

The Entrainment Influence on the Ocean Surface Layer in Tropical Latitudes

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ABSTRACT

A nonlinear, time-dependent, baroclinic model is developed for a zonally uniform, tropical, two-layer ocean on a north-south vertical section. The lower layer is infinitely deep, at rest, and at constant temperature. The dynamics of the well-mixed surface layer are described in terms of the components of horizontal mass transport, the specific mass, and the specific enthalpy. The forcing functions of the model are the zonal wind stress, the vertical entrainment of cold water from the lower layer into the surface layer, and the surface thermal energy input. The concept of entrainment forcing is based on the approach of Kraus and Turner for parameterizing the vertical motion of the seasonal thermocline.

Since zonal gradients of all quantities are neglected, the model applies only to the ocean's interior. This is rationalized by oceanographical observations. In particular, the east-west pressure gradient term is one order of magnitude smaller than the wind stress; it may be considered as an additional forcing function and, as such, absorbed in the zonal wind stress. Scale analysis reveals two time scales inherent in the model: a short scale of 0.2 day governing the mass transport equations, and a long scale of several years governing the conservation equations for mass and enthalpy. Short-term climatic fluctuations may be controlled by the latter scale.

Solutions for the steady state with Rossby number zero are presented. For wind stress and thermal energy input, simple analytic functions similar in shape to observed patterns are used. For the entrainment function three different possible distributions are investigated, all of which have an equatorial maximum attributed to strong vertical mixing in the equatorial undercurrent region. The principle responses of the model are: 1) a meridional pattern of zonal mass transport exhibiting the main observed features, particularly an equatorial countercurrent; 2) a thickness of the mixed layer similar to the observations; and 3) a surface temperature profile with an equatorial minimum. The tendency of this model to develop an equatorial countercurrent is caused by the entrainment forcing. It is shown that entrainment and energy balance are not entirely independent of one another.

1. Introduction

Kraus and Turner (1967) and Turner and Kraus (1967) have presented a one-dimensional parameterization of the seasonal thermocline in the ocean, in terms of both a laboratory and a mathematical model. Their work was based on the finding of Rouse and Dodu (1955) that a density discontinuity surface in a fluid can be shifted by a vertical mass flux.

In the experiments of Rouse and Dodu the upper layer of a stably stratified two-layer fluid was constantly stirred, whereas the lower layer was at rest. The input of small-scale turbulent kinetic energy into the upper layer caused vertical mixing; the lower layer became entrained into the upper layer. The density discontinuity did not tend to become smeared but remained sharp and well-defined. The authors described the entrainment process in terms of a moder-

ate downward velocity of the density interface. Turner and Kraus observed similar behavior in their laboratory experiment and they likened it to the rising and falling of the seasonal oceanic thermocline (Kraus and Turner). These models are strictly one-dimensional, expressing a balance of forces in the vertical.

In the present contribution we consider the large-scale dynamical implications of this entrainment mechanism. As entrainment is essentially a vertical mixing process, we expect it to have a maximum in regions with strong vertical current shear such as in the vicinity of the equatorial undercurrent (Knauss, 1963). Thus, we arbitrarily prescribe an entrainment function having these characteristics. We then investigate its influence on the horizontal motion, mass and temperature fields in a baroclinic ocean subject also to momentum and thermal energy input across its upper surface. The model is similar in many respects to that of Yoshida (1967); the primary difference lies in the manner of specifying the entrainment.

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The simplest possible model to do this seems to be a two-layer ocean on a north-south section with the lower layer at rest. In our model this is achieved by setting the pressure gradient equal to zero in the lower layer and by assuming that the stress at the interface affects only the upper layer. These conditions filter out the barotropic mode of motion (O'Brien and Reid, 1967). Therefore, the motion of the upper layer exhibits a baroclinic response.

We neglect the zonal derivatives of all quantities, retaining only meridional gradients. Hence the model is applicable only to the interior ocean region; sidewall effects are not taken into account. It seems that meridional oceanic boundaries are not crucial for the dynamics of this model. To rationalize this, we may restrict ourselves to some remarks about the zonal pressure gradient which does not vanish in the actual ocean. The zonal pressure gradient term would enter the zonal mass-transport equation in the approximate form

$$\int_i^s \frac{\partial p}{\partial x} dz \approx g \Delta \rho h_1 \frac{\partial h_1}{\partial x}, \quad (1.1)$$

where h_1 is the thickness of the upper layer and $\Delta \rho$ the density difference between it and the lower layer; a complete list of symbols is given in the Appendix. Employing $\Delta \rho = 10^{-3} \text{ gm cm}^{-3}$, $h_1 = 10^4 \text{ cm}$, and $\partial h_1 / \partial x = 10^{-5}$ for the mixed surface layer, the right-hand side of Eq. (1.1) becomes 0.1 dyn cm^{-2} , which is one order of magnitude smaller than the zonal wind stress. Moreover, this zonal pressure gradient term (1.1) is practically independent of latitude and longitude in the interior of the Pacific Ocean. On the other hand, since meridional gradients of the mixed layer thickness are of the order $\partial h_1 / \partial y = 10^{-4}$, the associated pressure gradient term has the same order as the wind stress. These estimates are based on oceanographical observations involving the depth of the 24.40 σ_T -surface³ of the Pacific Ocean (Barkley, 1968). If we now treat the zonal pressure gradient as a forcing function, we can absorb it into the zonal wind stress; therefore, the forcing function τ introduced in the following shall be interpreted as the sum of the zonal pressure gradient and the zonal wind stress, though we will call it simply the wind stress.

The characteristics of the model are shown in Fig. 1. The entrainment process which tends to deepen the thermocline is counterbalanced by the mass flux divergence in the upper layer which otherwise is treated like a slab. The poleward mass flux out of the slab is closed by subsidence processes in high latitudes which finally carries the water masses back into tropical regions (dashed arrows). However, the present model is considered to be independent of the particular kind of recirculation mechanism.

³ Along the 24.40 σ_T -surface the density of the ocean water would have the constant value $1.02440 \text{ gm cm}^{-3}$ for zero pressure.

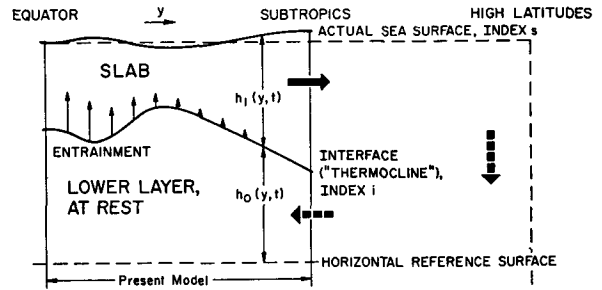


FIG. 1. Geometry of the two-layer ocean.

The forcing functions of the model are externally given. They are the zonal wind stress, the vertical entrainment mass flux into the upper layer, and the thermal heat input into the ocean from the atmosphere.

2. The model

The equations for momentum conservation in the slab, integrated vertically from the interface i to the sea surface s , yield

$$\frac{\partial}{\partial t} \int_i^s \rho u dz + \frac{\partial}{\partial y} \int_i^s v \rho u dz = \beta y \int_i^s \rho v dz + \tau - \tau_{xi}, \quad (2.1)$$

$$\frac{\partial}{\partial t} \int_i^s \rho v dz + \frac{\partial}{\partial y} \int_i^s v \rho v dz = -\beta y \int_i^s \rho u dz - \tau_{yi} - P. \quad (2.2)$$

The x derivatives are neglected, and only the zonal wind stress component τ (which includes the zonal pressure gradient term) is taken into account; τ_{xi} , τ_{yi} are the components of the stress vector at the interface between the upper and lower layer. In order to evaluate the meridional pressure gradient term P , we stipulate a linear dependence of density upon temperature; the influence of salinity is neglected. With the dimensionless temperature variable

$$\vartheta = \alpha(\theta - \theta_0), \quad \alpha \equiv \text{constant}, \quad (2.3)$$

the equation of state is

$$\rho = \rho_0(1 - \vartheta). \quad (2.4)$$

The condition that the pressure gradient vanishes in the lower layer leads to a well-known relationship between the thickness of the slab and the elevation h_0 of the interface above an arbitrary horizontal reference surface (Fig. 1). Applying this relationship as well as the condition of vertically constant temperature, and assuming that the sea level pressure vanishes, the pressure gradient term becomes approximately

$$P = \int_i^s \frac{\partial p}{\partial y} dz = \frac{g}{\rho_0} \frac{\partial}{\partial y} \left[\frac{(\rho_1 h_1)^2 \vartheta_1}{2} \right]. \quad (2.5)$$

We set

$$Q = \rho_1 h_1 \quad (2.6)$$

for the specific mass of the upper layer and

$$E = \rho_1 h_1 c_p (\theta_1 - \theta_0) = Q g a \delta_1 \quad (2.7)$$

for the corresponding enthalpy, where

$$a = c_p / (g\alpha). \quad (2.8)$$

The pressure term (2.5) then becomes

$$P = \frac{1}{\rho_0 a} \frac{\partial}{\partial y} \left(\frac{QE}{2} \right). \quad (2.9)$$

The term "specific" in the present text refers to a column of unit horizontal area in the upper layer. The quantity a has the dimension of a length of the order 3000 km, corresponding to the meridional length scale of the model (see Section 3).

We now introduce the horizontal mass transport components

$$M = \int_i^s \rho v dz, \quad N = \int_i^s \rho v dz, \quad (2.10)$$

and replace the function v in the nonlinear terms of Eqs. (2.1) and (2.2) by its vertical average

$$\bar{v} \approx \frac{1}{\rho_1 h_1} \int_i^s \rho v dz = N/Q. \quad (2.11)$$

The approximation (2.11) is equivalent to neglecting the correlation terms $u'v'$, $v'v'$, where the bar operator denotes a vertical mean. We finally parameterize the interface stress vector by

$$\begin{pmatrix} \tau_{xi} \\ \tau_{yi} \end{pmatrix} = k \begin{pmatrix} M \\ N \end{pmatrix}. \quad (2.12)$$

With Eqs. (2.9)–(2.12), the mass-transport equations take on the forms (2.14), (2.15) given below.

The continuity equation, vertically integrated over the slab, yields

$$\frac{\partial Q}{\partial t} + \frac{\partial N}{\partial y} = \rho_0 w^*, \quad (2.13)$$

where w^* denotes the entrainment velocity. If $w^* > 0$, the internal interface is shifted downward. Since the horizontal slopes of density discontinuities in the ocean on a global scale are not bigger than 10^{-4} , one can neglect the horizontal component of the entrainment velocity. The entrained water does not affect the equations of the total mass transport of the slab because it carries no momentum with itself. However, it affects the continuity equation (2.13) by acting as a mass source. Likewise, it would affect the equations of the average velocity in the slab, because the entrained water has to be accelerated at the expense of this average velocity; this is the reason why we prefer to use integrated mass transport equations instead of mean

momentum equations. The magnitude of w^* is of the order 5×10^{-5} cm sec $^{-1}$ (compare, e.g., Overstreet and Rattray, 1969). We shall call the quantity $\rho_0 w^*$ the entrainment mass flux σ .

In order to close the system, we add Eq. (2.17) for the specific enthalpy E of the slab; $\zeta = \zeta(y, t)$ is the net thermal energy flux into the sea surface (algebraic sum of radiation balance, sensible and latent heat flux). The entrained water does not affect the specific enthalpy because the reference temperature for E is just the temperature of the lower layer. The final set of model equations is now

$$\frac{\partial M}{\partial y} + \frac{\partial NM}{\partial y Q} = \beta y N + \tau - kM, \quad (2.14)$$

$$\frac{\partial N}{\partial t} + \frac{\partial NN}{\partial y Q} = -\beta y M - \frac{1}{\rho_0 a} \frac{\partial}{\partial y} \left(\frac{QE}{2} \right) - kN, \quad (2.15)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial N}{\partial y} = \sigma, \quad (2.16)$$

$$\frac{\partial E}{\partial t} + \frac{\partial NE}{\partial y Q} = \zeta. \quad (2.17)$$

3. Scale considerations

We shall now perform a simple scale analysis. Using the nondimensionalization notations

$$\left. \begin{aligned} M &= M' m' & N &= M' n' & Q &= Q' q' \\ t &= T' t' & y &= Y' y' & E &= E' e' \\ \tau &= D' \tau' & \sigma &= S' \sigma' & \zeta &= Z' \zeta' \end{aligned} \right\}, \quad (3.1)$$

the mass transport equations become

$$\frac{\partial m'}{\partial t'} + \text{Ro} \frac{\partial}{\partial y'} \left(\frac{n' m'}{q'} \right) = y' n' + \epsilon \tau' - \gamma m', \quad (3.2)$$

$$\begin{aligned} \frac{\partial n'}{\partial t'} + \text{Ro} \frac{\partial}{\partial y'} \left(\frac{n' n'}{q'} \right) \\ = -y' m' - \delta \kappa^{-1} \frac{\partial}{\partial y'} \left(\frac{q' e'}{2} \right) - \gamma n', \end{aligned} \quad (3.3)$$

with

$$\left. \begin{aligned} T' &= 1/(\beta Y') & \text{Ro} &= M' / (\beta Y'^2 Q') \\ \epsilon &= D' / (\beta Y' M') & \gamma &= k / (\beta Y') \\ \delta &= H' / Y' & \kappa &= (a Y' \beta M') / E' \end{aligned} \right\}. \quad (3.4)$$

The term H' denotes the scale of the thickness of the upper layer, i.e.,

$$H' = Q' / \rho_0. \quad (3.5)$$

We shall try to relate all scale of the model to the scales of the forcing functions. The latter may be given by

$$\left. \begin{aligned} \text{Wind stress:} & \quad D' = 1 \text{ dyn cm}^{-2} \\ \text{Entrainment:} & \quad S' = \rho_0 \times 5 \times 10^{-5} \text{ cm sec}^{-1} \\ \text{Energy input:} & \quad Z' = 5 \times 10^{-4} \text{ cal cm}^{-2} \text{ sec}^{-1} \end{aligned} \right\}, \quad (3.6)$$

values which may be compared, for example, with those of Hellerman (1967), Overstreet and Rattray (1969) and Sellers (1965). For the *a priori* given geometrical scale we find by inspection that

$$Y' = (D'/\beta S')^{1/2} \approx 3 \times 10^8 \text{ cm.} \quad (3.7)$$

Hence, the time scale, defined in (3.4), may be expressed by

$$T' = (S'/\beta D')^{1/2} \approx 0.2 \text{ day.} \quad (3.8)$$

As D'/S' has the dimension of a velocity of the order 10^4 cm sec^{-1} , Eqs. (3.7), (3.8) are in accordance with the scaling of Matsuno (1966) for equatorial latitudes. We shall interpret the expression (3.7) by rewriting it in the form $\beta S' Y' = D'/Y'$ which, anticipating Eq. (3.9), corresponds to the scaling of the Sverdrup transport in terms of the horizontal wind-stress gradient.

We require the forcing function of the mass transport equations (the wind stress) to be unity, i.e., $\epsilon = 1$; this results in

$$M' = (S'D'/\beta)^{1/2} = S'Y'. \quad (3.9)$$

Likewise, we require the pressure gradient term in Eq. (3.3) to be of order unity which stipulates equality of the aspect ratio δ and the parameter κ leading to

$$\rho_0 Y'^2 \beta S' = Q'E', \quad (3.10)$$

where equality of the two length scales a and Y' has been assumed. Eq. (3.10) relates the two scales Q' , E' to externally given quantities. In order to gain a second relation for Q' , E' , we assume that the time scales of the continuity equation (2.16) and the thermal energy equation (2.17) may be determined by the respective source functions σ , ζ , and we stipulate that these time scales are equal. This eliminates the time scale and yields

$$Q'/E' = S'/Z'. \quad (3.11)$$

Combining Eqs. (3.10) and (3.11), we find

$$Q' = \left(\frac{\rho_0 D'^2}{\beta Z'} \right)^{1/2}, \quad (3.12)$$

$$E' = \left(\frac{\rho_0 D'^2 Z'}{\beta S'^2} \right)^{1/2}. \quad (3.13)$$

We note that this scaling of Q' and E' is independent of the relation (2.7) between Q and E ; however, we are led back to (2.7) by inserting the values of Eq. (3.6) into (3.11). In so doing we have

$$E' = (Z'Q')/S' = Q' \times c_p \times 10\text{K.} \quad (3.14)$$

The order of the specific mass of the slab which actually determines the thickness of the slab is obtained from Eq. (3.12); thus,

$$Q' = \rho_0 H' \approx \rho_0 \times 10^4 \text{ cm.} \quad (3.15)$$

The order of the horizontal mass transport is

$$M' \approx \rho_0 \times 10^4 \text{ cm}^2 \text{ sec}^{-1}, \quad (3.16)$$

which corresponds to an average horizontal velocity of the order 1 cm sec^{-1} . Combining (3.4) for κ with Eqs. (2.7), (3.7) and (3.9), we find

$$\kappa = \delta \text{ Ro}^{-1} \text{ Fr, Fr} = (M'/Q')^2 / (g'H'), \quad (3.17)$$

where $g' = g\delta_1$ is the reduced gravity. The parameter ϵ should be understood as a Rossby number based on the wind stress (Pedlosky, 1969). However, it is not a free parameter in the present model.

The time scale T_1' of the continuity and thermal energy equations, typically much larger than T' , was assumed to be determined by

$$T_1' = Q'/S' = E'/Z' \approx 2 \times 10^8 \text{ sec.} \quad (3.18)$$

It might be interesting to note that this time constant of some 6-7 years seems to govern short-term climatic fluctuations of the tropical Hadley cell due to atmosphere-ocean interactions (Namias, 1969). We now see by combining Eqs. (3.4), (3.9) and (3.18) that

$$\text{Ro} = T'/T_1' \approx 10^{-4}. \quad (3.19)$$

Thus, the continuity and thermal energy equations take on the dimensionless forms (3.22) and (3.23), respectively. The nondimensional equations of the model are

$$\frac{\partial m'}{\partial t'} + \text{Ro} \frac{\partial}{\partial y'} \left(\frac{n'm'}{q'} \right) = y'n' + \tau' - \gamma m', \quad (3.20)$$

$$\frac{\partial n'}{\partial t'} + \text{Ro} \frac{\partial}{\partial y'} \left(\frac{n'n'}{q'} \right) = -y'm' - \frac{\partial}{\partial y'} (q'e'/2) - \gamma n', \quad (3.21)$$

$$\frac{\partial q'}{\partial t'} + \text{Ro} \frac{\partial}{\partial y'} n' = \text{Ro} \sigma', \quad (3.22)$$

$$\frac{\partial e'}{\partial t'} + \text{Ro} \frac{\partial}{\partial y'} \left(\frac{n'e'}{q'} \right) = \text{Ro} \zeta'. \quad (3.23)$$

Forcing functions are the wind stress τ' , the entrainment transport σ' , and the energy input ζ' . Parameters of the model are the Rossby number Ro and the friction number γ , the latter having the approximate magnitude 0.1.

We shall briefly relate the present model to Stommel's (1948) original approach of a nondivergent ocean. Let us consider the vorticity equation obtained from Eqs. (3.20) and (3.22) for the stationary state without advection but with entrainment; in this case, we have⁴

$$n' = \text{curl } \tau' - y'\sigma' - \gamma \text{curl } \mathbf{m}'. \quad (3.24)$$

⁴ In (3.24) curl denotes the two-dimensional operator which is a scalar.

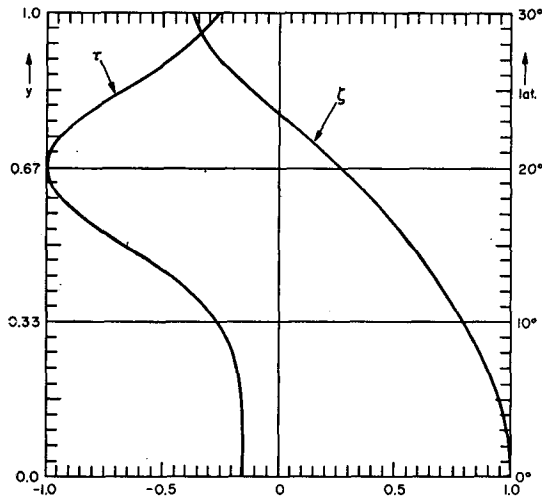


FIG. 2. Latitudinal profiles of analytical sea surface zonal wind stress $\tau'(y')$ and thermal energy input $\zeta'(y')$.

Here, $\tau' = (\tau', 0)$, $\mathbf{m}' = (m', n')$. In the case *without* zonal symmetry, which is gained by adding the pressure term $-\partial(q'e'/2)/\partial x'$ to the right-hand side of Eq. (3.20) and by replacing the stationary continuity equation (3.22) by

$$\frac{\partial m'}{\partial x'} + \frac{\partial n'}{\partial y'} = \sigma', \tag{3.25}$$

the vorticity equation (3.24) still holds. Eqs. (3.24) (3.25) with $\sigma' \equiv 0$ depict Stommel's model, which can be solved with the boundary conditions $m=0$ at the meridional and $n=0$ at the zonal walls. Eqs. (3.24) and (3.25) with $\sigma' \neq 0$ are a simple generalization of Stommel's model, in which the boundary conditions have to be modified in order to eliminate the mass increment caused by entrainment. This can be done, e.g., by requiring

$$\int_0^1 n'(x', 1) dx' = \int_0^1 \int_0^1 \sigma'(x', y') dx' dy',$$

provided curl τ' vanishes along the southern and northern boundaries.

For the model *with* zonal symmetry, however, the special case $\sigma' \equiv 0$ is trivial. Applying the condition of no meridional motion across the equator, i.e., $n'(0) = 0$, we find $n' \equiv 0$, $m' = \tau'/\gamma$. This describes a purely zonal current in which the wind stress is exactly balanced by interface friction.

We finally note that Fofonoff (1962, p. 338) briefly discusses the generalized continuity equation (3.25). However, he understands the mass source function σ as an input at the sea surface rather than from the abyss. Yoshida (1967), on the other hand, does employ (3.25) with entrainment dependent upon the layer thickness anomaly.

4. Analytic solution for the steady state

The special case of Eqs. (3.20)–(3.23) in which the motion is steady and the advective terms in the momentum equations vanish is described by

$$0 = y'n' + \tau'(y') - \gamma m', \tag{4.1}$$

$$0 = -y'm' - \frac{\partial}{\partial y'} \left(\frac{q'e'}{2} \right) - \gamma n', \tag{4.2}$$

$$\frac{\partial n'}{\partial y'} = \sigma'(y'), \tag{4.3}$$

$$\frac{\partial}{\partial y'} \left(\frac{n'e'}{q'} \right) = \zeta'(y'). \tag{4.4}$$

This set of equations is nonlinear. However, it can be

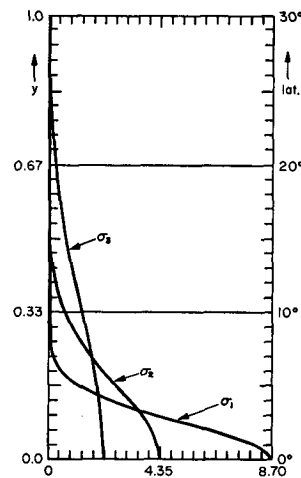


FIG. 3. Latitudinal profiles of assumed vertical entrainment $\sigma'(y')$ of cold water from lower layer into slab.

solved by simple integrations of the form

$$n'(y') = \int_0^{y'} \sigma'(y^*) dy^*, \tag{4.5}$$

$$m'(y') = \frac{1}{\gamma} [y'n'(y') + \tau'(y')], \tag{4.6}$$

$$q'(y') = \left\{ q_0 e_0 - 2 \int_0^{y'} [y^* m'(y^*) + \gamma n'(y^*)] dy^* \right\} n'(y') / \int_0^{y'} \zeta(y^*) dy^*, \tag{4.7}$$

$$e'(y') = \frac{q'(y')}{n'(y')} \int_0^{y'} \zeta'(y^*) dy^*, \tag{4.8}$$

with the boundary conditions

$$n'(0) = 0, \quad q'(0) = q_0, \quad e'(0) = e_0, \tag{4.9}$$

where q_0 and e_0 are not independent of one another. Both (4.7) and (4.8) lead to indeterminate expressions for $y=0$. However, employing l'Hospital's rule, we find

$$e_0' = \frac{\zeta'(0)}{\sigma'(0)} q_0'. \tag{4.10}$$

The argument y' varies from $y'=0$ (equator) to $y'=1$ (30° latitude). There is no singularity in the solution for q' as long as the latitudinal integral of the energy balance ζ' does not vanish (except for $y'=0$); this is valid for the observed energy balance (Sellers, 1965). Likewise, there is no singularity in the solution for e' as long as $n'(y')$ does not vanish (except for $y'=0$); this is also valid because we assume the entrainment function $\sigma'(y')$ to be maximum at the equator and nowhere negative.

An important aspect of the stationary model (4.1)–(4.4) is the fact that the forcing functions $\sigma'(y')$ and $\zeta'(y')$ are not entirely independent of one another. Assume, for example, that there is no entrainment at all, i.e., $\sigma'(y') \equiv 0$. This causes a pattern $n'(y') \equiv 0$ which would not be consistent with an arbitrary profile $\zeta'(y') \neq 0$. Conversely, if one prescribes an entrainment distribution $\sigma'(y') \neq 0$ with $n'(1) > 0$, it would not be consistent to set $\zeta'(y') \equiv 0$. Both cases may be formally verified by inspection of Eqs. (4.3) and (4.4). They can also be interpreted from obvious physical reasons.

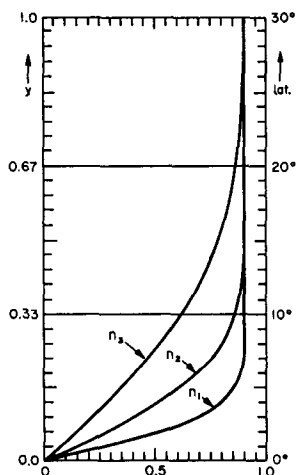


FIG. 4. Latitudinal profiles of north-south horizontal mass transport $n'(y')$ due to entrainment.

In the first case there is no horizontal mass transport across the poleward boundary. If, on the other hand, thermal energy is continuously fed into the slab, a steady-state temperature profile is not possible. In the second case the entrained water, though not directly affecting the thermal energy of the slab, determines the outflux across the lateral boundary; the poleward mass transport carries thermal energy with itself. Hence, in this case, too, no steady state

can develop. Obviously, the first case considered here is the trivial case mentioned in Section 3.

We shall now discuss three special examples of the solutions just obtained. For τ' and ζ' we choose analytical latitudinal profiles similar in shape to the observed distributions [cf. Hellerman (1967) for the stress and Sellers (1965) for the energy balance], i.e.,

$$\tau'(y') = -0.15 - 0.85 \exp[-2(3y'-2)^2], \tag{4.11}$$

$$\zeta'(y') = 0.5(1 + \cos 2.8y') - 0.4 \exp[-1.6(3y'-3)^2]. \tag{4.12}$$

Fig. 2 shows these two forcing functions.

No direct observations are available for the entrainment function. There is some evidence, however, that vertical mixing reaches maximum intensity in equatorial latitudes due to the equatorial undercurrent (Knauss, 1963). For this reason we shall investigate the response of the steady model to three different σ' distributions which are Gaussian functions centered on the equator. We use the form

$$\sigma'(y') = \sigma_0 \exp\left[-0.69\left(\frac{y}{y_3}\right)^2\right], \tag{4.13}$$

where the half-widths of these functions are $y_3 = 3^\circ, 6^\circ$ and 12° latitude; we will refer to them as σ_1', σ_2' and σ_3' , respectively (Fig. 3). Appreciable values of σ_1' are restricted to the immediate vicinity of the equatorial undercurrent region, whereas σ_3' exhibits a much broader distributed, though a weaker, influence of entrainment. The σ' functions are normalized in order to always produce the same latitudinal mass transport $n'(1)$ across the poleward boundary (Fig. 4). We note again that the function n' is not influenced by the wind stress [Eq. (4.5)]. Therefore, n' has nothing to do with the meridional Sverdrup transport.

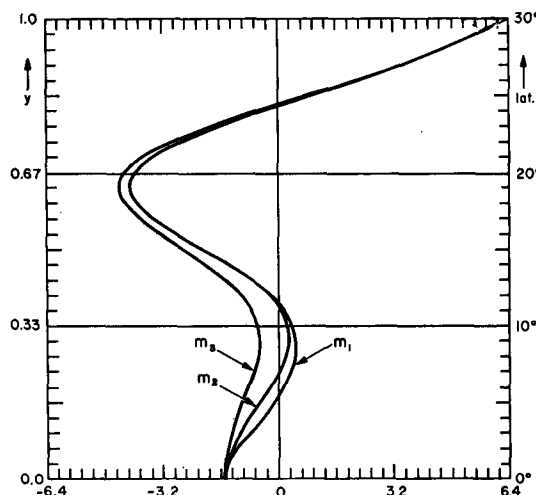


FIG. 5. Latitudinal profiles of east-west horizontal mass transport $m'(y')$ due to wind stress and entrainment.

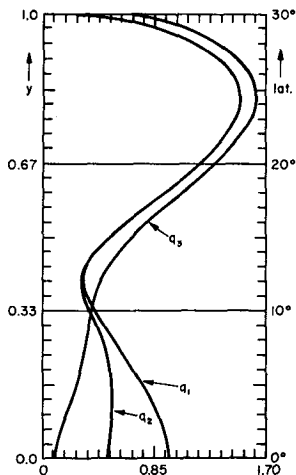


FIG. 6. Latitudinal profiles of specific mass $q'(y')$ of slab, proportional to depth of slab.

Fig. 5 shows the zonal mass transport for the three cases. The striking fact is the tendency of the model to develop an equatorial countercurrent, embedded within equatorial currents. This is due to entrainment forcing as can be seen by inspection of Eqs. (4.5) and (4.6), but is nearly independent of the particular entrainment profile. We conclude, therefore, that the entrainment concept, together with the wind stress, is able to produce a fairly realistic latitudinal pattern of the zonal mass transport in the ocean surface layer. Since the present model is symmetric with respect to the equator, there should be a tendency of the actual ocean to generate an equatorial countercurrent in the Southern Hemisphere as well. There is, in fact, evidence for a Southern Hemispheric countercurrent in the Pacific Ocean (Reid, 1961) and in the Atlantic Ocean (Reid, 1964).

Fig. 6 shows for the three cases the distribution of specific mass which is proportional to the depth of the slab. The observed depth distribution of the mixed

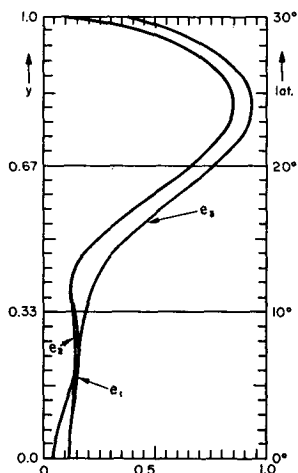


FIG. 7. Latitudinal profiles of specific enthalpy $e'(y')$ of slab.

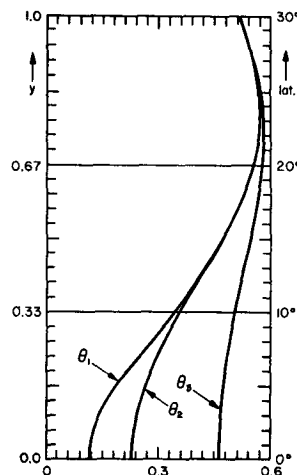


FIG. 8. Latitudinal profiles of $e'(y')/q'(y')$. This ratio is proportional to the temperature difference ϑ between slab and lower layer.

layer in the northern Pacific Ocean is characterized by a pronounced equatorial minimum and a maximum at 5° latitude; in the subtropics strong seasonal fluctuations prevail (Wyrski, 1964). As a rule, the mixed layer is shallow in the inner tropics and deep in the subtropics (Defant, 1961, p. 120). The depth profiles of Fig. 6 are in accordance with this general picture.

In Fig. 7 the thermal energy, or the specific enthalpy of the slab with respect to the lower layer, is plotted as a function of latitude. The patterns are similar to the depth profiles. Evidence for low heat content of the slab in the equatorial region seems to be supported by satellite observations which show a relatively cloud-free equator (Kornfield *et al.*, 1967). The latter statement can also be seen to follow from Fig. 8 which shows the ratio $e'(y')/q'(y')$. Combining Eqs. (4.5) and (4.8) yields

$$\frac{e'(y')}{q'(y')} = \int_0^{y'} \zeta'(y^*) dy^* / \int_0^{y'} \sigma'(y^*) dy^* \propto \vartheta_1. \quad (4.14)$$

The temperature difference between the slab and the lower layer is thus governed by the thermal heat input at the sea surface and the entrainment function; Fig. 8 exhibits a relatively cool equator and relatively warm subtropics.

5. Conclusions

This study has shown that an extremely simple model of an ocean surface slab gives a plausible description of some of the gross features of the oceanic surface circulation. In particular, the modelling of the equatorial countercurrent as a response to the combined action of wind stress and entrainment of cold water from below, as well as the depth distribution of the mixed layer and its latitudinal temperature pattern, may warrant interest. It seems that the concept of

entrainment velocity, first recognized by Rouse and Dodu (1955) in the laboratory, and further investigated and applied to oceanic conditions by Kraus and Turner (1967), and Yoshida (1967) is applicable to large-scale processes in the ocean. The present approach is different from thermocline methods. In models like the one-dimensional model by Overstreet and Rattray (1969), or the three-dimensional model of Niiler and Dubbeldam (1970), the heat flux across the sea surface goes directly through the mixed layer to supply the heat flow required to maintain the thermocline. In the present model the entrainment function, instead of being determined by the divergence of the Ekman layer, is a mixing velocity; it is governed by the heat flux across the sea surface and the generation of turbulent kinetic energy due to wind stress and current shear. Keeping in mind the fairly low order of magnitude of the horizontal velocity (1 cm sec^{-1}), the response of this model should not be thought of as describing the actual surface circulation of the ocean very closely but rather as a significant baroclinic contribution to it.

If we assume that the temperature of the mixed layer is generally a smooth function of latitude, it follows from Eq. (4.14) that σ' and ζ' must exhibit patterns that are not too different. If this conclusion is correct, it provides a means of estimating the entrainment function σ for which measurements are not yet available. It also shows the close correspondence between dynamic and energetic processes in the upper ocean in which most of the sun's energy driving both oceanic and atmospheric motions is processed.

The latter viewpoint may have some implications for air-sea interaction models. There the driving forces of the present model are, in fact, interacting functions. In order to incorporate this, we can parameterize 1) the momentum exchange τ between air and water by the surface wind speed, 2) the entrainment function σ in terms of the turbulent kinetic energy input and of the heat balance at the sea surface (Kraus and Turner, 1967), and 3) the net energy input ζ into the ocean in terms of wind stress, sea surface temperature, and some atmospheric surface variables (see, e.g., Kraus and Rooth, 1961). According to these possible parameterizations, the present model can be extended to an hierarchy of models of increasing complexity.

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APPENDIX

List of Symbols

a	length parameter, $c_p/(g\alpha)$
c_p	specific heat per unit mass of water
e'	nondimensional specific enthalpy
g	earth's acceleration
g'	reduced earth's acceleration
h_0	level of interface ("thermocline") relative to reference level
h_1	thickness of slab
i	index of interface between slab and lower layer
k	linear friction constant
m'	nondimensional zonal mass transport
\mathbf{m}'	nondimensional horizontal mass-transport vector
n'	nondimensional meridional mass transport
p	pressure
q'	nondimensional specific mass
s	index of sea surface
t	time
t'	nondimensional time
u	zonal velocity component
v	meridional velocity component
w^*	entrainment velocity
x	zonal coordinate, positive eastward
y	meridional coordinate, positive northward
y'	nondimensional meridional coordinate
y^*	meridional integration variable
z	vertical coordinate, positive upward
D'	scale of zonal wind stress
E	specific enthalpy of slab
E'	scale of specific enthalpy
Fr	Froude number
H'	scale of thickness of slab
M	zonal mass transport
M'	scale of horizontal mass transport
N	meridional mass transport
P	vertically integrated meridional pressure gradient
Q	specific mass of slab
Q'	scale of specific mass
Ro	Rossby number
S'	scale of entrainment mass flux
T'	time scale of mass-transport equations
T_1'	time scale of continuity and enthalpy equations

Y'	horizontal scale
Z'	scale of thermal energy input
α	thermal expansion coefficient
β	meridional derivative of Coriolis parameter
γ	friction number
δ	aspect ratio
ϵ	Rossby number based on wind stress
ζ	thermal energy input
ζ'	nondimensional thermal energy input
ϑ	nondimensional temperature
κ	nondimensional parameter
ρ	density
$\Delta\rho$	density difference between slab and lower layer
σ	entrainment mass flux
σ'	nondimensional entrainment mass flux
τ	zonal wind stress plus integrated zonal pressure gradient
τ'	nondimensional zonal wind stress
τ'	nondimensional horizontal wind stress vector
τ_{xi}, τ_{yi}	stress components at interface ("thermocline") level
Θ	actual temperature

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