GRAPHICAL METHOD FOR DETERMINING ATMOSPHERIC PRESSURE FROM ROCKETSONDE OBSERVATIONS

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ABSTRACT

A graphical method of determining pressure from rocketsonde temperature-measurements is presented, and information for the construction and use of such a graph is given. This graphical method is shown to be simple and rapid. Comparison with results from computer processing of the same temperature data yields pressure differences of only 0.5 percent in 72 percent of the cases, and 1 percent in 90 percent of the cases.

1. INTRODUCTION

The rocket-boosted meteorological instrument package represents a valuable tool for measuring atmospheric parameters in the region between balloon and satellite levels. Before the Meteorological Rocket Network (MRN) was established in 1959 the acquisition and processing of rocketsonde data were carried out by many groups with similiar objectives but without complete knowledge of each other's activities. With the founding of the MRN, the U.S. Army Electronics Research and Development Activity (ERDA)—formerly the U.S. Army Signal Missile Support Agency—of White Sands Missile Range, N. Mex., voluntarily undertook the task of processing and publishing data acquired by the various groups. Presently the responsibility of publishing the monthly digest lies with the Environmental Science Services Administration's (ESSA) Environmental Data Service, National Records Center, Asheville, N.C. This effort has been an important aspect of these groups' participation in the MRN, and allowed:

(a) all data to be archived, and available in a monthly digest [1] for use by the participants, and

(b) a single computer program to be utilized for processing all data received for inclusion in the digest [2].

The uniform reduction of the rocket data was possible through the cooperation of the participating rocket ranges in sending their data to ERDA at White Sands. These data records contained temperature, tabulated rawinsonde, and radar plotboard recordings. The radar plotboard records were considered to be a common denominator to all ranges and were used for determining altitude and wind data. When some stations lost their radar plotting board facilities and others developed specialized techniques unique to their requirements, it was decided that each contributor should be responsible for the accurate reduction of its own data. The finished product is still forwarded to ERDA at White Sands for processing and subsequent publication by ESSA in the monthly digest. The processing will also be undertaken by ESSA with the July 1966 data.

In order to satisfy the needs of the real-time user of rocketsonde data, a rapid and efficient method of data reduction is needed. If computers are not readily available, an alternate approach must be used. The graphical method described here for determining pressure from observed temperature is one answer to the problem. This technique consists of a straightforward solution of the hydrostatic equation and equation of state. Construction and utilization of the graph are relatively uncomplicated and rapid.

2. PRESSURE COMPUTATION

In order to determine atmospheric pressure from a given temperature-height profile the hydrostatic equation

$$dp=-\rho g dz \tag{1}$$

and equation of state

$$p=\rho RT \tag{2}$$

are combined to obtain the relation

$$dp/p=-(g/RT)dz. \tag{3}$$

Integration of (3) from the pressure p_0 at height z_0 to pressure p_1 at height z_1 yields

$$\ln (p_1/p_0)=-(1/R)\int_{z_0}^{z}(g/T)dz. \tag{4}$$

This relationship may be replaced by the excellent approximation

$$\ln (p_1/p_0)\cong -g'\Delta z/RT' \tag{5}$$

or

$$p_1/p_0\cong \exp (-g'\Delta z/RT') \tag{6}$$

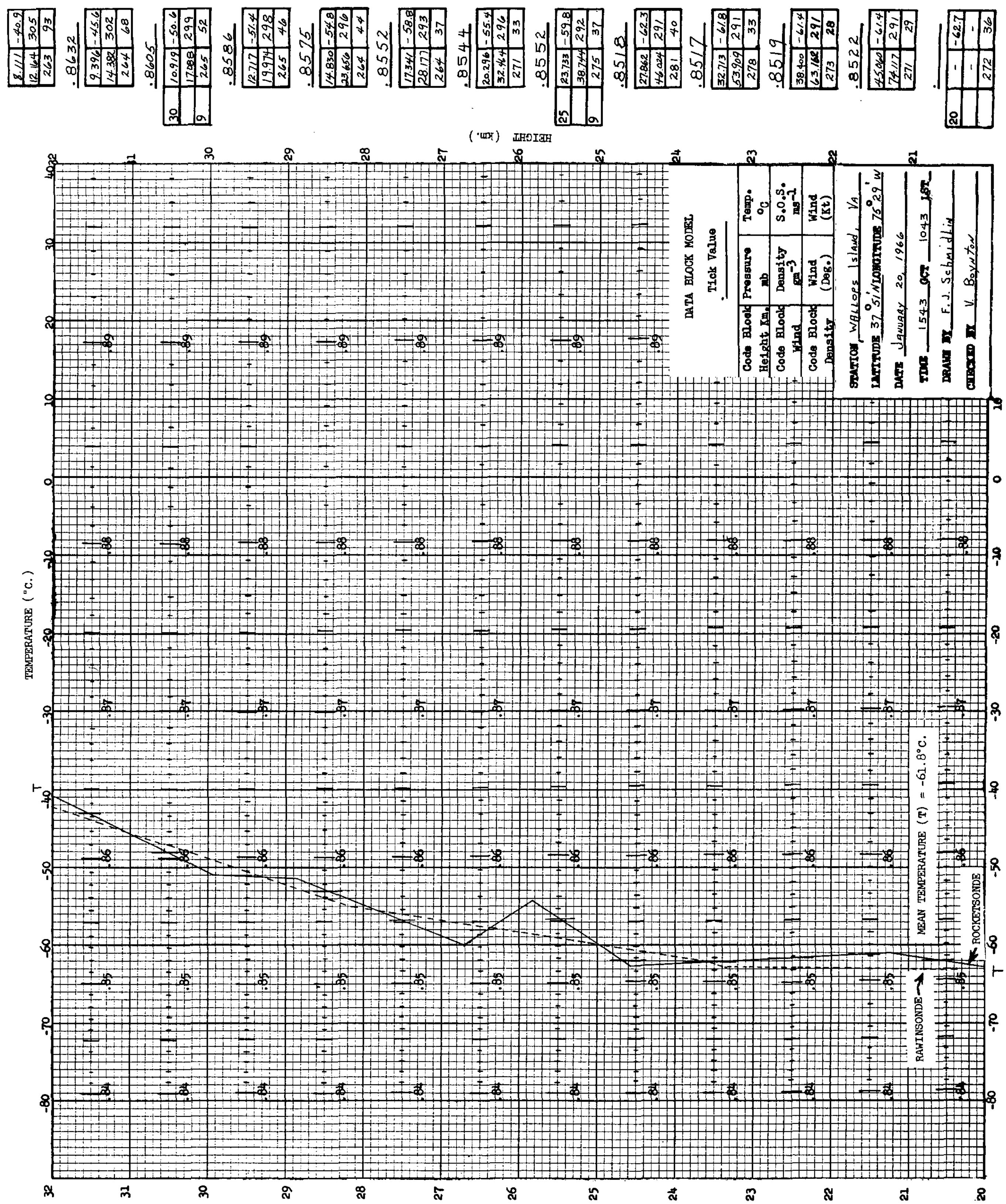

FIGURE 1.—Pressure computation graph for altitude range 20–32 km.

where R is the gas constant for dry air, g' is the acceleration due to gravity at the mean height of the stratum Δz and at 45° Lat., and T' is the mean (absolute) temperature of the stratum. The pressure profile is generated by employing the last-obtained p_1 as the initial pressure p_0 in each successive stratum. This procedure is employed both in the graphical method to be described, and in the computer reduction of rocketsonde temperature data [3].

3. CONSTRUCTING THE DIAGRAM

The lower portion of the graph developed for the determination of pressure is shown in figure 1. Three parameters determine the graph: temperature, height, and the pressure ratio p_1/p_0. The scales of 1 km. and 10° C. per in. for the ordinate and abscissa, respectively, were selected to give both adequate resolution for plotting and reasonable overall diagram size. However, although a temperature range of $-80°$ C. to 40° C. can conveniently be placed on one graph, an altitude range of 20 km. to 62 km. (the interval normally obtained in a rocket sounding) requires a representation four times larger. Therefore a complete set of diagrams includes three graphs in addition to that shown in figure 1. Furthermore, it is desirable to allow for a 1- to 2-km. overlap between successive charts.

Values of the pressure ratio (p_1/p_0) were computed for 1-km. layers. The extreme values of the temperature scale ($-80°$ C. to 40° C.) yield a range for p_1/p_0 of approximately 0.840 to 0.899. The location of the tick marks, which represent p_1/p_0 at intervals of 0.001, is then obtained as a function of temperature from the relation

$$T' = -g'\Delta z/[R \ln (p_1/p_0)] \qquad (7)$$

which is simply a rearrangement of equation (5).

4. USING THE DIAGRAM

The rocketsonde temperature and height parameters required for graphical determination of the pressure are generally obtained from direct temperature sensor measurements and radar evaluation, respectively. As temperatures and corresponding times are selected from a telemetry record, it is convenient to enter values on a form such as the Thermodynamic Data Tabulation Form (fig. 2). The altitude versus time record obtained from radar is correlated with the temperature-time information to give temperature as a function of height, which is then plotted on the pressure-computation diagram (fig. 1).

The initial pressure value required for developing a pressure-altitude profile is extracted from the rawinsonde observation (fig. 3) closest in time to the rocketsonde launching. It should be noted that in order for the procedure to be valid, reasonable agreement must exist between rocketsonde and rawinsonde temperatures at the base level.

In the case to be discussed, rocketsonde temperatures are available down to 20 km. and agree quite well with the rawinsonde temperatures. Therefore, rawinsonde data

MET ROCKET THERMODYNAMIC DATA TABULATION FORM

STATION WALLOPS ISLAND, VA. Model No. L1 - 2257 Motor Type ARCAS

Lat. 37° 51′ N Long. 75° 29′ W Payload ARCASONDE 1-A Sensor Type 10 mil bead

Ground Tracking Equip. GMD-1B & FPS-16

	Year	Month	Day	Zero Time
75th Mer.	1966	Jan.	20	1043
GCT	1966	Jan.	20	1543

Calibration Check #

Temp. Ord. 78.3 Ref. Ord. 89.9 Freq. Ratio 0.871

Calibration Temp. 24.4 °C Ambient Temp. 24.6 °C Corr. % °C

% Not required for differences less than ± 1.0° C.

Level No.	Time MIN. & Sec.	REF. ORD.	TEMP. ORD.	FREQ. RATIO THERM RESIS.	TEMP. °C	CORR. # TEMP. °C	PRESS. * mb.	DENSITY * gm -3	SOS ms -1	ALTITUDE (geometric) feet X10 2	ALTITUDE (geometric) meter X10 1	Signif. Level
1	2:39	90.4	57.3	0.634	- 7.2					1534	4676	
2	2:49	90.5	57.3	.633	- 7.2					1510	4602	
3	2:58	90.5	56.0	.618	- 8.7					1494	4554	
4	3:36	90.6	59.4	.657	- 5.0					1430	4359	
5	3:41	90.6	57.3	.632	- 7.2					1420	4328	
6	3:57	90.6	58.7	.648	- 5.5					1400	4267	
7	4:08	90.7	56.7	.625	- 8.0					1383	4215	
8	4:30	90.9	59.1	.650	- 5.3					1350	4115	
9	4:54	91.0	55.0	.604	-10.0					1322	4029	
10	5:01	91.0	50.7	.557	-14.7					1312	3999	
11	5:28	91.0	53.4	.587	-11.7					1280	3901	
12	5:39	91.0	51.4	.565	-13.8					1269	3868	
13	5:49	91.0	46.7	.513	-18.8					1259	3837	
14	6:49	91.1	57.3	.629	- 7.7					1198	3652	
15	6:57	91.1	56.4	.619	- 8.5					1192	3633	
16	7:17	91.1	43.3	.475	-22.3					1172	3572	
17	7:24	91.1	43.0	.472	-22.6					1164	3548	
18	7:28	91.1	40.7	.447	-25.0					1162	3542	
19	7:58	91.1	33.4	.367	-32.5					1138	3469	
20	8:18	91.1	32.3	.355	-33.7					1122	3420	
21	8:40	91.1	32.5	.357	-33.8					1104	3365	
22	8:43	91.1	30.8	.338	-35.6					1100	3353	
23	8:52	91.1	32.2	.353	-34.1					1095	3338	
24	11:42	91.2	20.3	.223	-50.9					982	2993	
25	12:56	91.2	20.1	.220	-51.4					947	2886	
26	15:24	90.7	15.4	.170	-60.0					876	2670	
27	17:26	90.3	18.1	.200	-54.2					847	2582	
28	19:03	90.0	14.4	.160	-62.6					806	2457	
29	26:41	88.2	14.8	.168	-60.9					697	2124	
30	30:17	87.5	14.0	.160	-62.7					656	2000	

REMARKS:

If applicable * Derived Values

FIGURE 2.—Sample thermodynamic data tabulation form.

RAWINSONDE OBSERVATIONS FOR METEOROLOGICAL ROCKETS
WEATHER BUREAU SUPPORT FACILITY
WALLOPS ISLAND, VIRGINIA

Prepared By: M. Powell Lat: 37° 51′ N Long. 75° 29′ W

Model No. L1 - 2527

R/S No. 79

		Year	Month	Day	Time Release	Time At Top
75	Mer.	1966	Jan.	20	0615	0752
	GCT	1966	Jan.	20	1115	1252

Distance to Rocket Launch: Miles 0.48 Dir. 206°

Pressure Tenths Mb.	Height Geopot.	Height Geometric	Wind Deg. S/= 180	Wind Spd. kts	Wind Components from N-S (mps)	Wind Components from E-W (mps)	R. H. %	Tem. C
1020.6	00003	00000	330	10	- 4	3	69	- 1.7
1000.0	165	170	332	16	- 7	4	73	- 3.0
850.0	1437	1440	005	20	-10	- 1	80	- 7.7
700.0	2924	2920	353	26	-13	2	40	-16.0
500.0	5371	5380	031	39	-12	- 11	32	-32.7
400.0	6921	6930	325	36	-15	11	26	-37.5
300.0	8878	8890	299	62	-15	28		-44.2
250.0	10095	10120	288	58	- 9	29		-46.0
200.0	11585	11610	276	64	- 3	33		-44.5
150.0	13476	13510	278	66	- 4	34		-53.0
100.0	16036	16080	279	59	- 5	30		-63.2
81.0	17350	17400	284	58	- 7	29		-61.7
72.0	18090	18140	286	54	- 7	27		-66.0
70.0	18230	18280	287	52	- 8	26		-65.3
65.0	18690	18750	289	39	- 6	19		-63.2
50.0	20293	20360	281	35	- 3	18		-63.0
31.0	23230	23320	272	35	- 1	18		-62.7
30.0	23436	23530	272	35	- 1	18		-62.2
20.0	25960	26070	273	41	- 1	21		-58.2
14.5	28040	28160	277	54	- 3	28		-55.0
10.0	30406	30560	268	80	1	41		-47.0
7.9	31994	32150	-	-	-	-		-41.8

FIGURE 3.—Sample tabulation of data from rawinsonde observation in support of meteorological rocket firing.

for the 50-mb. level (approximately 20 km.) provide the base for the determination. The computed geopotential height of the 50-mb. surface is converted to a geometric height [4, 5]. The difference between this height and 21 km. is employed, together with the mean temperature between 50 mb. and 21 km., to obtain the pressure at the latter level (table 1). This pressure (45.060 mb.) represents the initial value, or p_0, for obtaining the complete pressure-altitude profile graphically. The procedure is as follows (refer to fig. 1):

1. enter the 21-km. pressure in the appropriate block at the right-hand edge of the diagram (45.060);
2. determine the mean temperature in the layer between 21 and 22 km.;
3. read, to the highest possible accuracy (four decimal places are desirable), the tick value corresponding to the mean temperature, and enter this value on the line above the block (0.8522);
4. multiply the tick value by the initial pressure, and enter the new pressure value (three decimal places) in the 22-km. data block (38.400).

Steps (2) through (4) are then repeated for each successive 1-km. layer.

As presented, table 1 requires double interpolation for obtaining the pressure value at 21 km. The table can be improved by presenting temperature in 0.1° C. increments and Δz in 10-m. increments. For operational application additional tables are required to include the maximum possible range for the 50-mb. level. Also, in the event rocketsonde data do not extend down to the 50-mb. level, additional tables can be developed for the 30-, 20-, and 10-mb. levels. However, the case may arise where the above tables do not satisfy the user's requirements, or where greater resolution is desired at the initial level. In this event it is recommended that equation (4) be solved manually.

It is convenient to extract the temperature value for each 1-km. level from the plotted profile and enter these data in the appropriate position in the right-hand margin as the sounding is evaluated. Temperatures may then be combined with the derived pressure values to determine density, and may also be employed in the calculation of the speed of sound. All of the thermodynamic parameters ordinarily derived from a rocketsonde observation are thus available in the data blocks of the pressure diagram.

5. ACCURACY OF METHOD

A brief study was conducted to assess the relative accuracy of the graphical method for determining pressure. For this purpose data from the Meteorological Rocket Network Data Reports [1], processed by computer, were used as a standard. These data were selected for various stations during the months of October 1963, and January, April, and July 1964. The pressure at the base level of the rocketsonde report was used as the initial pressure for the graphical technique. A level by level comparison of pressure was then accomplished for all available soundings. For 90.2 percent of the 327 levels compared, the difference between the graphically reduced and computer-derived pressures was less than 1 percent; for 72.4 percent, the difference was less than half of 1 percent. The percentage of agreement undoubtedly could have been increased by the exercise of extreme care in plotting of data and selection of tick mark values.

6. CONCLUSIONS

The graphs described above have been developed to provide an alternate method of determining pressures at rocketsonde levels. Other methods in general use involve either manual or computer solutions of basic equations. Only slightly less accuracy is realized with the graphical method in comparison with the computer results. However, it is quite probable that manual solution of the equations is more susceptible to error than either the graphical or computer methods.

TABLE 1.—*Pressure at 21 km. as a function of distance from 50 mb. to 21 km. (Δz), and mean temperature between these levels.*

Δz	MEAN TEMPERATURE (°C.)																							
	−45	−46	−47	−48	−49	−50	−51	−52	−53	−54	−55	−56	−57	−58	−59	−60	−61	−62	−63	−64	−65	−66	−67	−68
50	49.64	49.63	49.63	49.62	49.62	49.62	49.62	49.62	49.62	49.61	49.61	49.61	49.61	49.61	49.60	49.60	49.60	49.60	49.60	49.60	49.59	49.59	49.59	49.50
100	49.26	49.26	49.25	49.25	49.25	49.25	49.24	49.24	49.23	49.23	49.23	49.22	49.22	49.22	49.21	49.21	49.20	49.20	49.20	49.19	49.19	49.18	49.18	49.18
150	48.90	48.89	48.89	48.88	48.88	48.87	48.86	48.86	48.86	48.85	48.85	48.84	48.84	48.83	48.83	48.82	48.81	48.81	48.80	48.80	48.79	48.79	48.78	48.78
200	48.53	48.52	48.52	48.51	48.51	48.50	48.50	48.49	48.48	48.47	48.47	48.46	48.45	48.44	48.43	48.43	48.42	48.42	48.41	48.40	48.39	48.39	48.38	48.37
250	48.18	48.17	48.16	48.15	48.15	48.14	48.13	48.12	48.11	48.10	48.09	48.08	48.08	48.07	48.06	48.05	48.04	48.03	48.02	48.01	48.00	47.99	47.98	47.97
300	47.82	47.81	47.80	47.79	47.78	47.77	47.76	47.75	47.74	47.73	47.72	47.71	47.70	47.69	47.68	47.67	47.66	47.65	47.64	47.63	47.62	47.61	47.59	47.58
350	47.46	47.45	47.44	47.43	47.42	47.41	47.40	47.39	47.37	47.36	47.35	47.34	47.33	47.31	47.30	47.29	47.28	47.26	47.25	47.24	47.23	47.21	47.19	47.18
400	47.11	47.10	47.09	47.08	47.06	47.05	47.04	47.02	47.01	47.00	46.98	46.98	46.96	46.95	46.93	46.91	46.90	46.88	46.87	46.86	46.84	46.83	46.81	46.80
450	46.77	46.76	46.74	46.72	46.71	46.70	46.68	46.66	46.65	46.64	46.62	46.60	46.59	46.58	46.56	46.55	46.53	46.52	46.50	46.48	46.47	46.46	46.44	46.42
500	46.42	46.40	46.38	46.37	46.35	46.34	46.33	46.31	46.29	46.28	46.26	46.24	46.23	46.21	46.19	46.17	46.15	46.14	46.12	46.10	46.09	46.07	46.05	46.03
550	46.10	46.08	46.06	46.04	46.03	46.01	45.99	45.97	45.96	45.94	45.93	45.91	45.89	45.87	45.85	45.83	45.81	45.80	45.78	45.76	45.74	45.72	45.70	45.69
600	45.73	45.72	45.70	45.68	45.66	45.64	45.62	45.61	45.59	45.57	45.55	45.52	45.50	45.48	45.46	45.44	45.42	45.40	45.38	45.36	45.33	45.30	45.28	45.26
650	45.39	45.37	45.35	45.33	45.31	45.29	45.27	45.25	45.23	45.21	45.19	45.17	45.15	45.13	45.11	45.09	45.07	45.05	45.03	45.01	44.99	44.95	44.92	44.89
700	45.06	45.04	45.02	45.00	44.98	44.95	44.93	44.91	44.89	44.87	44.85	44.82	44.80	44.77	44.75	44.73	44.70	44.68	44.66	44.63	44.60	44.57	44.54	44.51
750	44.73	44.71	44.69	44.67	44.65	44.63	44.61	44.58	44.55	44.52	44.50	44.48	44.45	44.42	44.40	44.37	44.34	44.32	44.30	44.27	44.24	44.21	44.18	44.15
800	44.40	44.38	44.36	44.34	44.32	44.29	44.27	44.24	44.21	44.18	44.16	44.14	44.10	44.07	44.04	44.02	43.99	43.97	43.94	43.90	43.87	43.84	43.81	43.79
850	44.07	44.05	44.03	44.00	43.97	43.95	43.93	43.90	43.87	43.84	43.82	43.79	43.76	43.73	43.70	43.67	43.64	43.62	43.59	43.55	43.52	43.49	43.47	43.44
900	43.74	43.72	43.70	43.68	43.65	43.61	43.59	43.56	43.53	43.50	43.48	43.45	43.42	43.39	43.35	43.32	43.29	43.27	43.24	43.20	43.17	43.14	43.11	43.08
950	43.41	43.38	43.36	43.33	43.30	43.28	43.25	43.22	43.19	43.16	43.13	43.10	43.07	43.04	43.01	42.98	42.95	42.92	42.89	42.86	42.83	42.79	42.76	42.73

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