Some Observations of Baroclinic Diurnal Tides over a Near-Critical Bottom Slope

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ABSTRACT

The time-depth structure of the baroclinic diurnal tide has been examined with the aid of current and temperature profiles on the West Florida Continental Shelf. Of interest is the fact that the diurnal frequencies (e.g., the K1 and O1 tides) are near the "critical frequency" corresponding to the bottom slope and density stratification at the experimental location.

The baroclinic semiannual tide was rather weak and most of the semidiurnal tidal energy was contained in the barotropic currents. This large ratio of barotropic-to-baroclinic, semiannual tidal energy is in agreement with the results obtained by Koblinsky (1979) from previous (current meter) measurements in the same area.

In contrast, the baroclinic diurnal tide is quite strong and exhibits appreciable structural variations with time. The diurnal oscillations are predominantly of low vertical modal order, and there is no evidence of the concentrated "beams" of internal tidal energy which have sometimes been observed in other areas (e.g., Torgrumin and Hickey, 1979). However, the diurnal structure is modulated in a fashion which seems to be more complicated than can be accounted for by a simple "beating" effect between the K1 and O1 constituents. This relatively rapid modulation in amplitude and vertical structure indicates that there was present a significant transient component in either the generation or propagation of the internal diurnal tide. It is shown that variations in the vertical shear of low-frequency currents which occurred were in the correct sense and were potentially of sufficient amplitude to produce a subcritical bottom slope for the diurnal constituents during one period of the experiment. In this same period, there is clear evidence of near-bottom intensification of the diurnal oscillations. The data also show that the internal diurnal oscillations are propagating up-slope, away from the shelf break.

1. Introduction

In recent years, theoretical concepts of the generation, propagation and dissipation of internal tides on the continental shelf have probably developed more rapidly than has the observational base on which these concepts can be examined. This is due in part to the specialized nature of many of these theories and also, undoubtedly, to the fact that few shelf experiments have been designed specifically to look at the tidal problem.

The gap between theory and observations is unfortunate in light of the oft-expressed view that shelf internal tides have properties which can be of importance in other aspects of shelf dynamics. For example, these tides often contribute an appreciable fraction of the energy observed at a point on the shelf and are capable of propagating this energy to locations far removed from their point of origin (often assumed to be the shelf break). If this energy is then lost from the wave field by some process such as internal breaking nearer the shoreline, then these waves could constitute an important path by which energy could be carried across the shelf and converted to other forms, such as that dissipated by mixing.

Given the complex nature of the problem it is not surprising that a "unified" theory of internal shelf tides does not exist, particularly since the combined effects of mean-flow interaction and topography can influence the internal tides in a complicated manner. Some theoretical work has been directed primarily to describing the propagation of internal waves (with tides as a special case) in the presence of a geostrophic current (Healey and LeBlond, 1969; Mooers, 1975a,b). Another important area of theoretical work can be classed under the heading of "the generation and propagation of internal tides under the influence of topography." This group includes the generation of internal tides by a barotropic (surface) tide passing over a depth discontinuity such as the shelf break (Prinsenberg et al., 1974; Rattray et al., 1969; Rattray, 1960), generation and propagation over fairly generalized topography (Baines, 1973; Sandstrom, 1976), and the propagation of internal waves in a wedge (Wunsch, 1968, 1969; Mooers, 1972).

Observationally, recent studies of internal tides on
the shelf are not especially numerous. We can refer to the study of the internal wave band during a coastal upwelling event off Oregon by Hayes and Halpern (1976), the analysis by Koblnsky (1979) of tides on the West Florida Continental Shelf, and the profiles of Johnson et al. (1976) off Oregon as examples. Cases in which the detailed vertical structure of internal tides can be resolved (e.g., Johnson et al., 1976) are even less common, although a recent study of barotropic and baroclinic tides, again off the coast of Oregon (Torgrimson and Hickey, 1979), was reasonably successful in this regard.

Koblnsky (1979), in an analysis of current meter records from this same general area, found that the depth-dependent diurnal $K_1$ currents were characterized by significant energy in high vertical mode numbers, and larger current magnitudes near the surface than near the bottom. Also, coherence between current meters in the depth-dependent diurnal tide was found to be intermittent. In contrast, the semidiurnal $M_2$ tidal currents were found to be predominantly depth-independent.

The purpose here is to examine a set of current and temperature data, obtained on the West Florida Continental Shelf, in which the vertical structure of the internal tide can be clearly observed. These data were obtained in July 1975 by CTD/current profilers, or Cyclesones (Van Leer, 1974), at the location shown in Fig. 1 in a water depth of ~103 m. Data were sampled densely (every 5 m) in the vertical. The overall duration of the experiment was about 11 days. In summer the area is strongly stratified. STD casts made at the mooring site during this experiment showed fairly uniform vertical stratification with stability periods ~ 6-8 min.

The bottom topography in this area is approximately "wedge shaped" from the shelf break at ~220 m depth to a point somewhat east of the experiment site, with a slope of $2 \times 10^{-3}$ (±15%). Farther east the topography levels off and finally blends in with the amorphous coastline of the Everglades. This bottom slope is of particular interest in view of the "wedge solutions" obtained by Wunsch (1975). In these solutions the vertical structure of internal waves is shown to depend on the bottom slope ($\gamma_0$), and the slope of the wave characteristics, $\gamma_c$, the latter being given by

$$\gamma_c = \pm \left( \frac{\omega^2 - f^2}{N^2 - \omega^2} \right)^{1/2},$$

where $\omega$ is the wave frequency, $f$ the Coriolis parameter and $N$ the stability frequency. Two cases can be identified: 1) $\gamma_c > \gamma_0$, or "subcritical" propagation; and 2) $\gamma_c < \gamma_0$, or "supercritical" propagation. The properties of horizontal energy propagation across the wedge are substantially different in these two cases, and in addition the vertical modal structure is quite sensitive to values of $\gamma_c$ when $\gamma_c = \gamma_0$. It is known from nearby coastal data that $K_1$, $O_1$ and $M_2$ are the major tidal constituents in this area. At Naples, Florida, the amplitudes and local phases ($H, \omega$) are $K_1$ (0.150 m, 286.3°); $O_1$ (0.133 m, 280.3°); $M_2$ (0.273 m, 339.5°). The speeds (periods) in degrees/mean solar hour (hours) are 15.041 (23.93), 13.943 (25.82) and 28.984 (12.42), respectively. If the

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**Fig. 1.** Bathymetry of the experiment site. The location of the Cyclesones is shown as a solid circle.
$K_1$ and $O_1$ frequencies are used in (1), together with typical values of $N$ (stability period $\approx 6$–8 min) and $f$ (inertial period $= 27.37 \text{ h}$) it is found that $\gamma_e(K_1) = (2.0$–$2.7) \times 10^{-3}$ and $\gamma_e(O_1) = (1.3$–$1.7) \times 10^{-3}$. The sensitivity of $\gamma_e$ to $N$ makes it somewhat difficult to state precisely whether either constituent is supercritical or subcritical. However, it is clear that the bottom slope must be close to critical for both constituents, and this fact provides the motivation in what follows to look at the baroclinic diurnal tide in greater detail.

2. Data

Fig. 2 shows contours of the $U$ (positive eastward) and $V$ (positive northward) horizontal velocity components and temperature from 20 to 95 m depth as obtained by the profilers. Time is in hours referred to the beginning of the year. Three Cyclesondes were used, but the records obtained from the individual instruments did not exceed about four days’ duration. Fortunately, the three instruments were located close together (within 0.5 km) and the individual records overlapped so that a single record of about 11 days’ duration could be synthesized. For one small segment (60 to 95 m from 4535 to 4554 hours), data from two nearby Aanderaa current meters were used to bridge a gap in the profiler data.

The profilers completed a round trip (bottom-top-bottom) within 30 min, and round trips were repeated every hour. The vertical sampling rate ranged from 3 to 5 m. To remove a systematic bias observed between velocities obtained by ascending and descending profiles (the result of an error in the assumed velocity calibration (Van Leer and Leaman, 1978)), adjacent up and down profiles were averaged after interpolating to 5 m depth increments. The resulting data set consists of 270 profiles at 5 m intervals from 20 to 95 m.

Although Fig. 2 represents eight separate data records obtained from three different profilers, the calibration in velocity was sufficiently accurate that records could be combined without difficulty. Due to calibration problems with temperature between profilers, the estimated uncertainty in temperature is $\pm 0.1^\circ\text{C}$. Conductivity was measured but has not been used owing to large uncertainties in accuracy. STD casts during the experiment show little variation of salinity with depth, so density is mainly determined by temperature. STD data were obtained only from a small area around the mooring location.

3. Extraction of the diurnal signal

There is probably no one best way to separate and focus on the diurnal signal. Although standard time-series techniques could be used (computing the spectra of $U$, $V$ and $T$ at each level and the cross-spectra between levels, etc.), the large volume of computations involved and the fact that such an approach reduces our ability to observe visually what is going on through contour plots make this method unattractive. The approach adopted instead has been to isolate the diurnal signal through the use of various temporal filtering techniques applied to the data at each depth. This approach has the advantage that no assumptions about the vertical structure of the diurnal tide are made. Based on the previous discussion, any approach to the analysis which made $a$ priori assumptions about the vertical structure (in the form of an assumed modal structure, for example) would be naive, at best.

Fig. 2 clearly shows the presence of both low-frequency and tidal-period motions. To obtain a clearer picture of the tidal oscillations, a low-pass Lanczos filter ($-6 \text{ dB at 40 h, \ -20 dB at 30 h}$) was used on the profiler time series ($U$, $V$, $T$) at each depth to separate these two frequency bands. The result of this separation is shown in the low-passed and high-passed contours in Figs. 3 and 4.

The low-passed data show generally weak ($< 10 \text{ cm s}^{-1}$) onshore-offshore flow throughout the record. In contrast, a strong burst of alongshore, southward flow (up to $\sim 30 \text{ cm s}^{-1}$) of several days’ duration appears in the middle of the record. The temperature remains fairly uniformly stratified, although a thin layer of heavily stratified water appears at the bottom after the onset of southward flow in the interior. This cold layer has been attributed to upwelling advection of cold water by the Ekman boundary layer (Weatherly and Martin, 1978).

Of greater interest is the picture of strong diurnal oscillations which emerges from Fig. 4. To focus on the depth-varying part of the high-passed velocity data, a depth-averaged horizontal velocity component has been subtracted from each profile. Therefore, the $U$ and $V$ contour plots in Fig. 4 show the depth-varying component of the high-passed current, while the bottom panel shows the depth-averaged components.

a. Depth-averaged components

The depth-averaged components show the presence of strong oscillations in both the diurnal and semidiurnal frequency bands. The $V$ component is mainly diurnal, while the $U$ component has an appreciable diurnal inequality.

The dominant tidal constituents at nearby coastal stations have already been mentioned. It is difficult to resolve the $K_1/O_1$ lines with 224 h record lengths. However, it is found that a least-squares fit of $U$ and $V$ at $M_2$ and the average period of $K_1$ and $O_1$ will account for 89% of the variance in the depth-averaged $U$ and $V$ components. Some parameters resulting from this fit are given in Table 1. An exact fit at the $K_1$
Fig. 2. Original contours of horizontal velocity (East-U; North-V) and temperature. Contour intervals: $U, V = 5 \text{ cm s}^{-1}; T = 5^\circ\text{C}$. 
and O₁ frequencies produces only a slight improvement (2%) in the amount of variance accounted for.

Since the inertial period (27.37 h) at this latitude is not far from the diurnal periods, it was also important to vary the diurnal frequency used in the least-squares fit while fitting at M₂ in order to determine whether the apparent diurnal energy in the depth-averaged components was not really inertial energy. The result of this experiment was that a distinct minimum (11%) in the residual variance occurs at a fit period of ~24.3 h (near the mean diurnal period of 24.8 h). As the fit period was moved toward the inertial period the percent residual variance rapidly rose to a large value (55% at the inertial period). This indicates that the depth-averaged oscillations are in fact primarily due to the diurnal tide.

b. Depth-varying component

Evidence for the presence of significant energy in baroclinic tidal oscillations can be seen in Fig. 4. Energy in the diurnal band is especially evident, while semidiurnal baroclinic energy is not apparent and, also, some high-frequency variations are present. Despite this high-frequency component, a distinct modulation both in the intensity and the vertical structure of the diurnal velocity oscillations can still be seen. Early in the record the signal is relatively weak with no clearly defined structure. Starting around 4500 hours, a more energetic wave appears with a single minimum in velocity at 50–60 m and with an apparent intensification of velocity near the bottom. Amplitudes again decrease toward the end of
Fig. 4. High-frequency (tidal and above) velocity and temperature contours. The upper two panels show contours for the depth-varying velocity components, while the bottom panel shows the time series for the depth-averaged components. Contour intervals: \( U, V = 5 \text{ cm s}^{-1}; T = 0.25^\circ\text{C}. \)
Table 1. Parameters of depth-averaged tidal currents.

<table>
<thead>
<tr>
<th></th>
<th>$K_i/O_i$</th>
<th>$M_2$</th>
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</thead>
<tbody>
<tr>
<td>Kinetic energy (ergs cm$^{-2}$)</td>
<td>29.6</td>
<td>26.8</td>
</tr>
<tr>
<td>Ellipse bearing (deg)</td>
<td>123</td>
<td>68</td>
</tr>
<tr>
<td>Ellipticity</td>
<td>0.71</td>
<td>0.98</td>
</tr>
</tbody>
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the record, and a more complicated vertical structure appears. There is also some evidence of organized vertical motion in the diurnal band in the temperature record.

To clarify visually the diurnal signal, some additional processing can be applied. At this point it would of course be possible to low-pass filter the high-passed data in time at each depth in order to keep only the diurnal band and reject semidiurnal and higher frequencies. For several reasons it was decided instead to compute a depth-averaged spectrum (i.e., to compute the $U$ and $V$ component spectra at each of 16 depths and average over depth at each frequency). First, the result gives an idea of the depth-averaged energy contributions in various parts of the high-frequency spectrum. Second, it was found in computing some trial cross-spectra between adjacent depths that velocities are essentially incoherent except in the diurnal band and (in a few cases) the semidiurnal band. Therefore, the depth-averaged spectrum gives a means of estimating a level of background noise on which the tidal signal is superimposed. Finally, one can choose a well-defined band of frequencies in which to reconstruct the diurnal band signal, based on a depth-averaged signal-to-noise ratio.

Fig. 5 shows the resulting depth-averaged spectrum (the plot is variance preserving). Errors bars have not been given for the tidal peaks since the diurnal tidal estimates are obviously correlated at different depths. Nevertheless, several important features can be seen in the spectrum. First, it is clear that the diurnal band is the dominant contributor to the depth-averaged variance; the variance contribution in the semidiurnal band is quite small. It should be recalled that an appreciable $M_2$ surface tide is present. Second, the diurnal band is not centered at the inertial frequency but rather at a point more appropriate to the average period of the $K_i$ and $O_i$ constituents. Finally, the fact that at high frequencies the vertical coherence was low, together with the flat appearance of the energy above 0.1 cycles per hour (cph) at a level of approximately 25 cm$^2$ s$^{-2}$ cph$^{-1}$ suggests that this value can be used as a reasonable estimate of the background noise in the data. [The reason for the poor coherence at non-tidal frequencies is unclear, although some degradation in coherence could be expected from the noise introduced by surface waves (Van Leer and Leaman, 1978).]

Using the above noise estimate, energy levels for the signal-to-noise (S/N) ratios S/N = 1 and 2 are shown in Fig. 5. Note that this is a depth-averaged ratio; at depths where the diurnal amplitudes are smaller, the actual S/N ratio will be smaller. The ratio S/N = 2 is used to define a definite bandwidth in the sense that for all frequencies whose energy levels are above S/N = 2, the Fourier coefficients can be used, as shown in Fig. 6, to reconstruct the depth-time structure of the diurnal-band oscillations. Using the bandwidth of the diurnal signal ($3 \times 10^{-2} - 6 \times 10^{-2}$ cph) and the noise level estimated above, the rms error for a given value of $U$ or $V$ in Fig. 6 is estimated to be $\pm 0.6$ cm s$^{-1}$.

4. Significant structural features of the internal diurnal tide

The modulation in amplitude and vertical structure with time of the internal diurnal tide is apparent from Fig. 6. The signal is initially weak but increases after about 4500 hours, when a "first mode" appears with a single velocity minimum between 50 and 60 m. The amplitude of the first mode decreases toward the end and a weaker but more complicated structure appears, with several velocity maxima in the vertical. The strongly baroclinic signal, when present, appears to be intensified near the bottom in both the $U$ (onshelf) and $V$ (alongshelf) components. Also, the $U$ component is somewhat more

![Fig. 5. Depth-averaged spectrum for the depth-varying velocity components shown in Fig. 4.](image-url)
energetic than the $V$ component. Certain simple hypotheses, based on known theoretical results and the dynamics of internal waves, can be examined in the light of these observations. The conclusions drawn from this must be tentative because of the limited temporal and geographical (one point) coverage and because of the complexity of generation and propagation of internal tides near a critical bottom slope.

a. Modulation envelopes

The predicted diurnal surface tide in Naples, Florida, is dominated by the $K_1$ and $O_1$ constituents. The height ($h$) of the surface tide is given by

$$h (m) = 0.150 \cos[15.041(t - 4421) + 191.2^\circ] + 0.133 \cos[13.943(t - 4421) + 343.5^\circ],$$

where $t$ is time in hours relative to the beginning of the year. The $K_1$ and $O_1$ constituents are of comparable magnitude. The modulation envelope of $h$ has a peak at 4496 hours and the time interval between maxima or minima in the envelope is 13.66 days (328 hours).

Clearly, if a single diurnal constituent were known to be dominant, or to be the only constituent present, then the strong modulation seen in Fig. 6 would
be unexpected. For example, any of the linear theories for shelf-break generation and upshelf propagation of internal tides (e.g., Prinsenberg et al., 1974), including the assumption of a time-invariant medium, can only produce a response (the baroclinic tide generated at the shelf break) which is at the same frequency as the forcing (the barotropic tidal constituent). If a single barotropic constituent were dominant, then little or no modulation would be seen in the baroclinic response on the shelf, and the strong modulation that is in fact observed could be attributed more easily to other causes, such as temporal variations in the wave propagation medium.

The picture is complicated here by the presence of two diurnal constituents of almost equal magnitude. In this case the linear theories predict that the baroclinic oscillations (in the first vertical mode, for example) produced on the shelf by the superposition of two barotropic tides (e.g., K₁ and O₁) should exhibit the same modulation characteristics as the superposition of the two barotropic tides. Note, however, that the peaks in the modulation envelopes for the barotropic tide and, for example, the first-vertical-mode baroclinic tide will not necessarily occur simultaneously at a given point on the shelf; the baroclinic tides will also be spatially modulated since the two constituents will have different horizontal wavelengths across the shelf.

It is instructive to examine the modulation envelopes of U and V at various depths in order to determine whether they are comparable to the barotropic modulation envelope. (Another approach is to fit vertical modes at each time and then see how the modes modulate. If the modal structure were better known, this approach would be quite attractive.)

Modulation envelopes for the U component, as shown in Fig. 7, were obtained by determining the times and magnitudes of positive and negative peaks in the velocity. The behavior of the V component is similar. The observed modulation clearly depends on depth and is relatively weak in the upper part of the water column. In contrast, near the bottom the modulation envelope exhibits a fairly narrow peak between approximately 4550 and 4600 hours. Since the K₁/O₁ modulation interval is 328 hours, this indicates that, especially near the bottom, the baroclinic diurnal tide is modulated much more rapidly than can be accounted for by a simple superposition of oscillations at the K₁ and O₁ frequencies. Note also that the depth-averaged V component (Fig. 4) had relatively weak diurnal modulation in comparison with the rapid modulation of the baroclinic diurnal tide.

b. Wave hodograph structure and propagation direction

Some additional information can be gained from examining the relative structure of the U and V components and temperature in Fig. 6. The clear diurnal baroclinic signal appears from roughly 4490 to 4600 hours. Since the U component is generally larger than the V component during this time, while U and V are out of phase by ~6 h (90°), one might guess that the tidal ellipses observed at a given depth would tend to be oriented with their major axes in the U (onshelf–offshelf) direction. To test this supposition, velocity components at each depth were fit over 48 h windows at the mean diurnal frequency, within the above time interval. The depth-averaged
ellipse orientations ($\phi$, considered to be positive clockwise from north) and ellipse stability ($E$) were then computed according to formulas derived by Gonella (1972). For the fit window 4514–4562 hours, $\phi = 84^\circ$ and $E = 0.79$, while for the window of 4562–4610 hours, $\phi = 83^\circ$ and $E = 0.80$. Therefore, during the period of strong baroclinic signals, the tidal ellipses are quite stable with depth and the mean ellipse orientation is slightly north of east. (The same procedure applied to 48 h segments at the beginning and end of the data set yielded appreciably lower values of ellipse stability, which is not surprising given the weak velocities at those times.)

As noted, in the middle of the record (particularly from about 4490 to 4550 hours) the vertical structure looks qualitatively like a first baroclinic mode, with a single velocity minimum in the vertical. The temperature structure seen in Fig. 6 tends to support this idea, since temperature oscillations that are essentially in phase with depth appear at the diurnal frequency. Since the magnitude of the temperature signal is only marginally above the level of accuracy inferred above, a detailed examination of the vertical temperature structure is probably not warranted. However, in the upper part of the water column (20–40 m, say), the $V$ component leads the temperature signal by ~6 h (or about one-fourth of the diurnal period) from ~4490 to 4550 hours.

In the simple case of a propagating baroclinic wave (ignoring complications introduced by topography), it is easily demonstrated that the ellipse orientation ($\phi$) is aligned along the direction of wave propagation. Since $\phi = 84^\circ$, the first mode observed during part of this study propagated approximately either upslope or downslope. The remaining ambiguity in direction can be resolved by noting that the interior temperature for the first vertical mode will lag the near-surface velocity component which is orthogonal to the ellipse orientation (in this case, the $V$ component, to within ~6$^\circ$) if the wave is propagating upslope, while the reverse will be true in the case of downslope propagation. The characteristics of the velocity and temperature signals described above are thus basically consistent with upslope propagation.

There is also a question of whether the baroclinic mode is vertically symmetric. The $U$ component (Fig. 6) phase lines move upward with time. This might indicate that a vertically asymmetric mode exists with a component which is propagating phase upward or energy downward (Leaman, 1976). But in several cases in the middle of the water column, the $V$ component shows the opposite phase progression. The reason for this is unclear, although in this depth range (40–60 m) and time interval velocity component magnitudes were usually ~1 cm s$^{-1}$.

Since this is near the noise level previously estimated, observations in this region are possibly subject to significant noise contamination and should be treated with caution.

5. Summary and discussion

The analysis described above was carried out to obtain a clear visual picture of the baroclinic diurnal tide without making any assumptions about the vertical structure. The sensitivity of this structure to a near-critical bottom slope, as well as to the effects of low-frequency variations in the medium through which the waves are propagating, warranted this approach.

The main observational results of this analysis are summarized below:

1) The baroclinic diurnal tide during this time exhibited appreciable variations both in amplitude and vertical structure. Since the predicted $K_1$ and $O_1$ constituents in the surface tide are of comparable magnitude, it is not surprising that modulation also appears in the internal tide. What is surprising is that whereas the predicted interval between energy minima in the diurnal-band modulation envelope is 328 h, the observed interval in the envelope of the baroclinic tidal energy is significantly shorter (<224 h), with distinct minima appearing near the beginning and end of the record.

2) The observed structure is basically low-order in the vertical. From ~4490 to ~4600 hours the vertical structure is qualitatively similar to a first baroclinic mode with a single velocity minimum in the vertical and a temperature oscillation that is vertically in phase. After 4600 hours the first-mode structure decays and a less energetic structure that is similar to a second mode, with two velocity minima in the vertical, appears.

3) When the first-mode structure was dominant, the orientation of the velocity ellipses and the velocity–temperature phase relations were consistent with upslope propagation.

4) For several days (~4550–4600 hours) the wave particle velocities were amplified near the bottom.

Several further points of comparison with available theories for the generation or propagation of shelf internal tides are suggested by these observations. First, the inferred upslope direction of propagation (for the lowest mode, at least) is consistent with the theoretical result (e.g., Prinsenberg et al., 1974) that the shelf break can act as a source for the internal tides, which then propagate some of their energy upslope away from the source region. However, the discontinuous depth profiles commonly used in these theories often lead to the
production of concentrated beams of energy within the water column with appreciable energy in high vertical wavenumbers. Such energy concentrations, with the possible exception of the near-bottom velocities, are absent in these observations.

Second, because of the properties of internal wave reflection from a sloping bottom, one might expect to see a concentration of wave energy near the bottom for a wave propagating over a near-critical bottom slope. This is one of the outstanding features of the wedge solutions derived by Wunsch (1968, 1969), for example. [The presence of frictional dissipation near the bottom will modify this picture. However, the results of another analysis of these data (Weatherly and Martin, 1978) suggest that owing to the heavy stratification observed at the bottom, the vertical eddy momentum diffusion coefficient was small.] Since the $K_1$ and $O_1$ constituents are both near-critical in this case, such a near-bottom concentration of energy might be expected, and it is tempting to conclude that the amplified near-bottom velocities observed from 4550 to 4600 hours are a manifestation of this property. However, it is puzzling that this intensification occurs only during this brief period of time.

The theoretical results discussed above are based on steady-state theories of internal tide generation and propagation, in which the propagation medium is not time-varying. The apparent discrepancy between the expected and observed modulation characteristics of the internal tide suggests, on the other hand, that a significant transient component, presumably brought about by low-frequency variations in the medium, might also be present. In this regard it is of interest to return briefly to the low-frequency contours shown in Fig. 3. This figure shows that a strong vertical negative shear in the low-frequency $V$ (alongshore) velocity component existed in the middle of the record. Magnitudes and vertical shears of the low-frequency $U$ component were appreciably smaller. For the case of internal waves propagating normal to a vertically sheared geostrophic current, Mooers (1975a,b) and others have shown that the slope ($\Gamma_c$) of the wave characteristics is given by

$$\Gamma_c = -\frac{M^2}{N^2 - \omega^2}$$

$$\pm \left[ \frac{M^4}{(N^2 - \omega^2)^2} + \frac{\omega^2 - f^2}{N^2 - \omega^2} \right]^{1/2}, \quad (3)$$

where $f$ is the Coriolis parameter, $\omega$ the wave frequency, $N$ the stability frequency, and the vertical shear of the low-frequency current $dV/dz$ enters through the term $M^2 = f dV/dz$. In the absence of low-frequency vertical shear, $M = 0$ and $\Gamma_c = \gamma_c$ [Eq. (1)]. Eq. (3) does not include terms arising from the effect of low-frequency, horizontal current shears on the wave propagation characteristics, since this information was not available from the present experiment.

To determine whether upslope reflection from the bottom was possible for the internal diurnal tides observed here, the upper (+) sign is chosen in Eq. (3). Using values of $N$ and $f$ appropriate to the experimental location, Eq. (3) can then be used to determine the smallest low-frequency vertical shear that will produce subcritical propagation ($\Gamma_c > \gamma_c$) for the diurnal tidal constituents. The result of this computation is that subcritical propagation can occur if negative vertical shears of 0.5 cm s$^{-1}$ m$^{-1}$ or greater are present. Vertical shears of this magnitude or greater were clearly observed in the lower 50 m in the middle of the record (Fig. 3), suggesting that time-varying propagation conditions could account for the rapid modulation observed in the internal diurnal tide.

However, it should be noted that $\Gamma_c$ also decreases with increasing values of $N$. In fact, the intense vertical stratification observed at the bottom toward the end of the experiment was possibly sufficient to again produce supercritical bottom reflection, even though significant vertical shear is still present in the low-frequency current. This possibility cannot be pursued further at this time, mainly because of the absence of other supporting data. Ideally, Eq. (3) could be integrated to determine ray paths on the shelf (Torgimson and Hickey, 1979). However, the detailed hydrographic sections across the shelf which would be needed to perform the integration are not available from this experiment.

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