60	−0.95	−0.55	−0.57	−0.447	−0.75	−0.13	−0.53	−0.17	−1.07	−0.570

Negative values of the phase departure in table 1 indicate that the observed wind reaches a maximum before the theoretical time: i.e., that the observed leads the theoretical or "frictionless" wind. (Throughout this article, "phase" is used in the sense of "time of maximum," in contradistinction to "phase angle.") The row averages in table 1 represent values obtained by taking unweighted, vector means of the harmonic components for the five stations; the column averages are scalar means of the values between 1000 and 300 mb., except in the case of Fort Worth where the surface value was substituted for the 1000-mb. value.

Inspection of table 1 reveals that with few exceptions the observed wind leads the theoretical wind, and that on the average the phase lead is about one-half hour. A phase lead of the observed over the frictionless wind means that the tidal wind tends to blow more nearly in the direction of low pressure than does a wind with no phase shift, while a phase lag would imply a flow more nearly in the direction of high pressure. On the average, the wind is lighter than the theoretical value; negative departures of the amplitude are particularly noticeable below the 750-mb. level. Thus, both phase and amplitude departures in table 1 indicate a wind distribution which is consistent with the presence of a frictional force, especially in the lower troposphere.

Other relatively systematic features of the wind distribution appear in table 1. On the average, there is an increase in the amplitude of the semidiurnal wind between the surface and the 650-mb. level. Similarly there is a tendency for the phase lead of the observed wind to show a maximum near the earth's surface and a minimum in the 850- to 750-mb. layer. Valparaiso alone appears to have a regime markedly different from that at other stations, the wind decreasing instead of increasing with height above the surface.

TABLE 2.—*Probable error of the semidiurnal wind as determined at (2) Fort Worth, Tex., (3) Valparaiso, Fla., (4) Terceira, Azores, and (5) St. George, Bermuda; and the error of the mean of all individual monthly determinations at these stations and Washington, D.C.*

Mean Pressure, mb.	Probable Error, cm. sec.$^{-1}$												Probable Phase Error, hours											
	Eastward Component						Northward Component						Eastward Component						Northward Component					
	(1)	(2)	(3)	(4)	(5)	Average	(1)	(2)	(3)	(4)	(5)	Average	(1)	(2)	(3)	(4)	(5)	Average	(1)	(2)	(3)	(4)	(5)	Average
SFC	-----	8	5	7	6	3.4	-----	6	7	6	7	3.4	-----	0.9	0.3	2.0	0.7	0.33	-----	1.0	0.3	0.6	1.0	0.33
1000	-----	-----	6	6	5	3.5	-----	-----	7	6	5	3.7	-----	-----	0.4	1.0	0.5	0.35	-----	-----	0.3	0.7	0.4	0.32
950	-----	9	7	8	6	3.8	-----	6	9	6	8	4.0	-----	0.6	0.6	1.2	0.4	0.32	-----	0.8	0.4	0.5	0.5	0.30
900	-----	9	8	6	7	3.7	-----	9	10	8	6	4.1	-----	2.1	0.6	0.6	0.6	0.37	-----	0.7	0.5	0.7	0.4	0.27
850	-----	6	8	7	6	3.7	-----	11	10	8	6	4.4	-----	1.4	0.6	1.0	0.4	0.45	-----	1.0	0.5	0.7	0.3	0.28
800	-----	7	7	8	8	3.9	-----	11	10	8	8	4.6	-----	0.8	0.5	0.7	0.9	0.37	-----	0.6	0.5	0.5	0.6	0.27
750	-----	7	8	10	7	4.2	-----	10	12	14	5	4.8	-----	0.5	0.4	0.9	0.8	0.30	-----	0.5	0.5	1.0	0.3	0.27
700	-----	9	7	8	8	4.0	-----	9	9	9	7	3.8	-----	0.5	0.6	0.9	0.9	0.33	-----	0.4	0.4	0.8	0.3	0.20
650	-----	6	10	5	7	3.8	-----	8	9	8	7	4.0	-----	0.3	0.5	0.5	0.9	0.27	-----	0.4	0.5	0.7	0.3	0.20
600	-----	7	10	6	8	4.1	-----	8	11	7	10	4.5	-----	0.4	0.5	0.6	0.6	0.27	-----	0.3	0.6	0.9	0.5	0.27
550	-----	9	11	9	8	4.8	-----	8	12	10	9	4.9	-----	0.7	0.5	0.6	0.5	0.30	-----	0.3	0.8	1.3	0.4	0.30
500	-----	9	9	10	7	4.4	-----	9	11	9	8	4.7	-----	0.5	0.6	0.8	0.6	0.33	-----	0.4	0.8	1.2	0.3	0.30
450	-----	9	12	10	9	5.1	-----	10	11	10	10	5.4	-----	0.6	0.6	0.8	0.6	0.42	-----	0.4	1.6	1.0	0.4	0.33
400	-----	11	14	10	8	5.4	-----	14	15	12	13	6.7	-----	0.8	0.7	0.7	1.6	0.48	-----	0.8	1.3	1.9	0.4	0.50
350	-----	10	12	9	11	5.3	-----	14	11	9	17	6.2	-----	0.9	0.7	0.4	0.7	0.40	-----	0.9	1.7	0.6	0.8	0.47
300	-----	11	13	14	13	6.5	-----	10	11	13	19	6.8	-----	0.5	6.0	0.5	1.0	0.47	-----	0.4	3.0	2.3	1.2	0.60

As a measure of the reliability of the semidiurnal wind determinations, the probable errors of the eastward and northward wind components are presented in table 2. The probable errors were computed according to the method outlined by Chapman [2]. Because of the short record for Washington, D.C. errors were not computed for that station; however, the Washington data are included in the results in the column marked "Average." In the latter column, the probable errors refer to the mean of all the individual monthly determinations for the five stations at each isobaric surface. Also included in table 2 are the phase deviations represented by the probable errors, which are vector quantities. Comparison of the data in tables 1 and 2 shows that the probable errors of the 5-station average are, in general, smaller in the lower troposphere and larger in the upper troposphere than the observed departures from the theoretical wind. Thus there is some indication that the apparently systematic deviation from frictionless flow in the lower troposphere is a real phenomenon; less significance can be attached to the deviations at the higher elevations.

Figure 1 illustrates further the tendency for a systematic departure of the observed from the frictionless wind. Since according to theory [9] the eastward and northward components of the tidal wind are in quadrature, the two may be averaged if the phase of the northward component is first increased by three hours. Figure 1 shows harmonic dials on which such averages of the vectors representing both the eastward and northward components of the wind are plotted for each of the five stations. The resulting hodographs, with the exception of that for Valparaiso, bring out the general tendency for an increase in amplitude and phase of the semidiurnal wind above the earth's surface. Seasonal differences (Washington compared with Fort Worth, for example) and differences probably associated with terrain (continental compared with ocean stations) are also suggested by figure 1. The tendency for the observed wind to lead the theoretical wind even at high elevations is brought out clearly. As shown by table 1, this trend is more marked in the case of the northward than in the case of the eastward component of the wind. Ideally, the semidiurnal oscillation at middle and low latitudes resembles a simple progressive wave which is symmetrical with respect to the equator and has constant phase. Differences between the actual oscillation and its ideal representation may account for the failure of the observed wind phase to coincide with the theoretical phase at high elevations where the existence of frictional effects is somewhat doubtful.

If the meridional variation of the amplitude of the semidiurnal pressure wave is assumed constant with height [9], and friction is absent, the tidal wind at any isobaric surface may be inferred from the observed height variation, to which, at any given latitude, the wind is proportional. In figure 2a, the ratio of the amplitude of the height variation, at the three stations for which this information is currently available, to the height variation at the earth's surface, is plotted as a function of the mean height. The ratio of the amplitudes of the vector averages of the observed wind for the same three stations is also shown in figure 2a. It is clear that the variation with height of the amplitude of the observed tidal wind is much greater, in the lower troposphere, than the approximately zero height change of the frictionless or "pressure-dependent" wind. Similarly, it can be seen from figure 2b that the phase of the observed wind changes rapidly in the first 1.5 km., while the phase of the pressure-dependent wind increases much more slowly. Above 3 km., the parallelism between the two curves of figure 2b suggests that the wind is controlled by pressure forces alone and indicates that the tidal oscillation lags with elevation. One might also infer from figure 2, with some idealization of the data, that the amplitude of the tidal wind is nearly constant above 3 or 4 km. (The gradual increase in the height oscillation above 3 km. reflects the Washington data and conceivably could be the result of slight radiation errors of the rawinsonde thermistor.) If the phases of the observed and frictionless wind are assumed to be the same above 3 km., the tidal wind at anemometer level leads the frictionless wind by about an hour and a half.

3. THEORY

The foregoing examination of the observed semidiurnal wind suggests that its distribution in the lower troposphere can be approximately accounted for by an extension of the Ekman theory [3]—long accepted as an explanation for the deviation of the mean wind from geostrophic flow in the friction layer—to the tidal wind.

An assumption customarily made in explaining the wind hodograph in the friction layer, as well as the height lag of the diurnal temperature wave—constancy of the exchange coefficient, K, for momentum or heat transfer, with height and time—is made here also. Although the limitations of this assumption have been well recognized, the form of the solutions illustrates in a general way the effect of turbulent exchange on the mean wind and on the height distribution of the daily temperature wave. Figure 2 suggests another plausible assumption, i.e., that the variation with height of the "frictionless" or "pressure-dependent" wind may be neglected in comparison with the height variation of the actual tidal wind. This assumption, in effect, permits a decoupling of pressure and frictional forces and makes possible a simple analytic solution of the equations of motion.

With the addition of eddy friction terms, the equations of motion appropriate to the semidiurnal tidal oscillation may be written

$$\frac{\partial u}{\partial t}-2\omega \sin \phi \cdot v+\frac{1}{a \cos \phi} \frac{\partial P}{\partial \theta}-K \frac{\partial^2 u}{\partial z^2}=0$$

$$\frac{\partial v}{\partial t}+2\omega \sin \phi \cdot u+\frac{1}{a} \frac{\partial P}{\partial \phi}-K \frac{\partial^2 v}{\partial z^2}=0 \qquad (1)$$

where u and v are the eastward and northward components

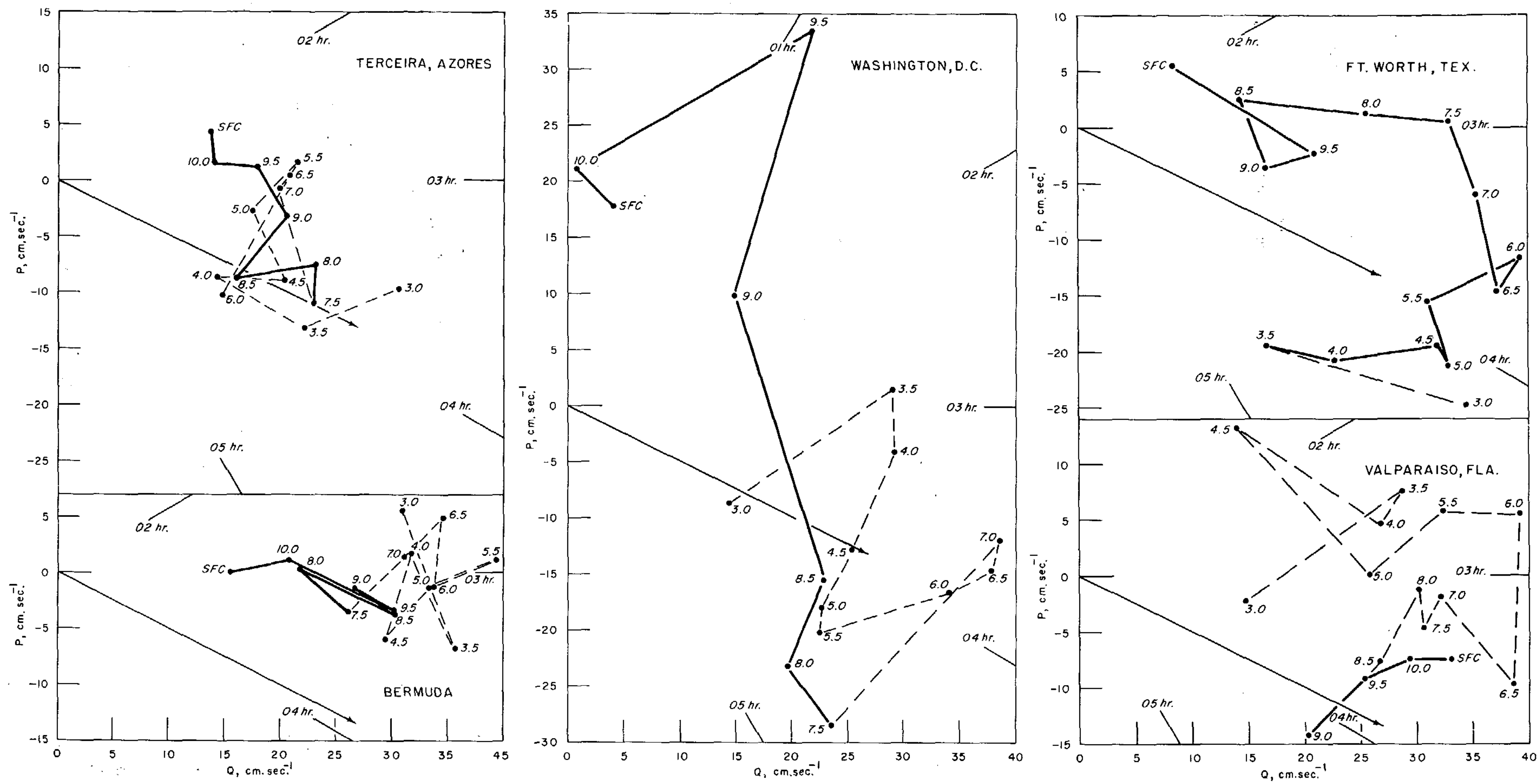

Figure 1.—Harmonic dials showing the average semidiurnal wind component for five stations at anemometer level (Sfc) and at isobaric surfaces between 10.0 and 3.0 centibars. The ordinate $P=p_u-q_v$ and the abscissa $Q=q_u+p_u$ where $u=p_u \cos 2\omega t+q_u \sin 2\omega t$ and $v=p_v \cos 2\omega t+q_v \sin 2\omega t$. The time of maximum is indicated in local time and the vectors drawn from the origin show the theoretical amplitude and phase of the average tidal wind-component $(u-iv)/2$.

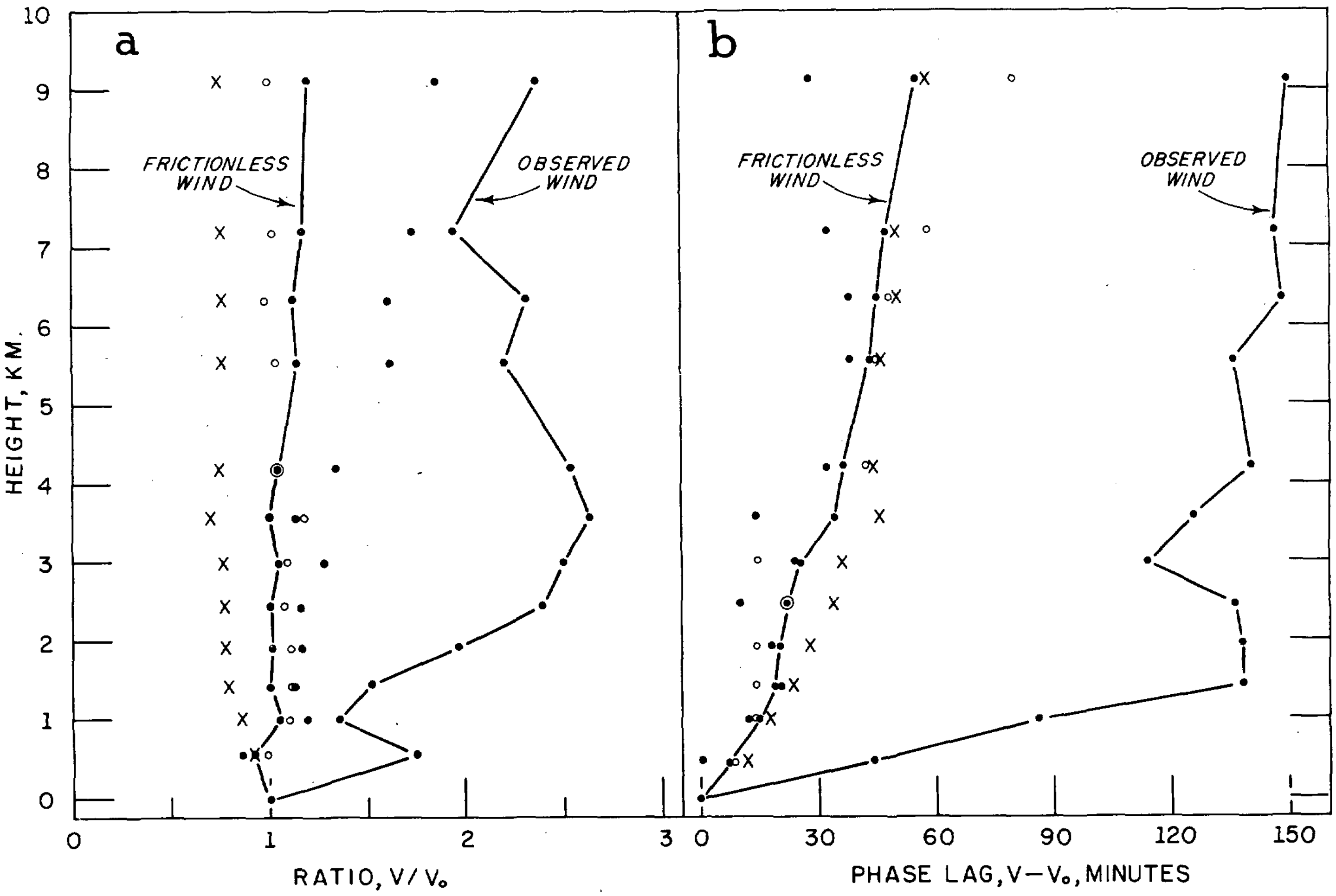

Figure 2.—Ratio of the amplitude of the pressure-dependent (frictionless) wind to its value at the earth's surface, and phase lag of the wind with elevation, at three stations: Washington, D.C. (filled-in circles); Terceira, Azores (open circles); and Fort Worth, Tex. (crosses). The average ratio and average phase lag are indicated by solid curves. The ratio of the average observed tidal wind at the same three stations, representing both eastward and northward components of the wind, and its phase lag with elevation are shown by the other pair of curves.

of the tidal wind, $P=p/\rho_0$ is the perturbation pressure p divided by the mean density ρ_0, ω is the angular velocity of the earth's rotation, a is the earth's radius, and $K=\mu_0/\rho_0$, μ_0 being the coefficient of eddy viscosity. The mean density is assumed independent of the latitude ϕ, of longitude θ (taken positive to the east), and of the time t; z is the vertical coordinate, measured positive upward above a height approximating anemometer level. The tidal potential, which is small in comparison with P, has been neglected since the theory presented here is primarily diagnostic; the solar semidiurnal tide is accepted as an empirical fact and the dilemma as to its thermal or gravitational origin is not relevant to the discussion.

In the theory of the mean wind distribution in the friction layer, it is hypothesized that some level near anemometer height separates the surface layer and the friction layer (see for example [10]). This level is not a physical boundary but simply represents an approximate height above and below which different laws hold for the wind distribution. It is assumed that the shear of the wind is here parallel to the wind itself; hence

$$u_a=\kappa\left(\frac{\partial u}{\partial z}\right)_a, \quad v_a=\kappa\left(\frac{\partial v}{\partial z}\right)_a, \tag{2}$$

the wind being continuous at the boundary $z=a$, where a indicates the height of the anemometer. The constant κ may be thought of as a coefficient of surface friction, and can be shown to be

$$\kappa=(z_a+z_0)\ln\frac{z_a+z_0}{z_0}, \tag{3}$$

where z_0 is the roughness parameter and z_a is the height of the anemometer level.

Equation (2) constitutes a lower boundary condition. It is now assumed as an upper boundary requirement that at some level representing the "top" of the friction layer the eddy friction terms become negligibly small.

These boundary conditions are of course identical with those generally assumed in the case of the mean wind.

Since equations (1) are linear, solutions for u and v may be represented as a sum of solutions

$$u=u_P+u_F$$
$$v=v_P+v_F \tag{4}$$

Here, u_P and v_P are taken to be functions of P alone, while u_F and v_F may be considered perturbations about u_P and v_P, respectively. It follows that u_P and v_P at the upper boundary, where the eddy friction terms are negligible, must satisfy the equation

$$\frac{\partial u_P}{\partial t}-2\omega\sin\phi\cdot v_P+\frac{1}{a\cos\phi}\frac{\partial P}{\partial\theta}=0$$
$$\frac{\partial v_P}{\partial t}+2\omega\sin\phi\cdot u_P+\frac{1}{a}\frac{\partial P}{\partial\theta}=0 \tag{5}$$

Since P has been assumed to be independent of height in the layer under consideration, equation (5) must hold true not only at the upper boundary but throughout the friction layer; its solution is likewise independent of height. It therefore follows that the remaining part of the solution must satisfy the equation

$$K\frac{\partial^2 u_F}{\partial z^2}-\frac{\partial u_F}{\partial t}+2\omega\sin\phi\cdot v_F=0$$
$$K\frac{\partial^2 v_F}{\partial z^2}-\frac{\partial v_F}{\partial t}-2\omega\sin\phi\cdot u_F=0 \tag{6}$$

It is now apparent that the assumption of the constancy of P with altitude allows pressure and frictional forces to be decoupled in so far as these affect the semidiurnal air motions. Partial solutions for u and v can now be readily obtained. Because the variables are periodic, and the oscillations occur twice daily, they may be represented by terms proportional to $e^{2i\omega t}$, where $i=\sqrt{-1}$. (Because of the slight difference in the length of the solar and sidereal days, the ω in $e^{2i\omega t}$ is slightly larger than the ω in the Coriolis force. This difference is important at latitudes near the poles but is ignored in the present discussion, which is not applicable at the poles in any case.) Thus the pressure-dependent part of the solution can be indicated [10, 11] as

$$u_P=\frac{1}{4a\omega^2\cos^2\phi}\left[\frac{1}{\cos\phi}\frac{\partial^2 P}{\partial t\partial}+2\omega\sin\phi\frac{\partial P}{\partial\phi}\right]$$
$$v_P=\frac{1}{4a\omega^2\cos^2\phi}\left[\frac{\partial^2 P}{\partial t\partial\phi}-\frac{2\omega\sin\phi}{\cos\phi}\frac{\partial P}{\partial\theta}\right] \tag{7}$$

$P=p/\rho_0$ is known from observations of the semidiurnal pressure wave to be approximately represented by the function

$$p=-P_E\cos^3\phi\sin(2\omega t+\beta) \tag{8}$$

where β is typically about 334° and is nearly independent of latitude, and P_E is 1.25 mb.

It may be verified that

$$u_F=Ae^{-\gamma z}\sin(2\omega t+\alpha-\gamma z)$$
$$v_F=Be^{-\gamma z}\cos(2\omega t+\alpha-\gamma z) \tag{9}$$

is a solution of equation (6), provided

$$\gamma=\pm\sqrt{\frac{\omega(1\pm\sin\phi)}{K}} \tag{10}$$

Because u_F and v_F are assumed to become negligibly small at the upper boundary, the positive sign must be chosen in front of the radical in equation (10). Since the expression $(1+\sin\phi)$ implies that $A=-B$ (an untenable result, as will be shown later), the positive sign must be discarded in this case, leaving

$$\gamma=+\sqrt{\frac{\omega(1-\sin\phi)}{K}} \tag{11}$$

It follows from the choice of signs that $A=B$ and the frictional contributions to the eastward and northward components of the semidiurnal wind have the same magnitude. The expression for γ also implies that friction is independent of height at the poles; but since equations (1) are not applicable close to the poles, this apparent contradiction of the upper boundary condition is assumed to be irrelevant.

The remaining parameters A and B, and the phase angle α, can be determined from the lower boundary condition (equation (2)).

Equation (4) at anemometer level becomes

$$u_a=u_P+(u_F)_a$$
$$v_a=v_P+(v_F)_a \tag{12}$$

It follows from the condition $z=0$ at z_a and from equation (9) that

$$(u_F)_a=A\sin(2\omega t+\alpha)$$
$$(v_F)_a=B\cos(2\omega t+\alpha) \tag{13}$$

From equations (2) and (9) it may be readily shown that

$$u_a=\sqrt{2}\gamma\kappa A\sin(2\omega t+\alpha-135°)$$
$$v_a=\sqrt{2}\gamma\kappa B\cos(2\omega t+\alpha-135°) \tag{14}$$

Hence the first part of equation (12) becomes

$$\sqrt{2}\gamma\kappa\, A\sin(2\omega t+\alpha-135°)=u_P+A\sin(2\omega t+\alpha) \tag{15}$$

with a similar equation for v. Equation (15) is sufficient to determine α.

In figure 3, β is the phase angle of the vector u_P, which is found from equation (7) and the empirical expression (8)

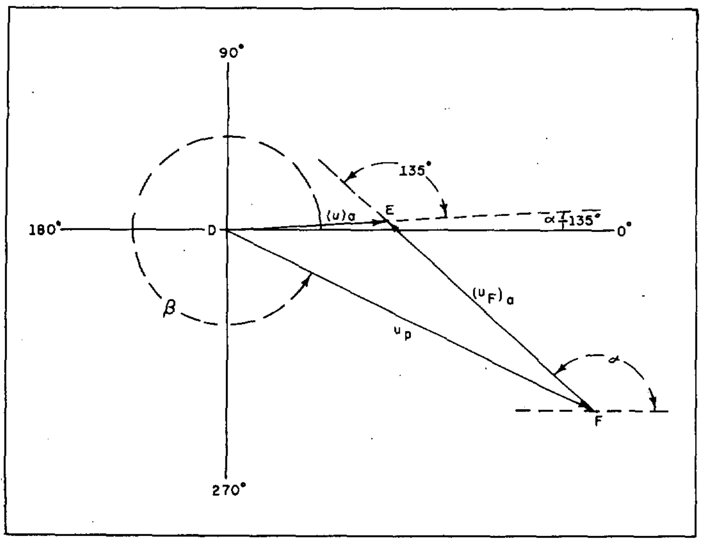

FIGURE 3.—Determination of constants α and A.

to be 334°. The vectors $(u_F)_a$ and u_a are drawn so that within the triangle DEF, angle E=135°, a condition which is necessary to satisfy equation (15). The magnitudes of the vectors u_a and $(u_F)_a$ are $\sqrt{2}\gamma\kappa A$ and A respectively, the amplitudes of the harmonic functions representing the quantities in equation (15). It is evident from figure 3 that

$$\alpha=\beta-135°+\angle D \tag{16}$$

or

$$\alpha=\beta-135°+\cot^{-1}(1+2\kappa\gamma) \tag{17}$$

since from the law of sines

$$\angle D=\cot^{-1}(1+2\kappa\gamma). \tag{18}$$

It also follows from the law of sines that

$$A=\sqrt{2}U_P \sin \angle D \tag{19}$$

where U_P is the amplitude of u_P, or that

$$A=U_P(1+2\kappa\gamma+2\kappa^2\gamma^2)^{-1/2} \tag{20}$$

and by analogy

$$B=V_P(1+2\kappa\gamma+2\kappa^2\gamma^2)^{-1/2}$$

where V_P is the amplitude of v_P. In order to insure the correct phase relationship [9] between the eastward and northward components of the frictionless wind, A and B must both be positive. (This is the consideration that led to the choice of the negative sign in the expression $(1\pm\sin\phi)$ in equation (10).) Equation (20), together with the earlier result, $A=B$, implies that $U_P=V_P$ in magnitude, a condition that is approximately satisfied empirically (equations (7) and (8)) only at latitudes poleward of 25°. At latitudes lower than 25°, V_P approaches zero rapidly, while U_P remains greater than 20 cm. sec.$^{-1}$. It is evident, therefore, that the assumptions and/or the boundary conditions discussed earlier are not appropriate at latitudes lower than 25°. Since there are as yet no data on the semidiurnal wind in the friction layer at low latitudes, speculation as to the probable form of a solution for u_F and v_F in this region cannot now be verified.

It is clear from the foregoing discussion that the constants γ, α, A, and B can be expressed explicitly in terms of the surface friction coefficient, the vertical exchange coefficient for momentum, and the observed magnitude and phase of the semidiurnal pressure wave. To summarize, the theory indicates that the semidiurnal wind in the friction layer, at latitudes poleward of 25°, can be approximated by

$$u=u_P+Ae^{-\gamma z}\sin(2\omega t+\alpha-\gamma z)$$
$$v=v_P+Be^{-\gamma z}\cos(2\omega t+\alpha-\gamma z) \tag{21}$$

where

$$\gamma=f(K,\omega,\phi),\ \alpha=g(\beta,\kappa,\gamma),\ A,B=h[(u_P,v_P),\kappa,\gamma] \tag{22}$$

The angle D, given by equation (18) or alternatively by

$$\angle D=\cot^{-1}\left[1+2\kappa\sqrt{\frac{\omega(1-\sin\phi)}{K}}\right] \tag{23}$$

is of some interest since it is analogous to the angle of cross-isobar flow at anemometer level in the case of the frictionally caused deviation from geostrophic flow. In the case of the tidal wind, this angle may be more conveniently interpreted as the phase lead of the observed over the pressure-dependent wind near the earth's surface. It can be seen from equation (23) that one limiting value for angle D (as K approaches infinity) is 45°. The frictional contribution to the wind is then equal and opposite to the contribution from pressure forces, and the wind at anemometer level is zero. For very small values of K, the angle D approaches zero; equation (19) shows that the contribution of friction is then also zero, and the wind at anemometer level is due entirely to pressure forces. In the case of friction, the time of maximum wind always precedes the time of maximum of the "frictionless" wind, since the phase angle and time of maximum, in the convention used here, are related by the expression

$$t_{max}=\frac{450°-\alpha}{30°\text{ hr.}^{-1}}$$

Thus, according to the theory, the maximum semidiurnal wind at anemometer level can occur as early as one and one-half hours before the time predicted by frictionless theory.

The frictional component of the wind, as may be seen from equation (21), describes an equiangular spiral and approaches zero at some height depending on the magnitude of γ. If h is chosen as the height at which the frictional component of the wind first becomes parallel to that due to pressure forces alone, then

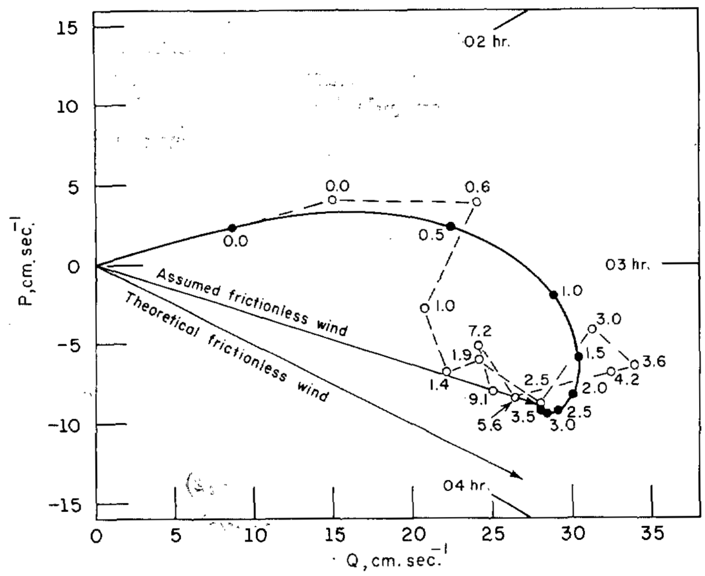

FIGURE 4.—Harmonic dial showing theoretical and observed amplitude and phase of the semidiurnal wind in the troposphere at various elevations (km.). The solid curve connects points computed on the assumption that the height of the "gradient" level is 2.5 km., and that the wind at anemometer level has the observed phase. The dashed line connects points showing the average of the eastward and northward wind components at five stations. Theoretical value of the frictionless wind is indicated for comparison.

$$h=(2\pi+\alpha-\beta)\sqrt{\frac{K}{\omega(1-\sin\phi)}} \tag{24}$$

Thus, h is analogous to the height of the gradient wind level, and approximates the maximum height to which the influence of surface friction is effective.

4. RESULTS

In order to apply the theory outlined above to the data of table 1, the harmonic functions representing both the eastward and northward components of the tidal wind were vectorially averaged, unweighted, in the manner already described, for all five stations. The observations averaged in this way and plotted on the harmonic dial of figure 4 show that the tidal wind is not distributed in the ideal fashion expected, i.e., it does not describe a hodograph spiralling through the friction layer and reaching a constant value coincident with the theoretical frictionless wind vector. Since the height h is thus somewhat indeterminate, it was decided to select as h that height at which the observed hodograph first makes its closest approach to the theoretical wind. With this point fixed as 2.5 km., and β chosen as 343° instead of the theoretical 334°, the angle D is 32°, α is 150°, and K is found to be 2.28×10^5 cm.^{2} sec.$^{-1}$. Then if the amplitude of the frictionless wind is taken as 30 cm. sec.$^{-1}$, the computed tidal wind is that shown in figure 4. The agreement clearly leaves much to be desired, but the discrepancies are perhaps not surprising in view of the imperfect data and the simplifying assumptions used in the derivation.

In particular, the assumption of a K-value constant with height throughout the friction layer is unrealistic and probably accounts for some of the difference between theory and observation. Nevertheless, this simplification makes it possible to predict the general features, if not the details, of the semidiurnal wind in the friction layer, and to assess in a qualitative way the effect of surface friction on the pressure oscillation. Since, according to the theory, surface friction should act to rotate the wind vectors toward earlier maxima (fig. 4), it may be inferred from the continuity equation that frictional divergence causes the semidiurnal pressure wave to reach a maximum earlier than in the absence of friction. With frictional divergence taking place in the lower atmospheric layer (which rests on a solid or fluid boundary), the pressure wave would necessarily show a lag with elevation, as suggested by the observations of figure 2b.

Although the magnitude of the frictional forces and the surface stress values implied by the above results have yet to be computed, preliminary estimates obtained as residuals between observed pressure forces and the observed accelerations suggest that the frictional effects may be appreciable and should probably be considered in any complete theory of the solar semidiurnal tide. Presumably the same should hold true for the lunar atmospheric tide, unless the observed effect on the semidiurnal motions is in reality the result of a periodic variation of the coefficient of eddy viscosity, which has been assumed constant in time. The general similarity between the observations at continental and ocean stations suggests that the time variation of K is not of overriding importance among the factors determining the wind distribution.

The theory thus implies that surface friction should advance rather than retard the time of high lunar tide in the atmosphere. Chapman's survey [2] of atmospheric tides shows that at only a small percentage of stations does the time of maximum lunar tide occur in advance of the moon's transit. On the other hand, his analysis of the seasonal variation [2] of the tide lends some support to the existence of a surface frictional effect of the type described above. In general, the maximum of the lunar tide occurs earlier during the Northern Hemisphere summer and during the equinoxes, when maximum coupling between the earth's major land masses and the atmosphere might be supposed to exist, than during the Northern Hemisphere winter; this advance in phase relative to the annual mean occurs simultaneously in both hemispheres.

5. CONCLUSIONS

An apparently systematic influence on the semidiurnal air motions in the lower troposphere has been examined in order to determine whether this effect can be attributed to surface friction. By means of an extension of the theory originally applied by Ekman [3] to the ocean currents, and by Åkerblom [1] to the atmosphere, an expression for the tidal wind has been obtained which depends not only on the pressure variation but also, within the friction layer, on the coefficient of surface friction and on the eddy exchange coefficient for momentum.

Although the observed semidiurnal wind variation in the friction layer does not follow closely the ideal representation indicated by theory, the agreement is sufficient to suggest that with additional data, and with the relaxing of the assumption of an eddy exchange coefficient invariant with height, the observed deviations from frictionless flow could be satisfactorily accounted for. It is therefore concluded that:

(1) Surface friction causes the actual tidal wind in the friction layer to lead the wind deduced from frictionless theory;

(2) This phase shift of the tidal wind must be accompanied by an advance in the time of maximum of the semidiurnal pressure wave, over the time which frictionless theory should predict; and therefore that

(3) The existence of surface friction should cause the semidiurnal pressure oscillation to show a phase lag with increase in elevation, in agreement with Gold's inference [5].

These conclusions of course need further verification. They would appear to be applicable to the lunar atmospheric tide as well as to the solar semidiurnal tide. The seasonal variation of the lunar tide, which reaches its maximum earliest during the months of strongest heating of the earth's major land masses, lends some support to this view. In future work on surface friction and the atmospheric tides, it would probably be well to stratify the observations by day and night in order to reduce the effect of variability in the eddy exchange coefficient.

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