

Reply

WARREN B. WHITE

*Scripps Institution of Oceanography, La Jolla, Calif.*

20 September 1971

As McKee (1971) points out, the stationary Rossby wake problem investigated by White (1971) is directly analogous to that of lee gravity waves in a stratified fluid solved by Miles (1968); however, the two methods of solution, that of White and that of Miles, are fundamentally different. To obtain a determinate solution for the perturbation streamfunction  $\psi_1$ , Miles first constructs a set of lee wave functions that satisfy the upstream boundary condition

$$\rho^{1/2}\psi_1 \rightarrow 0 \text{ for } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \text{ as } \rho \rightarrow \infty. \quad (1)$$

He completes the solution by requiring this new set of functions to satisfy the boundary condition at the cylinder wall. In contrast to this approach, White begins by constructing a set of functions that identically satisfy the island boundary condition. He then obtains a truncated solution by application of the less stringent upstream boundary condition

$$\rho^{1/2}\partial\psi_1/\partial\theta \rightarrow 0 \text{ for } \theta = \pi, \text{ as } \rho \rightarrow \infty. \quad (2)$$

In practical application Miles solution is also approximated by truncation of the series solutions. Miles' truncated solution exactly satisfies the upstream boundary condition (1) but not the island condition; White's solution exactly satisfies the latter, but not (1).

In a qualitative sense, one can see that the upstream boundary condition of Miles is approximated by that of White by considering the Taylor expansion

$$\begin{aligned} \psi_1(\infty, \theta) &= \psi_1(\infty, \pi) + \frac{\partial\psi_1}{\partial\theta}(\theta - \pi) + \dots + \frac{\partial^n\psi_1}{\partial\theta^n}(\theta - \pi)^n \dots \quad (3) \\ \theta = \pi & \qquad \qquad \qquad \theta = \pi \\ \rho = \infty & \qquad \qquad \qquad \rho = \infty \end{aligned}$$

White requires the first three terms of the right side of (3) to be zero and neglects the higher order terms in the expansion. The result is to approximately set the perturbation solution,  $\psi_1(\infty, \theta)$ , to zero in the region  $-\pi/2 \leq \theta \leq \pi/2$ , thereby approximating the Miles upstream boundary condition. The degree of accuracy of this approximation is unknown, but had higher order terms of the Taylor expansion (3) been retained, the

perturbation solution,  $\psi_1(\infty, \theta)$ , would have been set to zero a farther distance along the azimuth from  $\theta = \pi$ . However, even if one were to choose a sufficient number of terms in (3) to satisfy the Miles upstream boundary condition to within some preselected small error, the two solutions would still be different because the White solution would also be suppressed in the region downstream from the island except for a small range of  $\theta$  near zero.

McKee states that the *correct* upstream boundary condition should be

$$\rho^{1/2}\partial\psi_1/\partial\theta \rightarrow 0 \text{ for } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \text{ as } \rho \rightarrow \infty. \quad (4)$$

As can be seen, this condition is different from that of Miles given in (1). Had Miles applied the upstream condition to  $\partial\psi_1/\partial\theta$  rather than  $\psi_1$ , an entirely different set of lee wave functions would have been obtained. In particular, half of these will possess a discontinuity in  $\partial\psi_1/\partial\theta$  (i.e., radial velocity) at  $\theta = \pi/2$  in the far field, while the associated  $\psi_1$  will be continuous. On the other hand, half of the solutions satisfying (1) have discontinuities in  $\psi_1$  at  $\theta = \pi/2$  in the far field. One would accordingly expect a different solution to the problem if (4) is employed. Therefore, although the Miles problem as posed is mathematically unique, the question now arises in respect to physical uniqueness of the problem since the choice of the quantity to be suppressed in the upstream far field is still an arbitrary aspect.

In light of McKee's second point, the problem of instability in the White solution has been reviewed, and from more recent work the Kuo instability is found to occur whenever  $Is > 1.45$ , where  $Is$  is the Island number  $[(\beta/u)^{1/2}/a]$  given by White. Therefore, as McKee points out the circulation patterns in Figs. 3b and 3c in White, where  $Is = 2.0$  and  $3.0$ , respectively, are unstable and violate a basic assumption of the model, i.e., that there be no closed streamlines in the circulation pattern. A similar instability is found in the Miles lee gravity wave problem, where the analogous instability occurs whenever  $N\alpha/u > 1.27$  ( $N$  being the Väisälä frequency). To see if the difference in the two instability criteria is due to the first-order approximation in (3) made by White, the problem has since been solved by setting the third-order term in (3) to zero. This requires the

evaluation of the  $E_3$  and  $E_4$  terms in the general solution (21) given by White. The resulting streamfunction patterns are only slightly different from those given by White and the instability criterion is lowered only about 3% to  $Is > 1.40$ . It would appear from this that the difference between the Miles and White instability criteria is due chiefly to the difference in the form of the two solutions.

To see how well the White solution matches the Miles solution near the point of instability, the streamfunction pattern in the vicinity of the cylindrical island for  $Is = 1.40$  is given in Fig. 1 and can be compared with the figure showing the stratified shear flow over a semicircular obstacle for  $Na/u = 1.27$ , calculated in the appendix of Miles (1968) by Huppert. Upon examination of the two figures the flow patterns appear very much alike, including the way in which the perturbation streamfunction is suppressed upstream from the cylindrical obstacle. This suggests that the White (1971) solution gives nearly the same circulation pattern as the approximate Miles (1968) solution used by Huppert, who truncated the set of lee wave functions after the first two terms.

In summary, the Miles solution is the mathematically unique solution, although there is some doubt as to its

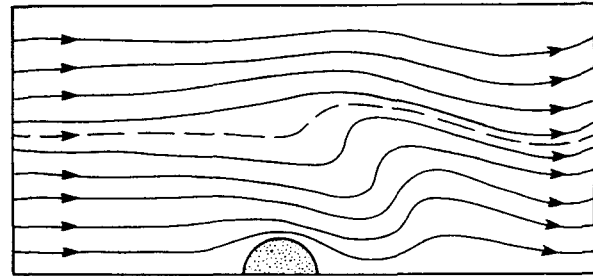


FIG. 1. Rossby wake flow pattern for  $Is = 1.40$ .

physical uniqueness. This solution is approximated by the White solution, which the author feels is much simpler, yet which can lead to physically satisfying results that are comparable with those of the truncated Miles solution.

#### REFERENCES

- Miles, J. W. 1968: Lee waves in a stratified flow, Part 2. Semicircular obstacle. *J. Fluid Mech.*, **33**, 803-814.  
 McKee, W. D., 1971: Comments on "A Rossby wake due to an island in an eastward current." *J. Phys. Oceanogr.*, **1**, 287.  
 White, W. B., 1971: A Rossby wake due to an island in an eastward current. *J. Phys. Oceanogr.*, **1**, 161-168.

## INDEX

No. 1. January.....	1- 64	No. 3. July.....	149-240
No. 2. April.....	65-148	No. 4. October.....	241-296

## SECTION A. Index to Contributions by Authors

Titles are in italics. Notes and Correspondence are indicated by asterisks.

- ALLISON, LEWIS J. (with Warnecke, McMillin and Szekiolda), *Remote sensing of ocean currents and sea surface temperature changes derived from the Nimbus II satellite*, 45-60
- BARCELON, VICTOR, *A simple model of the thermocline in a bounded ocean*, 7-11
- BERBERIAN, G. A. (with Frank Chew), *A determination of horizontal divergence in the Gulf Stream off Cape Lookout*, 39-44
- CALDWELL, DOUGLAS R. (with William P. Elliott), *Surface stresses produced by rainfall*, 145-148
- CHARNEY, JULE G. (with Stanley L. Spiegel), *Structure of wind-driven equatorial currents in homogeneous oceans*, 149-160
- CHEW, FRANK (with G. A. Berberian), *A determination of horizontal divergence in the Gulf Stream off Cape Lookout*, 39-44
- CSANADY, G. T., *Baroclinic boundary currents and long edge-waves in basins with sloping shores*, 92-104
- *On the equilibrium shape of the thermocline in a shore zone*, 263-270
- (with C. R. Murthy), *Experimental studies of relative diffusion in Lake Huron*, 17-24
- DÜING, WALTER (with Claes Rooth), *On the detection of "inertial" waves with pycnocline followers*, 12-16
- ECHTERNACHT, K. L. (with Warsh and Garstang), *Structure of near-surface currents east of Barbados*, 123-129
- ELLIOTT, WILLIAM P. (with Douglas R. Caldwell), *Surface stresses produced by rainfall*, 145-148
- GALT, J. A., *A numerical investigation of pressure-induced storm surges over the continental shelf*, 82-91
- GARRETT CHRISTOPHER (with Walter Munk), *Internal wave spectra in the presence of fine-structure*, 196-202
- GARSTANG, M. (with Warsh and Echternacht), *Structure of near-surface currents east of Barbados*, 123-129
- GARVINE, RICHARD W., *A simple model of coastal upwelling dynamics*, 169-179
- GROVES, GORDON W. (with Motoyasu Miyata), *A study of the effects of local and distant weather on sea level in Hawaii*, 203-213
- HANEY, ROBERT L., *Surface thermal boundary condition for ocean circulation models*, 241-248
- HANTEL, MICHAEL, *The entrainment influence on the ocean surface layer in tropical latitudes*, 130-138
- HART, J. E., *A possible mechanism for boundary layer: Mixing and layer formation in a stratified fluid*, 258-262
- HSUEH, Y. (with James J. O'Brien), *Steady coastal upwelling induced by an along-shore current*, 180-186
- HUANG, JOSEPH CHI KAN, *The thermal current in Lake Michigan*, 105-122
- JONES, IAN S. F. (with B. C. Kenney), *Relative diffusion as related to quasi-periodic current structures*, 224-232
- KENNEY, B. C. (with Ian S. F. Jones), *Relative diffusion as related to quasi-periodic current structures*, 224-232
- KOFFLER, R. (with Rao and Strong), *\*Gulf stream meanders and eddies as seen in satellite infrared imagery*, 237-239
- LANDIS, ROBERT C., *Early BOMEX results of sea surface salinity and Amazon river water*, 278-281
- LARUE, R. (with Niiler and Simco), *A two-layer model of the North Atlantic thermocline*, 271-277
- LEVANON, NADAV, *Determination of the sea surface slope distribution and wind velocity using sun glitter viewed from a synchronous satellite*, 214-220
- LINDBERG, WILLIAM R., *An upper bound on transport processes in turbulent thermohaline convection*, 187-195
- LIU, PAUL C., *Normalized and equilibrium spectra of wind waves in Lake Michigan*, 249-257
- McKee, W. D., *\*Comments on "A Rossby wake due to an island in an eastward current,"* 287
- McMILLIN, LARRY M. (with Warnecke, Allison and Szekiolda), *Remote sensing of ocean currents and sea surface temperature changes derived from the Nimbus II satellite*, 45-60
- MİYATA, MOTOYASU (with Gordon W. Groves), *A study of the effects of local and distant weather on sea level in Hawaii*, 203-213
- MONAHAN, EDWARD C., *Oceanic whitecaps*, 139-144
- MUNK, WALTER (with Christopher Garrett), *Internal wave spectra in the presence of fine-structure*, 196-202
- MURTHY, C. R. (with G. T. Csanady), *Experimental studies of relative diffusion in Lake Huron*, 17-24
- NAMIAS, JEROME, *The 1968-69 winter as an outgrowth of sea and air coupling during antecedent seasons*, 65-81
- NIILER, P. P. (with Simco and Larue), *A two-layer model of the North Atlantic thermocline*, 271-277
- O'BRIEN, JAMES J. (with Y. Hsueh), *Steady coastal upwelling induced by an along-shore current*, 180-186
- PHILLIPS, O. M., *On spectra measured in an undulating layered medium*, 1-6
- RAO, P. KRISHNA (with Strong and Koffler), *\*Gulf stream meanders and eddies as seen in satellite infrared imagery*, 237-239
- REED, R. K., *\*An observation of divergence in the Alaskan Stream*, 282-283
- REID, R. O., *\*A special case of Phillips' general theory of sampling statistics for a layered medium*, 61-62
- RODEN, GUNNAR I., *Spectra of North Pacific temperature and salinity perturbations in the depth domain*, 25-33
- ROOTH, CLAES (with Walter Düing), *On the detection of "inertial" waves with pycnocline followers*, 12-16
- SCHOOLEY, ALLEN H., *Diffusion sublayer thickness over wind-disturbed water surfaces*, 221-223
- SIMCO, E. (with Larue and Niiler), *A two-layer model of the North Atlantic thermocline*, 271-277