ON THE INFLUENCE OF THE VERTICAL DISTRIBUTION OF STATIONARY HEAT SOURCES AND SINKS IN THE ATMOSPHERE

M. SANKAR-RAO
The Travelers Research Center, Inc., Hartford, Conn.

ABSTRACT

Within the framework of a quasi-geostrophic model, the influence of different kinds of vertical profiles of diabatic heating on a stationary harmonic of the atmosphere is studied. Except in the cases in which there is a diabatic heating reversal in the upper layers of the atmosphere, the results show qualitative similarity, especially in phase. This lends support to the somewhat arbitrarily selected vertical profiles of diabatic heating used in many previous studies.

1. INTRODUCTION

In studies of the dynamical influence of the stationary heat sources and sinks (or popularly, monsoonal effects) on the time-average perturbations in the atmosphere, it has been common to assume special hypothetical vertical distributions of the heating (e.g., Smagorinsky [8], Gilchrist [4], Staff Members [9], Delisle and Harper [2], Döös [3], and Saltzman [5], [6]). In this note, we aim to study the degree to which different assumed vertical structures affect the solution for a single harmonic under otherwise similar conditions.

The model atmosphere for this study will be the same quasi-geostrophic model discussed in two previous papers published in this journal (Sankar-Rao [7] and Saltzman [6]). We shall refer to the first of these papers by its reference number, [7], and shall follow the notation of this paper.

2. FORMULATION OF THE PROBLEM

In view of our limited objective stated above, we shall neglect the effect of the lower boundary topography and the transient eddy effects on the stationary harmonic. At the upper boundary of the atmosphere we shall assume that the stationary perturbation meridional velocity is zero. Hence, the equations for our problem can be written in the following manner (see (2), (2a), and (2b) of [7]):

$$\frac{\partial^2 v_*}{\partial X^2}+\frac{\partial^2 v_*}{\partial Y^2}+\Xi\,\frac{\partial^2 v_*}{\partial \xi^2}+\theta\,\frac{\partial v_*}{\partial \xi}+\Lambda v_*=\frac{L^2 F_*}{U_0} \tag{1}$$

$$v_*=0 \qquad \text{at } \xi=\xi_T \tag{1a}$$

$$\frac{\partial v_*}{\partial \xi}+B_K v_*+\tau_K\left(\frac{\partial v_*}{\partial X}-\frac{\partial u_*}{\partial Y}\right)=N_K H_* \qquad \text{at } \xi=\xi_b \tag{1b}$$

To study exclusively the effects of the stationary nonadiabatic heat sources and sinks, we shall take

$$F_*=\frac{f}{p_s}\frac{\partial}{\partial \xi}\,(Q_* K_0)$$

The method of solution of the system (1), (1a), and (1b) was given in the earlier paper [7]. The same method is followed here.

The zonal mean state considered here is the same as given in [7] for winter at 45° N. Also all other constants required here are taken from [7]. Besides, we consider here a heating distribution of the form,

$$Q_*=Q_1\cos\frac{2\pi m X}{L}\cdot\cos\frac{2\pi n Y}{k}$$

Here Q_1, the amplitude of the heating function, is a function of m, n, and ξ. As in [7], we take $(m, n)=(3, 0)$. Regarding Q_1, our knowledge is very poor. We know from the observational studies (e.g., Clapp [1] and Staff Members [9]) the geographical distribution of only the vertical integral of Q_* over the Northern Hemisphere. Hence, in the theoretical studies, the vertical profile of Q_1 was, in general, chosen according to the subjective judgment of the author. The object of the present study is to investigate quantitatively to what extent such arbitrary choice of the vertical profile of Q_1 affects the results. To compare the responses due to different vertical profiles of Q_1, we shall take the vertical mean heating

$$\{Q_1\}=\frac{1}{(p_b-p_T)}\int_{p_T}^{p_b} Q_1\,dp=1.438\times 10^{-5}\ \text{deg. sec.}^{-1}$$

for all the profiles. Thus, maintaining the vertical mean heating constant, we shall study the following cases.

Case $1=Q_1=B_1$
Case $2=Q_1=B_2 p$
Case $3=Q_1=B_3 p\sin \pi p/p_b$
Case $4=Q_1=B_4 p^2$
Case $5=Q_1=B_5 p^4$
Case $6=Q_1=B_6 p\cos \pi p/p_b$
Case $7=Q_1=B_7(p-p_a)$ where $p_a=250$ mb.

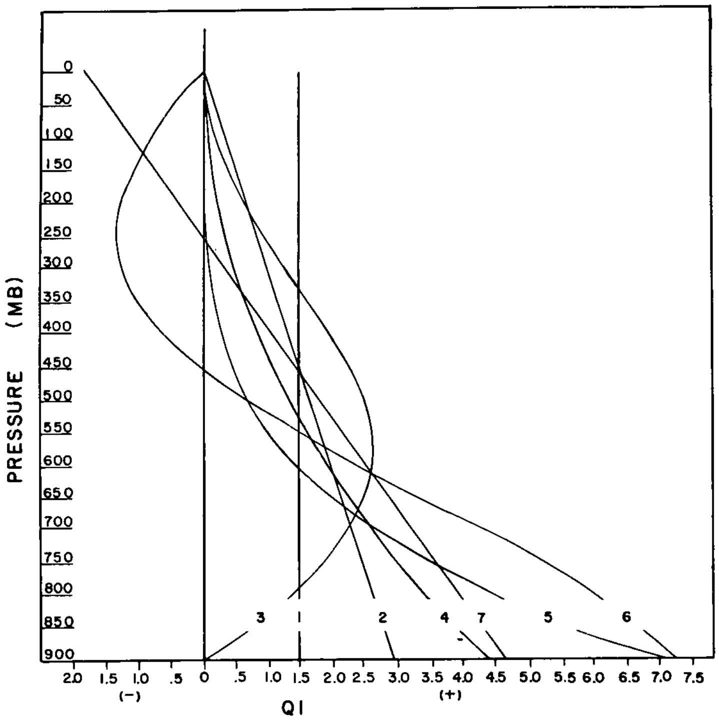

Figure 1.—Vertical distribution of different diabatic heating function amplitudes in 10^{-5} A° sec.$^{-1}$

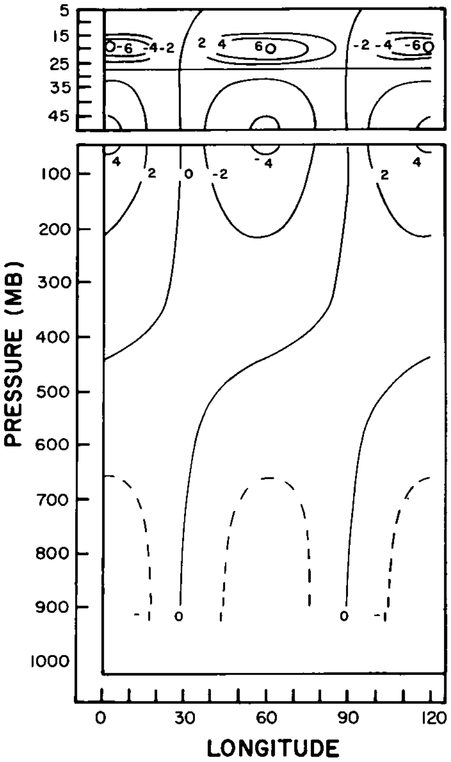

Figure 2.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 1. Above 50-mb. level the scale is changed for better pictorial representation.

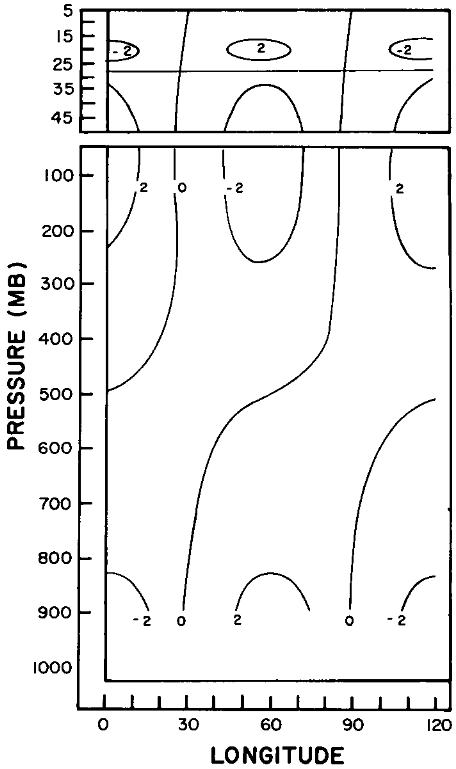

Figure 3.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 2.

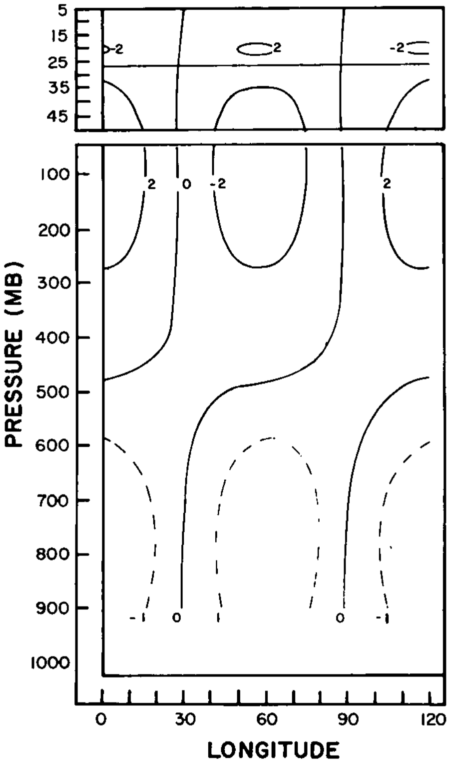

Figure 4.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 3.

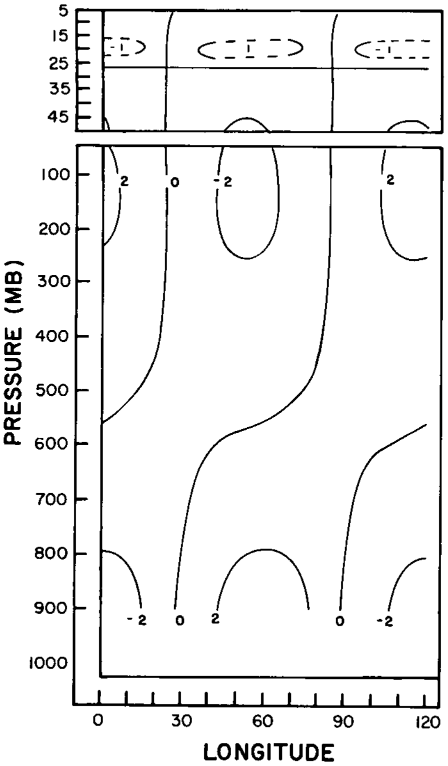

FIGURE 5.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 4.

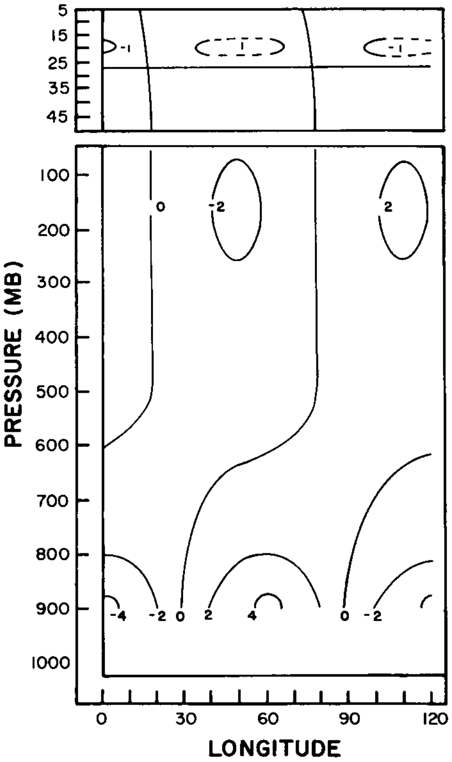

FIGURE 6.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 5.

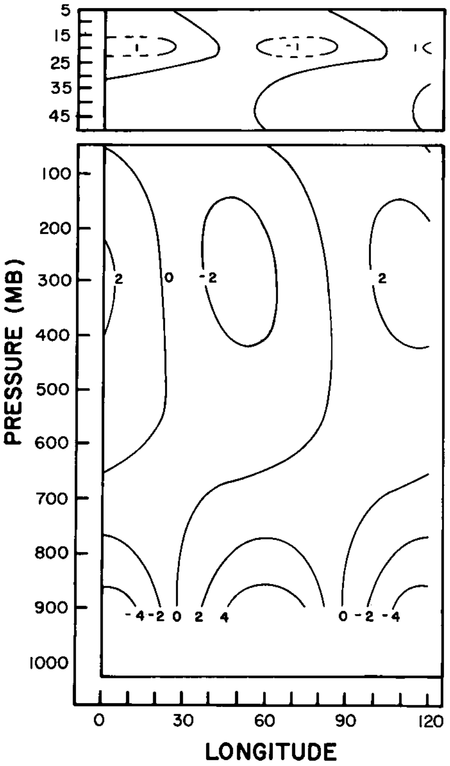

FIGURE 7.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 6.

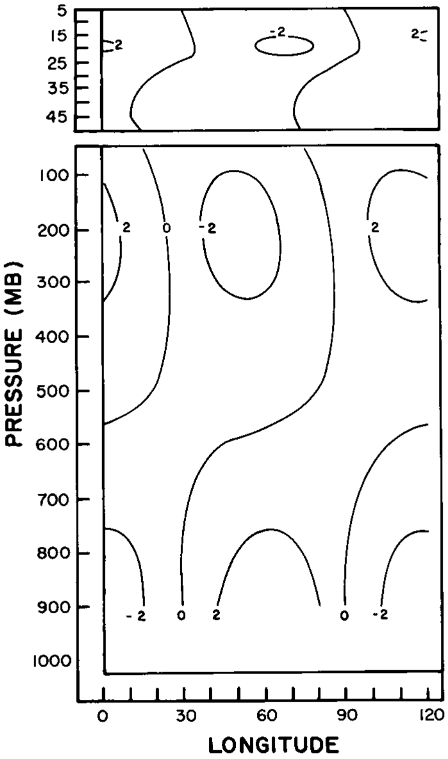

FIGURE 8.—v_* response in 10^2 cm. sec.$^{-1}$ due to heating given by Case 7.

Here B_i $(i=1, 7)$ are constants. The vertical distribution of Q_1 for all these cases is shown in figure 1. It can be seen from this figure that Cases 1 to 5 have no diabatic heating reversal in the atmosphere while Cases 6 and 7 have diabatic heating reversal in the upper layers of the atmosphere. It seems unlikely that such a heating reversal exists, except possibly in some atmospheres where the upper level constituents are functions of longitude. The results for Cases 1 to 7 are shown in figures 2 to 8 in that order.

3. RESULTS AND CONCLUSIONS

The results for Cases 1 to 5 show qualitative similarity, especially in phase. The results for Cases 6 and 7 show qualitative phase similarity with each other, but differ even in phase (especially in the upper half of the atmosphere) from those for Cases 1 to 5. However, as already pointed out, Cases 6 and 7 must be treated as purely hypothetical and unlikely in the real atmosphere. Even though the figures for Cases 1 to 5 look similar (because the phase is almost the same in almost all cases), we cannot altogether ignore the magnitude differences.

Here we showed that similar *qualitative* results are obtained with different vertical profiles of Q_1. This lends some support to the somewhat arbitrarily selected vertical profiles of heating functions in the previous works. On the other hand, we noticed, naturally, that for a significant advancement toward a quantitative theory, some of our future efforts should go into the three-dimensional mapping of Q_* over the globe.

Modeling simplifications as well as wrong speculations regarding the external forcings can lead us to unrealistic quantitative results. It may well be important to resolve the relative roles of these two factors. For this, we may have to study progressively some of the more sophisticated models.

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