Eddy Lateral Vorticity Transport and the Equilibrium of the North Atlantic Subtropical Gyre

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ABSTRACT

Oceanic gyres defined by the mean zero wind-stress curl lines have been the focus of wind-driven ocean circulation theory since its beginnings. In the face of single-signed vorticity input from the curl of the wind stress over a gyre, a mechanism to balance the wind vorticity input is required if an equilibrium is to be established. Traditional models have tended to restrict their allowable physics so that a dissipation mechanism is required for equilibrium. However, dissipation is not necessary, in principle, for equilibrium for a general fluid system. In fact, it has recently been shown that lateral eddy vorticity transport between gyres can provide an important part of the vorticity tendency required for equilibration in model oceanic systems. This note examines the possibility that in the North Atlantic subtropical gyre this process also might be important. After a brief review of the equilibration mechanisms possible in a primitive equation fluid, attention is focussed on estimates of the eddy lateral transport of relative vorticity in the North Atlantic. It appears that there could be sufficient eddy transport across the Gulf Stream to balance the wind vorticity input over the gyre. Equilibrium might thus be possible without any vorticity dissipation mechanism or without invoking higher order dynamical processes. If this mechanism is important in the ocean there should be interesting effects on the subpolar gyres, details of which depend on the circumstances surrounding the vorticity exchange process.

1. Introduction

One of the long standing questions of classical wind-driven ocean circulation modeling is how the midlatitude gyres attain an equilibrium circulation given that they are constantly receiving vorticity from the wind-stress curl field. The simplest solution is to invoke a frictional mechanism of some kind to act in the western boundary current—Stommel (1948) chose a particularly simple linear drag law while Munk (1950) adopted an eddy viscosity form. However, observations clearly indicate that nonlinear effects can be of dominant importance in the western boundary currents found in these gyres

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(Gulf Stream, Kuroshio, etc.) and that linear frictional models should not be taken seriously. Steady nonlinear boundary layers were described by Charney (1955) and Morgan (1956), but are only consistent south of the latitude of maximum wind-stress curl, where the boundary current transport is increasing as the current flows northward, and do not remove any of the vorticity put in by the wind. For the subtropical gyres, as Morgan (1956) has noted, something must happen north of this maximum curl latitude to modify the relative vorticity of a fluid parcel, if it is to be able to merge back into a linear Sverdrup interior circulation.

A number of numerical studies of idealized subtropical gyres using the barotropic vorticity equation have been carried out (e.g., Bryan, 1963, Veronis, 1966; Blandford, 1971) and clearly reveal that the equilibrium flow depends strongly on the way friction is modeled, even when nonlinear effects are strong (Blandford, 1971). The form and magnitude of the friction as well as the model boundary conditions are important. Since we do not know how to parameterize the processes in the ocean that are responsible for the transfer of vorticity from a solid boundary into the fluid, this is not a very satisfactory state of affairs.

Recently, it has been found that vorticity equilibrium is primarily reached in a numerical model ocean circulation by a transfer of vorticity across the zero wind-stress curl line that divides the model subtropical and subpolar gyres. This transfer is carried out by eddy lateral transport of eddy relative vorticity (Harrison and Holland, 1981; Holland and Rhines, 1980). Although the *ad hoc* model friction parameterization is responsible for some vorticity loss, it is of much smaller magnitude than the vorticity transport carried out by the eddy flux. Rhines and Holland (1979) and Holland and Rhines (1980) have devoted considerable attention to how eddy vorticity processes can transfer vorticity vertically in these quasi-geostrophic model oceans; now the role of horizontal eddy vorticity transport appears to deserve comparable attention. Here we ask if the lateral vorticity transport by eddies in the ocean might be able to provide an equilibrium mechanism for the North Atlantic subtropical gyre.

In Section 2 the basic vorticity integral constraints necessary for an equilibrium circulation are reviewed. Section 3 provides a discussion of the North Atlantic subtropical gyre vorticity constraints and of the ability of eddy vorticity transport to provide the required vorticity. Section 4 offers some summary comments about these results and their possible application to other gyres.

2. Equilibrium vorticity integral constraints

Take as model dynamical equations those of a primitive equation fluid (e.g., Bryan and Cox, 1968) but consider a Cartesian coordinate system on a beta plane for algebraic simplicity. There is no physical loss of generality in this simplification. The equation for the conservation of the time mean vertical component of relative vorticity ($\bar{\zeta}$) is

$$\frac{\partial}{\partial x_j}(\bar{u}_j\bar{\zeta}) + \beta\bar{v} + (f+\bar{\zeta})\frac{\partial \bar{w}}{\partial z} + \overline{TW} + \overline{ED} = \bar{V} + \bar{D}, \quad (1)$$

where

$$\bar{\zeta} \equiv \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y}, \quad (1a)$$

$$\overline{TW} \equiv \frac{\partial \bar{v}}{\partial z}\frac{\partial \bar{w}}{\partial x} - \frac{\partial \bar{u}}{\partial z}\frac{\partial \bar{w}}{\partial y}, \quad (1b)$$

$$\overline{ED} \equiv \frac{\partial}{\partial x_j}(\overline{u_j'\zeta'}) - \overline{\zeta'\frac{\partial w'}{\partial z}} + \overline{\frac{\partial v'}{\partial z}\frac{\partial w'}{\partial x}} - \overline{\frac{\partial u'}{\partial z}\frac{\partial w'}{\partial y}}. \quad (1c)$$

In the above f and β are the conventionally defined Coriolis parameter and its variation with latitude, $(u_1, u_2, u_3) = (u, v, w)$ are conventional and $\bar{V}$ and $\bar{D}$ represent the model processes by which the fluid is forced and vorticity is dissipated. If the domain of interest is assumed to have rigid boundaries, we require no normal flow boundary conditions in order to conserve the total mass of the system.

Let R denote some volume of fluid, $\mathcal{R}$ denote the entire fluid domain and $\{\ \}_R$ denote volume integration over the region R. Integrating (1) over $\mathcal{R}$ yields

$$\left\{\bar{\zeta}\frac{\partial \bar{w}}{\partial z}\right\}_{\mathcal{R}} - \left\{f\frac{\partial \bar{w}}{\partial z}\right\}_{\mathcal{R}} + \{\overline{TW}\}_{\mathcal{R}} + \{\overline{ED}\}_{\mathcal{R}} = \{\bar{V}\}_{\mathcal{R}} + \{\bar{D}\}_{\mathcal{R}}. \quad (2)$$

The mean flow advection of relative and planetary vorticity drop out and, if there is a rigid lid at the upper surface the planetary vortex stretching term, $\{f\partial\bar{w}/\partial z\}_{\mathcal{R}}$ has contributions only if there is variable bottom topography. The eddy advection of eddy relative vorticity also vanishes, $\{\partial(\overline{u_j'\zeta'})/\partial x_j\}_{\mathcal{R}} = 0$. From (2) it is clear that a variety of processes are in principle available to offset the vorticity source term, $\{\bar{V}\}_{\mathcal{R}}$.

With the exception of the possible net vortex stretching from $\{f\partial\bar{w}/\partial z\}_{\mathcal{R}}$ due to bottom topography, all of the remaining terms on the left-hand side of (2) require significantly non-geostrophic dynamics to be important. In the quasi-geostrophic approximation

$$\bar{\zeta} \ll f \quad \text{and} \quad \frac{\partial \bar{w}}{\partial z} = O\left[\frac{\bar{\zeta}}{f}\frac{\partial \bar{u}}{\partial x}\right],$$

so Eq. (2) reduces to

$$-\left\{f\frac{\partial\bar{w}}{\partial z}\right\}_R = \{\bar{V}\}_R + \{\bar{D}\}_R, \quad (3)$$

and if the domain bottom is flat, $\{f\partial\bar{w}/\partial z\}_R = 0$, so that

$$\{\bar{V}\}_R = \{\bar{D}\}_R. \quad (4)$$

Thus quasi-geostrophic systems with a rigid lid and flat bottom must rely on their dissipation mechanism to provide for the equilibration of the flow.

The interaction of the mean flow with bottom topography, $\{f\partial\bar{w}/\partial z\}_R$ offers another mechanism to offset the vorticity input from the forcing, but in the calculations reported to date (e.g., Gill and Bryan, 1971; Holland and Hirshman, 1972) this term seems to increase the vorticity input to the system rather than diminish it. Unfortunately, there is no way to determine the sign of $\{f\partial\bar{w}/\partial z\}_R$ *a priori* for a baroclinic fluid. The ageostrophic terms in (2) have not been systematically examined in any model calculation known to the author or in the ocean; traditional dynamical assumptions relegate them to unimportance except perhaps over special boundary layers.

When the region R is only a part of the domain $\mathcal{R}$ there can be boundary transports of vorticity from $\{\beta\bar{v}\}_R$, $\{\partial(\bar{u}_j\bar{\zeta})/\partial x_j\}_R$ and $\{\partial(u_j'\zeta')/\partial x_j\}_R$ in addition to the above internal or rigid-boundary processes. These transport terms can be present in a quasi-geostrophic fluid.

For our purposes the region of interest will be denoted by G and is a volume bounded by coasts on the east and west and by the zero wind-stress curl lines on the north and south, and extending from the bottom to the surface. The wind-stress curl is negative within the region if we are interested in the subtropical gyres. Because the region covers the entire depth of the fluid and we are only interested in equilibrium flows, $\{\beta\bar{v}\}_G = 0$, at least in the β-plane approximation. A variety of processes clearly are available, in principle, to balance $\{\bar{V}\}_G$; dissipation seems by no means essential.

3. Boundary eddy lateral vorticity transport as a gyre equilibration mechanism

In Section 2 it was shown that a number of processes exist to remove vorticity from a gyre forced by a negative wind-stress curl. Unfortunately, it is not possible to infer even the sign of the vorticity tendency from most of them, given our limited knowledge of the ocean circulation. The processes responsible for the equilibrium of the world's subtropical gyres are still unknown. In this section we consider the possibility that eddy lateral transport of vorticity, $\{\partial(\overline{u_j'\zeta'})/\partial x_j\}_G$, may provide the needed vorticity to offset the vorticity input by the wind, $\{\bar{V}\}_G$.

From Gauss' theorem

$$\left\{\frac{\partial}{\partial x_j}\overline{u_j'\zeta'}\right\}_G = \int_A \overline{u_j'\zeta'}n_j dA, \quad (5)$$

where A is the area bounding the region G and n_j is the unit outward normal vector. Since there is no eddy flow normal to any rigid boundary, only the open-ocean areas defined by the zero wind-stress curl lines and their projections on the ocean floor will have nonzero contributions to (5). The question of interest is whether these contributions can balance $\{\bar{V}\}_G$.

For definiteness consider the North Atlantic subtropical gyre. The mean wind-stress curl field is given in Fig. 1a, reproduced from Evanson and Veronis (1975) and the North Atlantic mean wind-stress field is given in Fig. 1b (Leetmaa and Bunker 1978). From these data it seems reasonable to take

$$\{\bar{V}\}_G \equiv \{\mathrm{curl}_z\boldsymbol{\tau}\}_G \sim (-5\times10^{-9})(5\times10^8)(4\times10^8)$$
$$\approx -1\times10^9\ \mathrm{cm^3\ s^{-2}}. \quad (6)$$

So that this is about the amount of vorticity that must be transported out of the region G in order to have vorticity equilibrium.

The question thus is whether the Gulf Stream could transport such a quantity of vorticity across the zero wind-stress curl line. Now, if the Gulf Stream mean path follows the zero curl line, we may assume that $\zeta' \sim f$ from observations of the instantaneous Gulf Stream. Further, we know that instantaneous velocities in the Gulf Stream are O (100 cm $\mathrm{s^{-1}}$), at least at upper thermocline and near surface depths. Certainly, if we take $u_0' \sim v_0' = 10$ cm $\mathrm{s^{-1}}$ as a conservative estimate of speeds above 500 m depth there can be no risk of over estimate. We also know that the Gulf Stream meanders vigorously along a considerable length (L) of its path $\mathscr{L}$.

Thus we may estimate the northern zero wind-stress curl line contribution to (5) by

$$\int_0^H\int_{\mathscr{L}} \overline{u_j'\zeta'}n_j d\mathscr{L}dz \sim u_0'f_0C_{u\zeta}(5\times10^4)L$$
$$\approx 50C_{u\zeta}L, \quad (7)$$

and we have ignored the contributions to (7) from depths below 500 m or away from the vigorously meandering area. If $C_{u\zeta} = \mathrm{O}(10^{-1})$ is the typical correlation between eddy velocity and eddy relative vorticity, then L must be O (2×10^8 cm) for (7) to give a total vorticity transport of O(10^9) $\mathrm{cm^3\ s^{-2}}$, the amount required by (6) for this transport to be important in the equilibrium of the gyre. In using

(7) to estimate the total vorticity transport through the gyre boundary by eddies (5), we have assumed that the transport across the southern zero curl line is negligible. As the currents in this region are much weaker and of broader scale than in the instantaneous Gulf Stream, this seems a plausible zeroth-order assumption.

Thus it appears that the Gulf Stream could, in principle, have sufficient eddy lateral vorticity transport to achieve vorticity equilibrium, even using the very conservative velocity estimate of O (10 cm s^{-1}) for eddy speeds, if the Gulf Stream follows the zero wind stress curl line for ~2000 km of its vigorously meandering path.

If the meandering path does not follow the zero curl line for this sort of distance then this transport process appears less likely to be important. The instantaneous vorticity somewhat away from the Gulf Stream is typically $<10^{-1}f_0$ (outside of strong Gulf Stream rings) and $u_0' \approx v_0' = 10$ cm s^{-1} is a rather high estimate of rms thermocline eddy speeds. Thus, even for $L = 5 \times 10^8$ (the width of the North Atlantic) we get an estimated eddy transport of only $O(10^8)$ cm^3 s^{-2}, or roughly one-tenth the needed transport. It is not clear how to make a satisfactory estimate of the average vorticity transport by warm core Gulf Stream rings north of the Gulf Stream, but a rough estimate using $\pi R^2 H f_0$ as the total relative vorticity in a ring of radius R having vorticity comparable to f_0 above a depth H gives the transport resulting from N rings annually as

$$\frac{N\pi R^2 H f_0}{3.2 \times 10^7} \; [\text{cm}^3 \, \text{s}^{-2}]. \qquad (8)$$

Using $f_0 \approx 10^{-4}$, $R = 10^7$ cm and $H = 5 \times 10^4$, it is necessary to have about 20 rings go northward across the zero curl line and decay within the subpolar gyre to get the total transport of 1×10^9 cm^3 s^{-1} required to balance the wind. This is a substantially greater number of rings than has been suggested from observations, but it must be noted that (8) is in no way a sharp bound; this is a difficult estimate to make reliably.

Thus, having the Gulf Stream follow the zero curl line for some distance is crucial to the possibility of getting enough eddy lateral vorticity transport across the northern gyre boundary to effect vorticity equilibration. However, the available data (Leetmaa and Bunker, 1978) suggest that the Gulf Stream does follow the zero wind-stress curl line for a considerable distance across the North Atlantic. It appears possible that the vigorously meandering Gulf Stream somewhere in the region from 35 to 70°W might transfer enough vorticity across the zero wind-stress curl latitude to offset the net wind-stress curl.

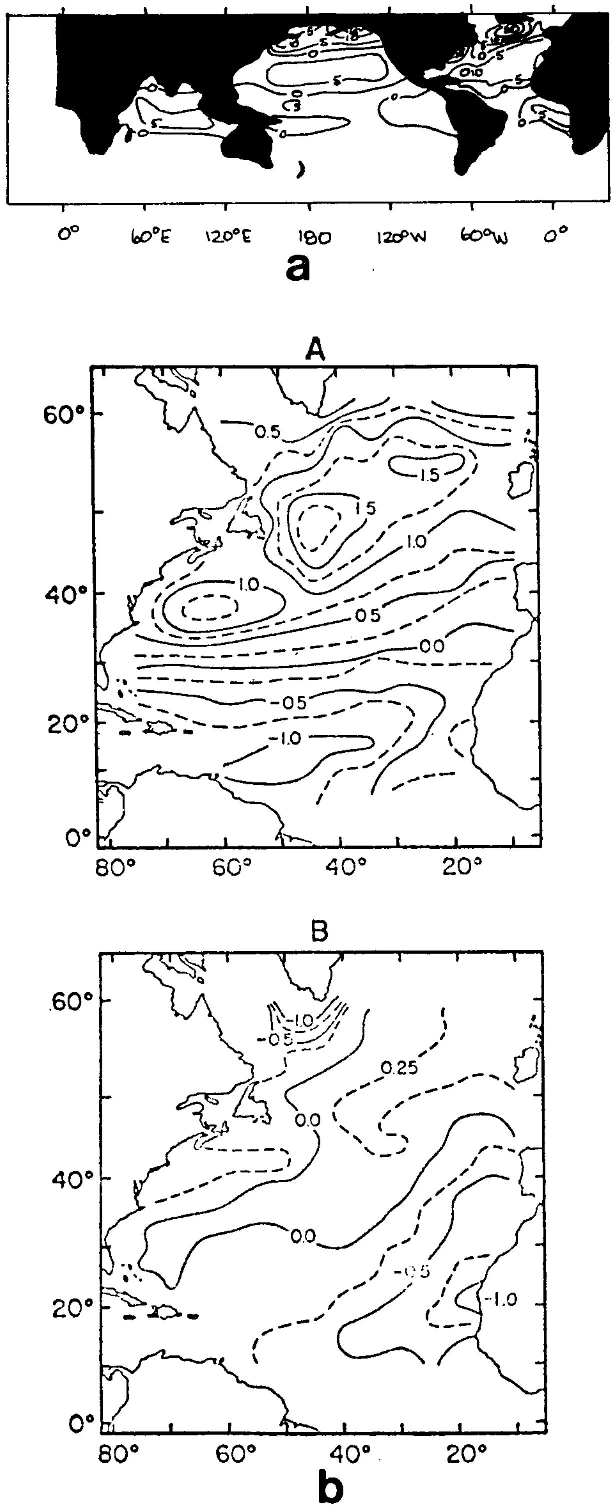

Fig. 1. (a) Annual mean wind-stress curl amplitude over the World Ocean as calculated with a spline interpolation method. Southern Hemisphere data have been largely omitted in this adaptation of Fig. 12 from Evanson and Veronis (1975). Units are 10^{-9} dyn cm^{-3}. (b) Contours of annual mean eastward (A) and northward (B) wind stress over the North Atlantic. Units are dyn cm^{-2}. (From Leetmaa and Bunker 1978).

4. Discussion

The results of Section 3 suggest that eddy lateral transport of vorticity across the northern zero wind stress curl line of the North Atlantic subtropical gyre can be of the same magnitude as the total wind stress curl input of vorticity over this gyre. Thus eddy vorticity transport could be a fundamental equilibration mechanism for this gyre.

In addition to the scale assumptions of Section 3, such a result assumes several other things. The first is that the net amount of negative vorticity supplied to the gyre is of the same order as the wind-stress curl vorticity input, $\{\bar{V}\}_G$. The second is that the sign of $\overline{u_j'\zeta' n_j}$ is relatively uniform across the distance L and negative as required to balance $\{\bar{V}\}_G$.

The first assumption requires that mean flow transport of mean relative vorticity into the gyre, that vortex stretching induced by the bottom topography and that the total vorticity tendency of the LHS terms of (2) together be of at most the same order as $\{\bar{V}\}_G$. That the Gulf Stream transport, properly defined, seems to agree pretty well with zonal integrals of the wind stress curl over much of the gyre (Stommel *et al.*, 1978; Leetmaa *et al.*, 1977) suggests that this assumption might be reasonable.

The second assumption is much more difficult to assess. The upper ocean time series of currents required to estimate the amplitude of $\overline{u_j'\zeta' n_j}$ in the Gulf Stream are not available. Measurement difficulties are formidable in this part of the ocean. A second possible problem concerns the scale of variability of $\overline{u_j'\zeta' n_j}$ in this region. To be important in the gyre balances, it is necessary to have $\overline{u_j'\zeta' n_j}$ negative over a significant fraction of the meandering Gulf Stream track. Again, no ocean data are available. The mechanism seems possible, but we cannot now say if it is likely.

Highly idealized EGCM numerical experiment results show length scales of $\overline{u_j'\zeta'}$ smaller than the $O(10^8$ cm) assumed in (8), but still have this vorticity transport process play a fundamental role in the flow equilibrium. In the EGCM experiment that has been most thoroughly studied, the great bulk of the transport occurs within a hundred kilometers or so of the western wall, as the two model western boundary currents collide head on at the zero wind-curl line (Harrison and Holland, 1981). But an eddy transport of only 2×10^8 cm^3 s^{-2} is needed for equilibrium of this small basin model, and nearly this much can be transported across this small area with reasonable eddy speeds. The North Atlantic must transfer much more negative vorticity northward than must this model flow.

A number of interesting questions arise if this mechanism is as important as suggested here. If the subpolar gyre has a strong western boundary current, again resulting from lack of a mechanism to get rid of sufficient positive vorticity along the western boundary, then the collision of the northward flowing subtropical gyre and southward flowing subpolar gyre boundary currents as they turn eastward could provide much of the vorticity flux needed to equilibrate each gyre, as is found in the numerical model flow discussed above. The Kuroshio/Oyashio system would be a prospect for this type of behavior. Another possibility is that the eddy vorticity transport into the subpolar gyre from the separated, meandering, eastward flowing subtropical gyre boundary current, profoundly alters the Sverdrup relation in part of the subpolar gyre and that the southward flowing subpolar gyre western boundary current has much less transport near the zero wind-stress curl line than it would otherwise. The Gulf Stream/Labrador Current system might fall into this category.

Acceptance of the possibility that the major wind-driven gyres might laterally exchange substantial amounts of vorticity forces reexamination of the single gyre focus that has been basic to wind driven ocean circulation theory from its beginnings. Morgan (1956) understood that what he called Region II, north and east of the maximum wind stress curl latitude at the western coast, was essential to the equilibrium of the subtropical gyre and explicitly considered the possibility that turbulence might play an important role in this region. But many of us since have tended to think of oceanic turbulence on the mesoscale as simply a way of providing access to a small-scale dissipation mechanism, usually assumed connected to boundary processes.

It seems useful to expand the range of possible important roles of the oceanic mesoscale to include the transport process discussed here and to reexamine the world's subtropical and subpolar gyres from this somewhat larger perspective. Hopefully, it will soon be possible to make long-term measurements in the Gulf Stream and the very crude results of Section 3 will be replaced by better estimates of $\overline{u_j'\zeta' n_j}$. Then a much better appraisal of the likelihood that this lateral eddy vorticity transport process plays a fundamental role in the vorticity equilibrium of the North Atlantic will be possible.

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