$$\frac{\partial}{\partial t} I_1 = -\sum_{j=1}^{J} \sum_{k=1}^{K} V_{k,j}(\alpha_j + \alpha_k) A_{k,j}/2. \tag{8}$$

It can be seen that the terms on the right side of (8) fall into two groups. Those terms which involve interfaces between subvolumes occur in couples which are equal and opposite. These terms cancel. The remaining terms are due to surfaces which lie on the outside of R. These terms are zero since the normal velocity along the outer boundaries of R is zero.

On the other hand, the change of I_2 is given as

$$\frac{\partial}{\partial t} I_2 = -\sum_{j=1}^{J} \sum_{k=1}^{K_j} V_{k,j}(\alpha_j^2 + \alpha_k \alpha_j) A_{k,j}. \tag{9}$$

This may be rewritten as

$$\frac{\partial}{\partial t} I_2 = -\sum_{j=1}^{J} \alpha_j^2 \sum_{k=1}^{K_j} V_{k,j} A_{k,j} - \sum_{j=1}^{J} \sum_{k=1}^{K_j} \alpha_k \alpha_j A_{k,j} V_{k,j}. \tag{10}$$

The first term on the right vanishes through the continuity relation (7). The same argument applied to (8) is also true for the second term of (10). All contributions for interfaces will occur as couples, which are equal and opposite. The remaining terms lie on the outer boundaries of the region R.

To see how the general formula (5) and (6) might actually be applied, consider the following equations of motion for an incompressible, homogeneous fluid under rotation,

$$\frac{\partial}{\partial t} u + \nabla \cdot (\mathbf{V}u) = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + F^x + fv \tag{11}$$

$$\frac{\partial}{\partial t} v + \nabla \cdot (\mathbf{V}v) = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + F^y - fu \tag{12}$$

$$\rho_0 g = -\frac{\partial p}{\partial z} \tag{13}$$

$$\nabla \cdot \mathbf{V} = 0. \tag{14}$$

Application of (6) to the left side of (11) and (12) will guarantee that the finite difference expressions for the advective terms will not alter the finite difference equivalent of the kinetic energy integral

$$\text{K.E.} = \int_0^L \int_0^L \rho_0 \frac{(u^2+v^2)}{2} \, dxdy$$

provided the continuity relation (6) is used as the diagnostic relation to determine the vertical component. This will eliminate the possibility of spurious changes in the energy level associated with the nonlinear numerical instability described by Phillips [4].

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CORRECTION

Vol. 93, October 1965, p. 582: The first of the expressions appearing three lines below equation (25) should be:

$$\frac{u_g}{fy}$$