Reply

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As McKee (1971) points out, the stationary Rossby wake problem investigated by White (1971) is directly analogous to that of lee gravity waves in a stratified fluid solved by Miles (1968); however, the two methods of solution, that of White and that of Miles, are fundamentally different. To obtain a determinate solution for the perturbation streamfunction ψ_1, Miles first constructs a set of lee wave functions that satisfy the upstream boundary condition

$$\rho^{\frac{1}{2}}\psi_1 \rightarrow 0 \quad \text{for} \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}, \quad \text{as} \quad \rho \rightarrow \infty. \tag{1}$$

He completes the solution by requiring this new set of functions to satisfy the boundary condition at the cylinder wall. In contrast to this approach, White begins by constructing a set of functions that identically satisfy the island boundary condition. He then obtains a truncated solution by application of the less stringent upstream boundary condition

$$\rho^{\frac{1}{2}}\partial\psi_1 \;\; \partial\theta \rightarrow 0 \quad \text{for} \quad \theta = \pi, \quad \text{as} \quad \rho \rightarrow \infty. \tag{2}$$

In practical application Miles solution is also approximated by truncation of the series solutions. Miles' truncated solution exactly satisfies the upstream boundary condition (1) but not the island condition; White's solution exactly satisfies the latter, but not (1).

In a qualitative sense, one can see that the upstream boundary condition of Miles is approximated by that of White by considering the Taylor expansion

$$\psi_1(\infty,\theta) = \psi_1(\infty,\pi) + \left.\frac{\partial\psi_1}{\partial\theta}\right)_{\substack{\theta=\pi\\ \rho=\infty}} (\theta-\pi) + \cdots + \left.\frac{\partial^n\psi_1}{\partial\theta^n}\right)_{\substack{\theta=n\\ \rho=\infty}} \frac{(\theta-\pi)^n}{n!} \cdots. \tag{3}$$

White requires the first three terms of the right side of (3) to be zero and neglects the higher order terms in the expansion. The result is to approximately set the perturbation solution, $\psi_1(\infty,\theta)$, to zero in the region $-\pi/2 \leqslant \theta \leqslant \pi/2$, thereby approximating the Miles upstream boundary condition. The degree of accuracy of this approximation is unknown, but had higher order terms of the Taylor expansion (3) been retained, the perturbation solution, $\psi_1(\infty,\theta)$, would have been set to zero a farther distance along the azimuth from $\theta = \pi$. However, even if one were to choose a sufficient number of terms in (3) to satisfy the Miles upstream boundary condition to within some preselected small error, the two solutions would still be different because the White solution would also be suppressed in the region downstream from the island except for a small range of θ near zero.

McKee states that the *correct* upstream boundary condition should be

$$\rho^{\frac{1}{2}}\partial\psi_1/\partial\theta \rightarrow 0 \quad \text{for} \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}, \quad \text{as} \quad \rho \rightarrow \infty. \tag{4}$$

As can be seen, this condition is different from that of Miles given in (1). Had Miles applied the upstream condition to $\partial\psi_1/\partial\theta$ rather than ψ_1, an entirely different set of lee wave functions would have been obtained. In particular, half of these will possess a discontinuity in $\partial\psi_1/\partial\theta$ (i.e., radial velocity) at $\theta = \pi/2$ in the far field, while the associated ψ_1 will be continuous. On the other hand, half of the solutions satisfying (1) have discontinuities in ψ_1 at $\theta = \pi/2$ in the far field. One would accordingly expect a different solution to the problem if (4) is employed. Therefore, although the Miles problem as posed is mathematically unique, the question now arises in respect to physical uniqueness of the problem since the choice of the quantity to be suppressed in the upstream far field is still an arbitrary aspect.

In light of McKee's second point, the problem of instability in the White solution has been reviewed, and from more recent work the Kuo instability is found to occur whenever Is > 1.45, where Is is the Island number $[(\beta/u)^{\frac{1}{2}}/a]$ given by White. Therefore, as McKee points out the circulation patterns in Figs. 3b and 3c in White, where Is = 2.0 and 3.0, respectively, are unstable and violate a basic assumption of the model, i.e., that there be no closed streamlines in the circulation pattern. A similar instability is found in the Miles lee gravity wave problem, where the analogous instability occurs whenever $Na/u > 1.27$ (N being the Väisälä frequency). To see if the difference in the two instability criteria is due to the first-order approximation in (3) made by White, the problem has since been solved by setting the third-order term in (3) to zero. This requires the evaluation of the E_3 and E_4 terms in the general solution (21) given by White. The resulting streamfunction patterns are only slightly different from those given by White and the instability criterion is lowered only about 3% to Is > 1.40. It would appear from this that the difference between the Miles and White instability criteria is due chiefly to the difference in the form of the two solutions.

To see how well the White solution matches the Miles solution near the point of instability, the streamfunction pattern in the vicinity of the cylindrical island for Is = 1.40 is given in Fig. 1 and can be compared with the figure showing the stratified shear flow over a semicircular obstacle for $Na/u = 1.27$, calculated in the appendix of Miles (1968) by Huppert. Upon examination of the two figures the flow patterns appear very much alike, including the way in which the perturbation streamfunction is suppressed upstream from the cylindrical obstacle. This suggests that the White (1971) solution gives nearly the same circulation pattern as the approximate Miles (1968) solution used by Huppert, who truncated the set of lee wave functions after the first two terms.

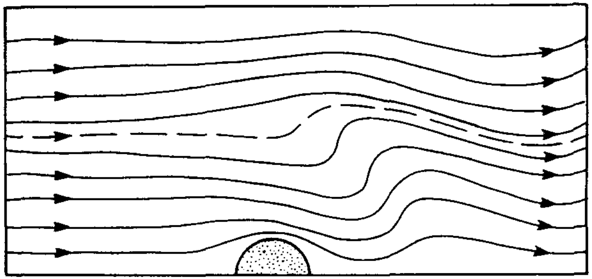

FIG. 1. Rossby wake flow pattern for Is = 1.40.

In summary, the Miles solution is the mathematically unique solution, although there is some doubt as to its physical uniqueness. This solution is approximated by the White solution, which the author feels is much simpler, yet which can lead to physically satisfying results that are comparable with those of the truncated Miles solution.

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