Some Eulerian-Scale Analysis Results: Eddy Terms in the Mean Heat, Momentum and Vorticity Equations

D. E. HARRISON

Department of Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge 02139

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ABSTRACT

The question of the importance of mesoscale motions in the long time averaged ocean circulation is examined from the viewpoint offered by Eulerian scale estimates of the magnitudes of the explicit eddy and largest inviscid mean flow terms in the mean heat, momentum and vorticity equations. Comparisons of these estimates reveal the quantities that must be known to obtain reliable estimates of the importance of eddy terms in the mean balances. Using historical information and long time series of data from the western North Atlantic, two distinct regimes ("near field" and "mid-ocean") are identified for this ocean region and the appropriate term comparisons are made for each regime. From estimates of the reliability of the ocean values used in these comparisons the robustness of the comparisons is examined. The momentum and vorticity equation estimates suggest that terms based on the eddy Reynolds stress can generally be neglected compared to terms involving f_0 and β in both the near field of the Gulf Stream and the mid-ocean. In the near field, mean advective terms appear to be at least as important as the eddy terms, but the eddy terms dominate these advective ones in the mid-ocean. The heat equation comparisons suggest that the eddy term is comparable to the mean horizontal advection of heat in the mid-ocean but is of somewhat reduced importance in the near field. Some remarks on the generality of results from numerical ocean models that contain mesoscale motions to the question of eddy importance in the ocean are offered.

1. Introduction

Over the last decade much research effort has been directed toward study of the mesoscale variability [period $\geqslant$ 10 days, horizontal scale O(100 km)] of the world ocean. Historical data analyses, large and small observation programs, theoretical and numerical modeling work have all been carried out in efforts to increase our knowledge of these motions. A review of much of the work in the North Atlantic may be found in MODE Group (1978), and a detailed bibliography of research on mesoscale motions has recently been prepared by McWilliams (1979).

These motions are of interest from many different perspectives. From the perspective of the climatological ocean circulation (long time average) the most significant questions concern the importance of their averaged transports, compared to the mean field transports, in the mean heat, salt, momentum and vorticity budgets. Because the eddy signal (instantaneous minus mean) is frequently much larger than that of the mean, and because there seems to be considerable eddy variance at periods of 100 days and longer, it is very time-consuming to measure the eddy and mean terms directly. In order to evaluate transports spatial integrals of these terms must be computed; clearly, considerable effort would be required to obtain the necessary data. However, unless indirect methods of determining the characteristics of the eddy transports can be found, it is essential that these direct measurements be made if the role of these motions in the mean circulation is to be understood.

Because of the length of time and the expense of making direct observations of eddy and mean terms in the ocean, considerable work has been done with numerical model ocean circulations that produce mesoscale motions as part of their flows. In these models (the EGCM's) the fluid system is spun up to some type of statistical equilibrium and the eddy and the mean terms in the dynamical equations are directly evaluated and studied. The first careful analysis results of these numerical experiments are not yet thoroughly understood. In some parts of the flow in some of the experiments the eddy terms are as large as any other; in many other regions and experiments they appear to be of very limited importance, if at all. The role of eddy transports has generally not been reported. Some of these experiments appear to confirm the widely held expectation that it is necessary to explicitly know the eddy terms if the mean circulation is to be understood. However, it would be unwise to assume at this time that this result applies to the oceans because of the many factors that seem

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TABLE 1. Scale notation.

Quantity	Mean-scale amplitude	Eddy-scale amplitude
(x,y,z)	(L,L,H)	(η,η,H)
(u,v,w)	$(U_0,U_0,\bar{\gamma}U_0[H/L])$	$(u_0,u_0,\gamma' u_0[H/\eta])$
$T(x,y,z)$	T_0 in vertical, θ_0 in horizontal	t_0 in vertical, ϑ_0 in horizontal

to affect the importance of the eddies from experiment to experiment.

The question of whether or not the eddies are anywhere important in the mean ocean circulation and, if they are important, in which regions of ocean basin, remains open at present. The purpose of this work is to present some very simple Eulerian scale analysis results about the relative importance of eddy terms and mean terms in the time averaged heat, momentum and vorticity equations. These results, together with the available western North Atlantic data, raise some interesting questions about the basic importance of eddy terms in ocean budgets, and about the applicability of existing numerical model results to the midlatitude ocean.

The scale analysis notation, assumptions and results are presented in Section 2. Different ocean regimes are described and the scale analysis results for these regimes presented in Section 3. The sensitivity of these results is examined in Section 4. Some of the implications of this work on the importance of eddies in the ocean and in EGCM numerical models are considered in Section 5.

2. Some scale analysis estimates

In this section the scale analysis notation that will be used throughout is introduced and the basic scaling results presented. The approach taken here is that of conventional Eulerian scale analysis. Throughout it will be assumed that the reader is familiar with the time-averaged forms of the so-called primitive equations and the equation for the vertical component of vorticity that is derived from them [these are written out in detail in Robinson *et al.* (1977)]. These will serve as the basic equations for this work. An overbar will be understood to indicate a time-averaged quantity, while a prime will indicate the instantaneous departure of a quantity from its time mean value.

The explicit eddy terms of interest are thus:

$\partial/\partial x_j(\overline{u_j' u_\lambda'})$ in the mean horizontal momentum equations

$\partial/\partial x_j(\overline{u_j' T'})$ in our model heat equation

$\epsilon_{3\lambda\delta}(\partial^2/\partial x_j \partial x_\delta)(\overline{u_\lambda' u_j'})$ in the vorticity equation.

Standard Cartesian tensor notation is used with summation implied by repeated subscripts, with $j = 1,2,3$; $\lambda, \delta = 1,2$; ϵ_{ijk} is the permutation tensor; $(u,v,w) = (u_1,u_2,u_3)$ are conventionally defined with east, north and upward velocity components positive. This notation is but a convenient shorthand to avoid cumbersome labels for these eddy terms. The term $\partial/\partial x_j(\overline{u_j' u'})$ is simply $(\partial/\partial x)\overline{u'^2} + (\partial/\partial y)\overline{u'v'} + (\partial/\partial z)\overline{u'w'}$, the zonal component of the divergence of the Reynolds stress tensor; the other terms may be written out easily into their expanded forms if desired.

We shall examine the size of these eddy terms compared with the non-dissipative and adiabatic mean field terms that might plausibly be important in the mean balances. Specifically, we inquire into the values of $\partial/\partial x_j(\overline{u_j' u'})$ compared to $f_0\bar{v}$, $\beta y \bar{v}$ and $\bar{u}_j(\partial/\partial x_j)\bar{u}$ (and the corresponding meridional momentum equation values), $\partial/\partial x_j(\overline{u_j' T'})$ compared to $\bar{u}_\lambda(\partial/\partial x_\lambda)\bar{T}$ and $\bar{w}(\partial\bar{T}/\partial z)$ and $\epsilon_{3\lambda\delta}(\partial^2/\partial x_j \partial x_\delta)(\overline{u_\lambda' u_j'})$ compared to $\beta\bar{v}$, $f_0(\partial\bar{w}/\partial z)$ and $\bar{u}_j(\partial/\partial x_j)\bar{\zeta}$. Here $\bar{\zeta}$ is defined to be $\partial\bar{v}/\partial x - \partial\bar{u}/\partial y$ and f_0 and β are conventionally defined.

Scale notation is needed for these various quantities and is given in Table 1. Capital letters generally indicate mean quantities; lower case letters indicate eddy quantities. The same vertical scale of variation is assumed for the mean and the eddies for convenience, and a factor $\bar{\gamma}$ (or γ') is introduced into the scaling for $\bar{w}$ (or w') to allow differing degrees of horizontal divergence of the velocity fields. In order to scale the eddy terms of interest two more types of scales are needed—correlation coefficients for averaged products of eddy terms [so that, e.g., $\overline{u'v'} \rightarrow C_{uv}u_0^2$] and the length scale of variation of these averaged products of eddy terms [so that, e.g., $\partial/\partial y(\overline{u'v'}) \rightarrow C_{uv}(u_0^2/\mathscr{L})$]. *A priori*, the relationships between $\mathscr{L}$, L and η are not known. Under many types of turbulent motion one might expect $\mathscr{L} \gg \eta$ or $\mathscr{L} \sim L$ but there are circumstances where $\mathscr{L} \sim \eta$, as will be discussed below.

The scale estimates of interest can now be evaluated. They are given in Table 2. In each case only the largest component of the mean and eddy term is used, and $\gamma' < 1$, $\bar{\gamma} < 1$ have been assumed. No dynamical assumptions have been made up to this point, except to the extent that some mean terms in the vorticity equation have not been included in our comparisons.

Consider the results in Table 2. For the mean heat equation (Table 2c) only ratios of mean and eddy quantities appear, apart from C_T. The absolute magnitudes of u_0 or ϑ_0 are thus unimportant from this perspective. However, in the momentum and vorticity equation terms (Tables 2a and 2b), values may depend on $u_0/f_0\mathscr{L}$, $u_0/\beta L\mathscr{L}$ or $u_0/\beta\mathscr{L}^2$ in addition to ratios and correlation coefficients. The absolute scale of some eddy quantities affects the importance of eddy terms in the vorticity and momentum equation comparisons. Note that the eddy length

TABLE 2. Relative importance of eddy terms. $C_u = \max(C_{uu},C_{uv})$, $C_\zeta = \max(C_{uu},C_{uv},C_{uv})$, $C_T = \max(C_{ut},C_{vt})$.

		A:	$f_0\bar{v}$	$\beta y\bar{v}$	$\bar{u}_j \frac{\partial}{\partial x_j}\bar{u}$
(a)	$\left[\frac{\partial}{\partial x_j}\overline{(u_j'u')}\right]A^{-1}$		$C_u\left(\frac{u_0}{U_0}\right)\frac{u_0}{f_0\mathscr{L}}$	$C_u\left(\frac{u_0}{U_0}\right)\frac{u_0}{\beta L\mathscr{L}}$	$C_u\left(\frac{u_0}{U_0}\right)^2\frac{L}{\mathscr{L}}$
		A:	$f_0\frac{\partial\bar{w}}{\partial z}$	$\beta\bar{v}$	$\bar{u}_j\frac{\partial\bar{\zeta}}{\partial x_j}$
(b)	$\left[\epsilon_{3\delta\lambda}\frac{\partial^2}{\partial x_\delta\partial x_j}\overline{u_j'u_\lambda'}\right]A^{-1}$		$\frac{C_\zeta}{\bar{\gamma}}\left(\frac{u_0}{U_0}\right)\frac{L}{\mathscr{L}}\frac{u_0}{f_0\mathscr{L}}$	$C_\zeta\left(\frac{u_0}{U_0}\right)\frac{u_0}{\beta\mathscr{L}^2}$	$C_\zeta\left(\frac{u_0}{U_0}\right)^2\left(\frac{L}{\mathscr{L}}\right)^2$
		A:	$\bar{w}\frac{\partial\bar{T}}{\partial z}$	$\bar{u}_\lambda\frac{\partial\bar{T}}{\partial x_\lambda}$	
(c)	$\left[\frac{\partial}{\partial x_j}\overline{(u_j'T')}\right]A^{-1}$		$\frac{C_T}{\bar{\gamma}}\left(\frac{u_0}{U_0}\right)\frac{L}{\mathscr{L}}\left(\frac{\vartheta_0}{T_0}\right)$	$C_T\left(\frac{u_0}{U_0}\right)\frac{L}{\mathscr{L}}\left(\frac{\vartheta_0}{\theta_0}\right)$	

scale of importance is $\mathscr{L}$, the length scale of variation of eddy quadratic statistics, not η, the instantaneous eddy horizontal length scale. Every comparison depends on $\mathscr{L}$, and the vorticity equation comparisons depend on $\mathscr{L}^2$. The coefficient γ' is seen to drop out of consideration, but $\bar{\gamma}$ is found in two places—mean vortex stretching in Table 2b and mean vertical advection of heat in Table 2c.

Neglecting correlation coefficients there are five nondimensional parameters in Table 2, other than the three given above, that measure the importance of eddy terms. It can be an interesting exercise to use the results of Table 2 to investigate the implications of different assumptions about u_0/U_0, $L/\mathscr{L}$, $\mathscr{L}$, *etc.*, on the relative importance of these eddy terms. Some examples are offered in a recent brief note (Harrison, 1979b). Fortunately, for our purposes here it is possible to significantly simplify the situation using ocean data.

3. Eddy importance under different flow regimes

Using the estimates from Table 2, it is now possible to examine the importance of eddy terms under different flow regimes, defined by different values of the various parameters of Table 2. Attention will be limited to cases similar to those believed to hold in the midlatitude North Atlantic.

According to Schmitz (1977, 1978), the meridional length scale of the near Gulf Stream region mean flow along 55 and 70°W is $\sim 10^7$ cm, while the meridional scale of variation of eddy statistics is larger (perhaps by a factor of 2–3). The flow between 28 and 35°N is not well resolved in latitude by his measurements but it is consistent with the data to assume that the mean is slowly varying ($L \sim 10^8$ cm) and that the eddy statistics have a comparable length scale. Although eddies have been observed with a range of scales, many observations suggest that $\eta \sim 5 \times 10^6$ cm is a reasonable value.

Estimates for u_0 and U_0 are also available from Schmitz' data. Near the Gulf Stream along 55°W but south of it (35–38°N) mean eddy kinetic energy is typically 50–150 $cm^2\ s^{-2}$ and decreases to just a few $cm^2\ s^{-2}$ at 28°N. The mean current speeds are typically 5–10 cm s^{-1} between 35 and 38°N and decrease to $\sim$1 cm s^{-1} at 28°N. In the latitude range 35–38°N there is little variation with depth of mean flow or eddy quantities (see Figs. 1 and 2 of Schmitz, 1978). Unfortunately, measurements from the Gulf Stream itself are not available above 4000 m depth. Other data from the North Atlantic are available, but none over as long a period of time as these. If we take $C_u = C_v = C_\zeta = O(1)$, then a reasonable upper bound for u_0 should be the square root of the mean eddy kinetic energy. Under this assumption we find $u_0 = 7$–12 cm s^{-1} for 35–38°N and $u_0 = 1$–7 cm s^{-1} between 28 and 35°N.

We now consider the problem of temperature amplitudes. From the Fuglister I.G.Y. atlas (Fuglister, 1960), one finds $T_0 \sim 10$°C at 36°N and $\sim$20°C at 24°N. Horizontal gradients are more difficult to estimate but $\theta_0 \sim 2°$ at 500 m and $\sim$0.1°C at 2000 m seem reasonable for both 24 and 36°N around 55°W. It is extremely difficult to estimate deep temperature gradients. Temperature statistics from Schmitz' POLYMODE data have not been published but much information is available from technical reports (Tarbell *et al.*, 1978; Spencer *et al.*, 1977). From Tarbell *et al.* (1978) we find $\vartheta_0 \sim 3°$ at 600 m, $\sim$0.2°C at 2500 m and $\sim$0.1°C at 4000 m depth for 36°N, while Spencer *et al.* (1977) show $\vartheta_0 \sim 0.6°$ at 500 m, $\sim$0.1°C at 2000 m and $\sim$0.05°C at 4000 m for 28°N. A value of $C_T = O(10^{-1})$ appears consistent with available data (Schmitz, personal communication).

TABLE 3. Ocean data scale estimates (western North Atlantic). See text for discussion of mid-ocean and near-field and specific scale values.

A:	$f_0\bar{v}$	$\beta y \bar{v}$	$\bar{u}_j \frac{\partial}{\partial x_j}\bar{u}$	
$\left[\frac{\partial}{\partial x_j}\overline{(u_j'u')}\right]A^{-1}$	$1 \to 7 \times 10^{-3}$	$0.5 \to 3 \times 10^{-2}$	10^2	mid-ocean
	$2 \to 4 \times 10^{-3}$	$1 \to 2 \times 10^{-1}$	3×10^{-1}	near-field
A:	$f_0 \frac{\partial \bar{w}}{\partial z}$	$\beta \bar{v}$	$\bar{u}_j \frac{\partial \bar{\zeta}}{\partial x_j}$	
$\left[\epsilon_{3\delta\lambda} \frac{\partial^2}{\partial x_j \partial x_\delta}\overline{(u_j'u_\lambda')}\right]A^{-1}$	$\frac{1}{\bar{\gamma}}(1 \to 7 \times 10^{-3})$	$0.5 \to 3 \times 10^{-2}$	10^2	mid-ocean
	$\frac{1}{\bar{\gamma}} \times 10^{-3}$	$4 \to 6 \times 10^{-2}$	10^{-1}	near-field
A:	$\bar{w} \frac{\delta \bar{T}}{\delta z}$	$\bar{u}_\lambda \frac{\partial \bar{T}}{\partial x}$		
$\left[\frac{\partial}{\partial x_j}\overline{u_j'T'}\right]A^{-1}$	$\frac{1}{\bar{\gamma}} \times 10^{-1}$	1		mid-ocean
	$\frac{3}{\bar{\gamma}} \times 10^{-2}$	3×10^{-1}		near-field

To oversimplify a little bit, these data suggest the existence of two different regimes. The first, which will here be identified by the label mid-ocean, is characterized by $u_0/U_0 = O(10)$, $L/\mathscr{L} = O(1)$, $\vartheta_0/\theta_0 = O(1)$ and $\vartheta_0/T_0 = O(10^{-1})$, while the second, to be called near field, is characterized by $u_0/U_0 = O(1)$, $L/\mathscr{L} = 0.5 \to 0.3$, $\vartheta_0/\theta_0 = O(1)$ and $\vartheta_0/T_0 = O(10^{-1})$. Because of the uncertainties involved in estimating some of the quantities that make up these ratios, it seems best to keep the discussion on an order of magnitude basis whenever there is doubt about a particular value. If these estimates err, it is generally in a direction to raise the importance of the eddy term in a given situation. We shall return to the robustness of our estimates below. Given these regimes it remains only to specify $\bar{\gamma}$, u_0 and $\mathscr{L}$ in order to make use of the estimates of Table 2.

The appropriate choice for $\bar{\gamma}$ in different oceanic regimes is not clear. A strict continuity equation scaling yields $\bar{\gamma} \sim 1$, while the assumption that $\beta\bar{v} \sim f_0 \partial w/\partial z$ yields $\bar{\gamma} \sim \beta L/f_0$, which is $O(10^{-1})$ for midlatitudes and $L \sim 10^8$ cm. Assuming quasi-geostrophic dynamics constrains the horizontal divergence more strongly, so that $\bar{\gamma} \sim U_0/f_0 L$ which is $O(10^{-4})$ for our mid-ocean values. Because of the range of possible variation, we shall carry $\bar{\gamma}$ through the estimates in which it occurs.

Using the western North Atlantic values of $u_0 = 1$–7 cm s^{-1} and $\mathscr{L} \sim 10^8$ cm ($L \sim 10^8$ cm) for the mid-ocean and $u_0 = 7$–12 cm s^{-1} and $\mathscr{L} \sim 3 \times 10^7$ cm ($L \sim 10^7$ cm) for the near field of the Gulf Stream, we get the scale estimates of eddy term importance shown in Table 3. Numbers less than unity indicate that the eddy term is small compared to the mean term against which it is being examined. We now consider the results of Table 3.

The momentum equation comparisons are given in Table 3a. The only comparison in which the eddy term is larger than the mean term involves mean advection of momentum in the mid-ocean. The next largest comparison values involve $\beta y \bar{v}$ and mean advection, but now the eddy terms are only a few tenths the size of the mean terms. In the other three comparisons the eddy terms are smaller still—a few times 10^{-2} compared to $\beta\bar{v}y$ in the mid-ocean and $O(10^{-3})$ compared to f_0 in both the near field and mid-ocean. By these estimates eddy Reynold stress divergence is unlikely to upset dynamics in the momentum equation at the level of f_0 or β in either the mid-ocean or the near field. Advection of momentum appears at least as likely as eddy processes to enter the next higher order level of dynamics in the near field, but appears negligible in the interior.

The situation is somewhat similar in the mean vorticity equation balances (Table 3b). The eddy term is large [$O(10^2)$] compared to mean advection of vorticity in the interior but is smaller than the mean terms considered in every other case. Even allowing $\bar{\gamma}$ to be as small as $O(10^{-2})$, vortex stretching remains larger by roughly a factor of 10 than the eddy term. However, it should be observed that these estimates require $\bar{\gamma} = O(10^{-1})$ if there is to be a consistent lowest order balance that does not involve forcing, dissipation or one of the other terms neglected in these comparisons. We shall return to this point below.

It is in the mean heat equation (Table 3c) that the eddy term is clearly of interest. The eddy heat flux divergence is the same order of magnitude as mean horizontal advection of heat, and the same order as (or larger than) mean vertical advection if $\bar{\gamma} < O(10^{-1})$ in the mid-ocean. The eddy term is of reduced importance, by roughly a factor of 10, in the near-field. If one uses $\bar{\gamma} = O(10^{-1})$ as required to get a lowest order vorticity equation balance, in these comparisons all three terms are of comparable magnitude in the mid-ocean and the mean advections provide the lowest order balance in the near field. Assuming $\bar{\gamma} < O(10^{-1})$ again leads to inconsistency—no lowest order balance with the terms being considered.

The sensitivity of these results to uncertainties in the values that go into them are considered in the next section.

4. Sensitivity of scale results

Taken at face value, the results of Section 3 suggest that the terms based on eddy Reynolds stress are not large enough to make the lowest order momentum and vorticity balances different from geostrophy. The eddy heat flux divergence, however, may be very important in the heat equation balances of the mid-ocean and marginally important in the near field. We now consider some estimates of the reliability of these results.

The many scale quantities that appear in Table 2 are known to various degrees. It is simple to obtain accurate (say to ~30%) values of u_0 and ϑ_0 given sufficiently long time series. So long as U_0 is a few centimeters per second or larger it can be estimated to similar accuracy. But it is difficult to obtain accurate estimates of U_0 when the mean flow is weak (≤ 1 cm s^{-1}) and of θ_0 if there are strong eddies present. Determination of L and $\mathscr{L}$ requires stable mean and eddy term values and adequate spatial resolution on the scales of variation.

The Schmitz (1977, 1978) results for L, $\mathscr{L}$, u_0 and U_0 north of 35°N are stable to the 30% level along 55°W [with the exception of one current meter location, 600 m at 35.5°N—see Schmitz (1978, Fig. 2)]. The temperature time series from these arrays also offer ϑ_0 values which appear accurate at the same level, and estimates of T_0 are better than this. However, it is not clear that the weak mid-ocean U_0 values can be distinguished from zero; C_T is known from few enough points and varies sufficiently within the available data that it may be only marginally determined in the order of magnitude sense we require; $\bar{\gamma}$ has not been determined by direct measurement; and θ_0 in the deep water is also only marginally known. However, the only consistent choice for $\bar{\gamma}$ has been shown to be $O(10^{-1})$ in Section 3. Clearly the momentum and vorticity estimates are more reliable than the heat estimates.

If our values are in error in every case in the direction to increase the eddy term importance, and are of 30% magnitude, then the comparison ratios can underestimate the eddy term magnitude by a factor of 5–10. This would make the Reynolds stress term comparable to $\beta y \bar{v}$, $\bar{u}_j(\partial/\partial x_j \bar{u}$ and $\bar{u}_j(\partial/\partial x_j)\bar{\zeta}$ in the near field but would leave this eddy term generally an order of magnitude smaller than the mean terms in the other momentum and vorticity comparisons. This much error leaves the flow f_0-geostrophic and sustains the $\beta\bar{v} \sim f_0(\partial\bar{w}/\partial z)$ lowest order vorticity balance.

The heat equation comparisons are difficult to defend even at the order of magnitude level in the deep ocean where C_T, U_0 and θ_0 are really not satisfactorily known. This is unfortunate since the estimates suggest that these balances are the ones in which the explicit eddy term is most likely to be O(1). Only with better data will one be able to determine if these estimates are satisfactory.

Despite the uncertainty in some of these estimates it is felt that they are of interest, since they are the best that can be made at present. They cannot conclusively establish that any particular relationship holds between an eddy and mean flow term, but have been as carefully made as the data allow and so represent good faith estimates of the eddy term importance.

5. Summary and comments

Based on the scaling formulas of Section 2 and the estimates of individual scale values from multiyear observations in the western North Atlantic, estimates of the importance of eddy Reynolds stress-based and heat flux-based terms in the long time average heat, momentum and vorticity equations have been made (Section 3). Two different regimes appear to characterize the data—a near-field regime immediately south of the Gulf Stream and a mid-ocean regime south of the near field area. The eddy term is found to be small compared with both the f_0 and β terms in the momentum and vorticity equations in both regions for all depths considered. It is also somewhat smaller than the mean flow advection terms in each equation in the near field, but is larger than mean flow advection in the mid-ocean. However, the eddy term is comparable to mean advection in the heat equation balance for the mid-ocean. In the near-field heat equation balance the eddy term is somewhat smaller than the mean advections [$O(10^{-1})$].

The estimates involving mean flow terms with $\bar{w}$, the mean vertical velocity, all assume $\bar{\gamma} = O(10^{-1})$; that is to say that the mean flow is horizontally nondivergent to $O(10^{-1})$. This rather classical scale result is found to be necessary in order to avoid inconsistency in the scale results (see Section 3). It is of interest to note that quasi-geostrophic scaling, which would lead to a value of $\bar{\gamma}$ one to two orders of magnitude smaller, is inconsistent with the available ocean data.

Some remarks about the statistical reliability of these scale estimates are offered in Section 4. The most reliable estimates appear to be for the near-field results, where the mean flow values are relatively large. Within plausible estimates of the error in each scale value, it is necessary for every scale value to be wrong, in the direction of increased eddy term importance, by the maximum assumed amount in order to make the eddy terms comparable to the β terms in the mean momentum and vorticity equations. In the mid-ocean, uncertainties about the mean flow values make the estimates less reliable, but if the mean flow is to be weak enough to make the eddy terms O(1) compared with the β-terms it is necessary to have $u_0/U_0 = \mathrm{O}(10^3)$ rather than O(10) as observed. The heat equation estimates have considerable uncertainty associated with estimates of mean horizontal advection. It is difficult to know how to assess their reliability; these results may over estimate or underestimate eddy term importance in the heat equation.

The magnitude of the horizontal length scale $\mathscr{L}$ of the variation of eddy quadratic statistics and its relative magnitude compared with its mean flow counterpart, L, are very important in determining the relative importance of eddy terms in the mean equation balances. If $\mathscr{L} < L$, eddy effects are of increased importance by at least the factor $L/\mathscr{L}$. Further, even if $L/\mathscr{L} = \mathrm{O}(1)$, if $\mathscr{L}$ is sufficiently small ($\mathscr{L} \leqslant 10^7$ cm) it can be possible for the eddy terms to be comparable to the β terms in the momentum and vorticity equations using the other North Atlantic values. However, the ocean data examined here do not offer support for either $\mathscr{L} < L$ or $\mathscr{L} \leqslant 10^7$ cm.

The EGCM flows mentioned in Section 1, however, do exhibit $\mathscr{L} < L$ and $\mathscr{L} \leqslant 10^7$ cm in some cases. Thus, even if they have oceanic values of u_0/U_0 and of u_0, eddy terms will tend to be more important in their mean balances than would the ocean eddies examined here. The reasons for this are very simple. In most published EGCM results (e.g., Holland and Lin, 1975; Robinson *et al.*, 1977; Holland, 1978) the basin used is 2×10^8 cm or less across a wind-driven gyre (1×10^8 is common for Holland's calculations) and the eddies in these systems tend to propagate due west and to be highly ordered in the meridional direction. [See Harrison and Robinson (1979) for more on the character of these motions.] The result of this regularity is often that $\mathscr{L} \sim \eta$ (at least in the meridional direction) in these systems and, because the basins are small to begin with, $\mathscr{L} \leqslant 10^7$ cm. Except in special locations of the ocean, perhaps where a significant mean flow impinges on a topographic feature and sheds eddies in a regular manner, one would not expect to find this degree of regularity in ocean motions. Less idealized basins, which include some bottom topography and are of oceanic dimensions may be essential if these apparently unrealistic features of the EGCM's are to be eliminated.

In the EGCM's in which the eddies are believed to be the most important in the mean balances (Holland, 1978), it has been assumed that quasi-geostrophic dynamics satisfactorily describes both the instantaneous and mean flows. As noted in Section 2, however, quasi-geostrophic dynamics assumes that the flow is horizontally nondivergent to order U_0/f_0L, so $\bar{\gamma}$ is typically small [$\mathrm{O}(10^{-3}$–$10^{-4})$]. There seems no way to be confident of the effects of this assumption on EGCM results at this time [see Harrison (1979) for some other constraints introduced by this assumption], but it clearly may encourage increased eddy importance in comparisons like those considered here. Further, such a small value for $\bar{\gamma}$ is not supported by our data.

Attention has here been focused on the various explicit eddy term amplitudes because some ocean data are available for these quantities. It should be kept in mind, however, that the eddy term estimates given here may be upper bounds for the actual term magnitudes, because of the assumed correlation coefficient values and because these estimates assume that the term amplitude is the same as the largest term component. It is quite possible that there will be some cancellation due to components of opposite signs. This factor, of course, may reduce the magnitude of some mean terms, e.g., $\bar{u}_\lambda(\partial \bar{T}/\partial x_\lambda)$, but will not affect comparisons with terms like $f\bar{v}$ or $\beta\bar{v}$.

One additional comment should be made at this point. Even if the time-mean vorticity balance is geostrophic to lowest order, this does not mean that the eddy terms are necessarily of negligible importance in determining the mean flow. As is well known, the geostrophic balance is dynamically degenerate—one may evaluate $\bar{v}$ knowing $\partial\bar{w}/\partial z$, but $\partial\bar{w}/\partial z$ must be determined by some other aspect of the flow dynamics. In the adiabatic quasi-geostrophic approximation, the mean vortex stretching is determined by the mean and eddy heat flux divergences divided by the assumed basic stratification. In thermocline-type theories the density equation is also the place where $\partial\bar{w}/\partial z$ is determined. Thus it is possible for the eddy heat flux divergence, if O(1) in the mean heat equation, to play a fundamental role in the lowest order vorticity balance. The scale results presented here are clearly not sharp enough to investigate effects beyond the lowest order balances, and so no comments have been offered about any higher order corrections to the basic balances.

Although this work has concentrated on pointwise term estimates, the most dynamically important quantities are transports of heat, momentum and vorticity. It should be noted that ratios of transports will not necessarily be proportional to the term ratio estimates given here. If $\mathscr{L} < L$ and an integration is done over a volume of characteristic dimension $< L$ but $> \mathscr{L}$, some cancellation of the eddy transport might be expected due to having the eddy term of one sign in some subregion of the volume and of the opposite sign in others. The mean transport would generally not be subject to this type of cancellation because the integration is done within the characteristic scale of variation of the mean term. This type of problem will not be of concern as long as $\mathscr{L} \sim L$, as in most of the ocean data discussed. But for the EGCM circulations in which $\mathscr{L} < L$, it is a factor that should be remembered when considering the implications of point balance results from these model experiments.

Only with much better data will it be possible to determine how eddy transports compare with mean transports in the ocean. Long-term monitoring is necessary to establish the statistical character of eddy correlations and the properties of the mean fields.

This analysis has many limitations, but raises some interesting questions about the extent to which explicit eddy effects appear likely to enter the lowest order dynamical balances of the mean ocean circulation and about the places where they are most likely to be important. In particular, it suggests that ocean eddies of the sort observed in the western North Atlantic south of the Gulf Stream may not alter a local geostrophic balance in the mean momentum and vorticity equations but may enter the basic heat equation balances. It also suggests that eddies may not be substantially more important in the near field of the Gulf Stream than in the mid-ocean.

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