On the Parameterization of Diapycnal Fluxes due to Double-Diffusive Intrusions

CHRISTOPHER GARRETT

Department of Oceanography, Dalhousie University, Halifax, N.S., Canada B3H 4J1

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ABSTRACT

An attempt is made to parameterize the large-scale average diapycnal (cross-isopycnal) mixing that presumably occurs in the thermohaline fronts that develop when large-scale epipycnal (along-isopycnal) gradients of T and S are stirred along isopycnals by mesoscale eddies. It is assumed that double-diffusive intrusions develop at the fronts and that their thickness is given by the formula of Ruddick and Turner (1979). This, combined with a crude estimate of the frontal width and a very over-simplified model of the eddy field, leads to a formula for the average diapycnal diffusivity for salt or some neutral tracer, and suggests that the mechanism is important in weakly stratified water with a large epipycnal gradient of salinity. The diapycnal eddy diffusivity for temperature is negative for a stably stratified temperature field. However, the opposite signs of the diapycnal diffusivities for salt and heat are unlikely to lead to observable consequences on account of the dominance, in fluxes across isopleths of T or S, of down-gradient epipycnal transports.

1. Introduction

A variety of processes have been proposed as possible causes of diapycnal[1] mixing in the ocean. The main ones are shear instability of internal waves, boundary mixing and double-diffusive processes. In some areas a significant flux across the mean position of isopycnals may be associated with baroclinic instability, but there is still a need for some other process, in series, that is directly linked to mixing at molecular scales.

The magnitude and correct parameterization of diapycnal mixing due to breaking internal waves or boundary mixing are still far from clear. Surprisingly, the best known values for diapycnal mixing coefficients seem to be those from the semiempirical work of Schmitt and Evans (1978) and Schmitt (1981) on the eddy diffusivities for salt and heat, as functions of the vertical gradients of temperature and salinity, in situations where the salinity gradient on its own would be unstable (i.e., salt fingers are possible).

For much of the ocean (particularly the abyssal Pacific) both the temperature and salinity are stably stratified, so that, at first sight, one might not expect double-diffusive processes to play any role in the vertical fluxes of heat and salt. However, CTD casts in such areas do frequently show inversions of T and S (C. S. Cox, personal communication), with vertical scales of tens of meters, and it is clear that stirring, by mesoscale eddies and mean flows, of epipycnal gradients of T and S, should lead to thermohaline fronts where double-diffusive intrusions can occur. The intrusions could arise kinematically if the vertical shear of the eddy or internal-wave field is adequate, but seem more likely to be associated with the dynamic instability mechanisms described by Stern (1967), Ruddick and Turner (1979) and Toole and Georgi (1981).

One can, perhaps, think of these double-diffusive frontal regions as being the means by which the ocean dissipates the large gradients of T and S, along isopycnals, that are produced by stirring. However, the epipycnal mixing process in these fronts is intimately linked with diapycnal mixing, raising the hope that the associated average diapycnal diffusivity, for heat, salt and other tracers, might be predictable from statistical properties of the eddy field combined with an understanding of certain features of the intrusions.

It is quite simple to see that the effect may be important by extrapolating from experience with the rather marked thermohaline fronts that have been studied (e.g., Horne, 1978; Joyce *et al.*, 1978). These tend to have estimated vertical diffusivities of up to 10^{-3} m^2 s^{-1} (though different for heat and salt) and widths of a few kilometers. Suppose that such fronts, with smaller cross-frontal differences but comparable diapycnal diffusivities, occur at the boundaries of eddies with a diameter of order 100 km. The average vertical diffusivity is then around 10^{-5} m^2 s^{-1}, which is big enough to suggest that the problem be investigated further.

In Section 2 the observations and theories of dou-

[1] In this paper "diapycnal" will be used to mean cross-isopycnal and "epipycnal" to describe processes occurring on an isopycnal surface. (The term "diapycnal," introduced by John Shepherd, seems etymologically sensible. The term "epipycnal" is perhaps less clearly required; "isopycnal" could be used instead.)

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ble-diffusive intrusions will be briefly reviewed. Section 3 deals with the parameterization of diapycnal mixing within any frontal region, based on differences between the initial state of the water before intrusions have developed and the final state in the intrusive zone after double-diffusive fluxes have run down unstable vertical gradients of T or S in the intrusive zone. Section 4 is an attempt to combine the results of Section 3 with estimates of the temporal and spatial intermittency of intrusive regions, based on characteristics of the mesoscale eddies and large-scale epipycnal gradients of T and S. Section 5 pursues the consequences of the formulas, from Section 4, for the large-scale average diapycnal diffusivities, and the paper concludes in Section 6 with a discussion of the implications of the paper and suggestions for further work.

2. Double-diffusive intrusions

Intrusions with reversing vertical gradients of T and S are well-documented features of fronts that have substantial epipycnal gradients of T and S (e.g., Gregg, 1975; Voorhis *et al.*, 1976; Gordon *et al.*, 1977; Horne, 1978), and in some recent work (e.g., Joyce *et al.*, 1978) measurements of T and S have been sufficiently accurate to show a decrease, in the direction of intrusions, of the density of warm, salty features. This is as expected if the vertical flux of salt in salt fingers is an important process.

In fact, if laboratory formulas for the vertical fluxes of salt and heat are applied to the intrusions, their lifetime is often found to be a matter of only a few days (e.g., Huppert, 1971; Horne, 1978), which suggests that actively mixing intrusions must be an intermittent phenomenon, and also suggesting that other processes must occur to sweep away the mixed water from persistent fronts if these are to be maintained.

Joyce (1977) has proposed a simple geometrical argument for estimating the lateral fluxes across fronts in terms of the vertical fluxes and the aspect ratio of the intruding features. The model is clearly sensitive to guesses at the diapycnal diffusivity in the intrusion region as well as requiring data on aspect ratios, so that although it is useful in a practical situation (e.g., Joyce *et al.*, 1978), one would like a dynamically-based and less data-dependent formula for general use.

Quite apart from the documented change in the density of intrusions, which suggests that they are dynamically active features, Georgi (1978) and Joyce *et al.* (1978) have shown that they extend much farther laterally than can be accounted for as a consequence of purely passive advection by internal waves. One thus seeks mechanisms by which double-diffusive intrusions could drive themselves across fronts.

The simplest situation envisaged would have flat sheets intruding some very small distance laterally on account of some random perturbation, and then continuing to intrude as double-diffusive fluxes cause lateral gradients of density and hence pressure. This is essentially the situation explored in the laboratory by Ruddick and Turner (1979, henceforth RT). However, if h is layer thickness, N the Väisälä frequency associated with the basic density stratification and f the Coriolis frequency, the intrusions could not be expected to intrude farther, without friction, than a distance of approximately the geostrophic adjustment scale Nh/f. As N/f is also the approximate aspect ratio (i.e., vertical shear of horizontal displacement) of internal waves (Gargett *et al.*, 1981), whose lateral displacements are inadequate, we conclude that a frictionless sheet model is also likely to be inadequate.

Including friction will clearly allow an intruding layer to spread farther than the internal Rossby radius, but the rate of intrusion is a sensitive function of the amount of friction. Stommel and Fedorov (1967) found that the rate of intrusion of a well-mixed layer into a stratified environment is small for small values of the eddy viscosity in a model, as the lateral flux is then confined to thin Ekman layers. The rate is also small, for obvious reasons, for large values of the eddy viscosity, and so must reach a maximum at some intermediate value for the viscosity. For any value of the eddy viscosity, the thickness of the intruding layer in fact satisfies a diffusion equation, with a variable lateral diffusivity. The model would have to be extended to include the changes in layer density due to double diffusion if it were to be applied to double-diffusive intrusions, but the predictions will, in any event, be rather sensitive to the parameterization of friction.

While RT's laboratory experiments, and the above discussion, relate to the movement of intrusions across an initially sharp thermohaline front, Stern (1967) has discussed an instability that would lead to cross-frontal motion in any region with epipycnal gradients of T and S. The basic mode of instability has inclined, rather than flat, sheets moving across the front, as illustrated in Fig. 1, where it is assumed that the mean salinity decreases in the x-direction. As the top layer in the sketch intrudes, it finds itself hotter and saltier than the layers above and below it. Given that fluxes of salt and heat through the bottom of the layer, associated with salt-fingering, are generally greater than the fluxes through the upper surface associated with the diffusive instability, the density of the layer will decrease. Similarly, the density of the layer below, coming from a region with cooler, fresher water, will increase. These changes in density, if not accompanied by vertical motion, cause a horizontal density gradient, which is compatible with the assumed motion of the layers through the thermal-wind equation

$$\partial u'/\partial z = g\rho_0^{-1}(\partial\rho'/\partial y), \tag{2.1}$$

where u' is the velocity component in the x-direction, and ρ' the density perturbation. In other words, the Coriolis force on the cross-frontal flow is balanced by a pressure gradient along the front.

The physical reasoning for this inviscid instability mechanism is supported by Stern's (1967) mathematical model, in which the double-diffusive fluxes are modelled in terms of a vertical eddy diffusivity for salt and a smaller one for heat. The vertical symmetry of such a parameterization is obviously not correct, but it does produce the appropriate lightening of a hot, salty intrusion.

Stern's (1967) linearized instability theory, and its extension by Toole and Georgi (1981, henceforth TG), assumes constant cross-frontal gradients of the mean temperature and salinity fields. However, with Stern's neglect of viscosity, the velocity component v' parallel to the front does not enter the governing equations, and so may be determined from u' even if the salinity gradient is not constant. For

$$u' = \mathrm{Re}\{u_0(x)\exp[\lambda(x)t + i(ly + mz)]\}, \tag{2.2}$$

the along-front flow is given by

$$v' = \mathrm{Re}\{il^{-1}[(du_0/dx) + u_0(d\lambda/dx)t]\exp[\lambda t + i(ly + mz)]\}. \tag{2.3}$$

The linearized inviscid theory does not predict any flow right across a front, but merely flow within the frontal region itself, with density gradients across the front being balanced by geostrophic flow along the front.

Stern's (1967) inviscid analysis shows that, for a given vertical scale of the intruding sheets, there is a horizontal scale along the front (i.e., an along-front slope of the sheets) which will maximize the growth rate of the instability. However, this maximum growth rate increases without bound as the vertical scale decreases, as discussed by TG. They extend Stern's theory to include viscous effects, and find that the maximum growth rate occurs at a finite vertical scale, but still with a finite horizontal scale, so that the momentum balance of the growing intrusion is still largely geostrophic.

The growth rate and wavenumbers of the fastest-growing mode, as calculated by TG, depend on the vertical as well as horizontal mean gradients, and also on the ratio σ of eddy viscosity to eddy diffusivity for salt. However, the sensitivity of their results to the vertical gradient of salinity is small, and to within less than a factor of 2 we may take the maximum growth rate as

$$\lambda_{\max} = 0.25(1 - \gamma)g\beta\bar{S}_x N^{-1}\sigma^{-1/2}, \tag{2.4}$$

where β is $\rho^{-1}(\partial\rho/\partial S)_T$, $\bar{S}_x$ the mean horizontal salinity gradient, and γ the ratio of heat diffusivity to

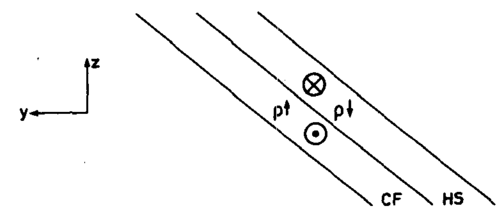

FIG. 1. Schematic of Stern's (1967) instability mechanism for double-diffusive lateral intrusions.

salt diffusivity in the intrusions. This is taken as the ratio of heat to salt fluxes (or, rather, the associated density fluxes) across a salt-fingering interface, with values from laboratory experiments ranging from 0.56 (Turner, 1967), to 0.7 (Schmitt, 1979). TG found the corresponding vertical wavenumber to be approximately

$$m = [0.5(1 - \gamma)g\beta\bar{S}_x N^{-1}\sigma^{-1/2}A_S^{-1}]^{1/2} = (2\lambda_{\max}/A_S)^{1/2}, \tag{2.5}$$

where A_S is the diffusivity for salt in the intrusions. They point out that this predicts a layer thickness that is inversely dependent on the salinity gradient $\bar{S}_x$ across the front (ignoring any dependence of A_S on $\bar{S}_x$ or layer thickness). As they remark, this contrasts with the linear dependence of layer thickness on salinity difference across a front, predicted by the energy and stability arguments of RT.

In fact, if h is the height of a pair of layers intruding across a sharp thermohaline front, equivalent to $2\pi/m$ in TG's theory, RT find

$$(1 - \gamma)g\beta\Delta S/N^2 < h < \tfrac{3}{2}(1 - \gamma)g\beta\Delta S/N^2, \tag{2.6}$$

where ΔS is the salinity difference across the front. In (2.6) the upper bound comes from the condition that, after salt fingers have removed salinity inversions caused by intruding layers, which are vertically mixed in the process, there should have been a net loss of potential energy. The lower bound comes from the condition that the lower layer of one pair of layers should not become denser than the upper layer of the pair of layers immediately below.

In summary, then, Stern (1967) and TG describe the initial instability of a diffuse front and find the growing intrusions to be significantly tilted in the along-front direction. RT examine the run-down state of flat intrusions across a sharp front.

As already remarked, the run-down time is typically only a few days, as is the growth time of instabilities estimated by TG for typical values of their parameters. Hence, on the time scale of stirring and frontal formation by mesoscale eddies (perhaps tens of days), we might expect intrusions to evolve to some run-down state to which RT's ideas apply, even if the intrusions originate as an instability of a fairly diffuse front. We will return to this problem in Section 4,

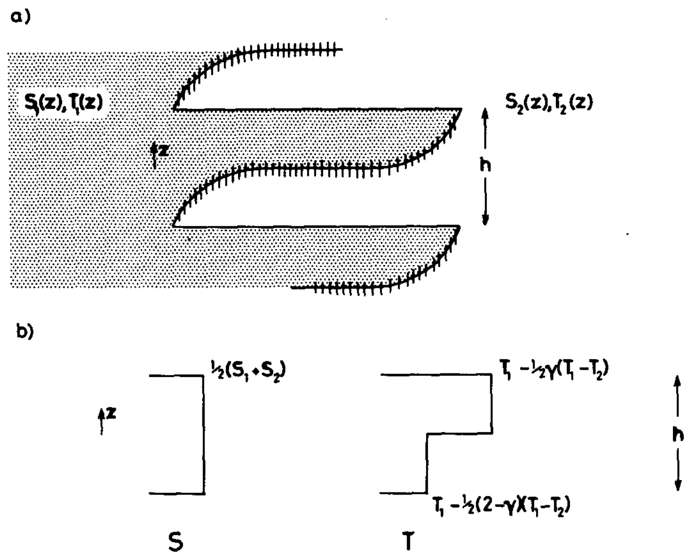

FIG. 2. Schematic (a) of double-diffusive intrusions across a thermohaline front, with salt fingers from the saltier waters (shaded) to the less salty (clear); and (b) the assumed profiles of S, T, in the run-down state, denoting $S_1(0) = S_1$, etc.

while discussing appropriate cross-frontal scales for intrusions, but for now we shall assume a scenario like that of RT and evaluate its consequences.

3. Parameterization of local mixing

We envisage an initially sharp thermohaline front, with flat isopycnals, with different profiles of temperature T and salinity S on each side, and a linear vertical density gradient. We assume that cross-frontal double-diffusive intrusions develop, but only consider for the moment the fluxes of salt and heat between intrusions associated with salt fingers. Following RT we assume that the salt fingers eventually remove the salinity differences within each double layer, but that the ratio of density fluxes associated with heat and salt is a constant γ so that the upper layer ends up with a temperature $T_1 - \frac{1}{2}\gamma(T_1 - T_2)$, and the lower layer, $T_1 - \frac{1}{2}(2-\gamma)(T_1 - T_2)$, as sketched in Fig. 2.

RT considered the potential energy of the region $|z| < \frac{1}{2}h$ before and after mixing. If the Väisälä frequency of the initial state is N, the initial P.E. is $-\frac{1}{12}\rho_0 N^2 h^3$ with respect to the level $z = 0$. The P.E. of the final state is $-\frac{1}{8}g\rho_0\delta(1-\gamma)h^2$, where $\delta = \alpha(T_1 - T_2) = \beta(S_1 - S_2)$ is the fractional density change across the front due either to temperature or salinity, $\alpha = -\rho^{-1}(\partial\rho/\partial T)_S$ and $\beta = \rho^{-1}(\partial\rho/\partial S)_T$ as before, and we assume $\delta > 0$. From the required decrease in P.E., RT found that $h < \frac{3}{2}(1-\gamma)g\delta N^{-2}$.

It is instructive to consider the changes in P.E. associated with T and S separately. For S the final P.E. is zero, and the initial P.E. is the average of that on the two sides of the front, i.e.

$$\frac{1}{2}\int_{-h/2}^{h/2} g\rho_0\beta[S_1(z) + S_2(z)]zdz.$$

For linear salinity gradients, this is $\frac{1}{24}g\rho_0\beta \times (S_{1z} + S_{2z})h^3$. If $S_{1z} + S_{2z} < 0$ (as for salinity stratification that would, on its own, be stable), we see that the P.E. associated with the salinity field has *increased.* This means that the source of energy to drive the double-diffusive intrusions must be the density stratification associated with the temperature field, even though the local source of energy within the intrusions must be the locally unstable salinity field that drives the salt fingers. Within the front as a whole, the upward flux of salt in the rising salty intrusions must exceed the downward flux in the salt fingers.

Another way of expressing this conclusion is that, if $S_{1z} + S_{2z} < 0$, the effective vertical eddy diffusivity for salt due to the frontal intrusions is positive. However, if $T_{1z} + T_{2z} > 0$ (a stable situation for temperature alone), the eddy diffusivity for heat is negative! The eddy diffusivity for density is, of course, negative.

The simple connections between eddy diffusivities and changes in P.E. are worth stating. Suppose C is some scalar with mean $\bar{C}(z, t)$ and fluctuations $C'(x, y, z, t)$, such that $\partial\bar{C}/\partial t = -\nabla\cdot(\overline{\mathbf{u}'C'})$. Integrating by parts over a volume within which mixing is occurring yields

$$\frac{\partial}{\partial t}\int \bar{C}zdV = \int \overline{w'C'}dV = -\int K_C(\partial\bar{C}/\partial z)dV. \quad (3.1)$$

If $C = g\rho$ we have

$$\partial(\text{P.E.})/\partial t = \int K_\rho \rho_0 N^2 dV; \qquad (3.2)$$

the changes in P.E. associated with changes in T and S may similarly be related to their eddy diffusivities.

The actual values for the diffusivities of salt, heat and density are given by

$$K_S = {}^1\!/_{12} h^2 \tau^{-1}, \qquad (3.3)$$

$$K_T = -{}^1\!/_{12} h[h_0(1 - \beta\bar{S}_z/\alpha\bar{T}_z) - h]\tau^{-1}, \qquad (3.4)$$

$$K_\rho = -{}^1\!/_{12} h(h_0 - h)\tau^{-1}, \qquad (3.5)$$

where $h_0 = {}^3\!/_2(1 - \gamma)g\delta N^{-2} > h$ and τ is the time interval between successive involvement in frontal intrusions for a given water parcel, assuming that in each episode mixing proceeds to the run-down state and then stops. (Actually, these formulas are not quite correct, as we are starting with linear gradients for S, T and ρ and finishing with layers.) We note that K_S is always positive and K_ρ negative; K_T is negative if $\bar{T}_z > 0$ and $\bar{S}_z < 0$, but may be positive if either the salinity or temperature is unstably stratified (see Fig. 3). We also point out that the above formulas satisfy the equation

$$K_\rho N^2 = -K_S g\beta\bar{S}_z + K_T g\alpha\bar{T}_z, \qquad (3.6)$$

which says that the total change in potential energy is the sum of the changes associated with the density stratification due to S or T.

If either S or T has a background stratification which is unstable, then of course double-diffusive fluxes can occur without requiring intrusions as a trigger. The total diapycnal diffusivity will then be that associated with the basic state, together with that occurring in the intrusive frontal regions.

In the scenario described so far it has been assumed that only the salt-finger fluxes from hot salty water down into cold fresh water below are important, and that the diffusive fluxes from hot salty water up into cold fresh water above may be ignored. If the reverse were true, most of the signs of the results discussed above would change. The salinity field would now be the overall energy source, although the temperature gradient would be driving the mixing locally, and the signs of K_S and K_T would change. In some intrusive situations (e.g., Horne, 1978), the profiles of S and T, when combined with laboratory flux laws, do suggest a dominance of fluxes across diffusive rather than salt-fingering interfaces, but in general the reverse is true. We shall thus assume that the scenario described so far reasonably describes the initial development of intrusions.

However, it should be noticed that even after the salt fingering has removed the salinity difference in each pair of layers, the resulting stratification is still suitable for diffusive instability to occur. This is clear from the sketch in Fig. 4 of S, T and ρ profiles through

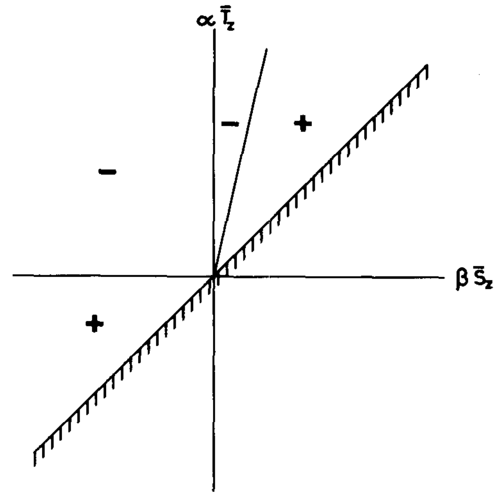

FIG. 3. Sign of the effective vertical diffusivity for heat as a function of $\beta\bar{S}_z$ and $\alpha\bar{T}_z$.

two double layers. If $\Delta'S$, $\Delta'T$ and $\Delta'\rho$ denote the differences in T, S and ρ between the upper layer of the lower pair of layers and the lower layer of the upper pair, we have

$$\Delta'S = -\bar{S}_z h, \qquad (3.7)$$

$$\Delta'T = (1 - \gamma)(T_1 - T_2) - \bar{T}_z h, \qquad (3.8)$$

$$\Delta'\rho/\rho_0 = \beta\Delta'S - \alpha\Delta'T = g^{-1}N^2 h - (1 - \gamma)\delta, \qquad (3.9)$$

where $N^2 = -g(\beta\bar{S}_z - \alpha\bar{T}_z)$ is the square of the Väisälä frequency of the ambient fluid and $\delta = \alpha(T_1 - T_2)$. As pointed out by RT, static stability requires $\Delta'\rho > 0$, so that $h > (1 - \gamma)g\delta N^{-2}$. Adding RT's other condition, $h < {}^3\!/_2(1 - \gamma)g\delta N^{-2}$, we may write (3.8) as

$$1 - {}^3\!/_2 g\alpha\bar{T}_z N^{-2} < \alpha\Delta'T(1 - \gamma)^{-1}\delta^{-1} < 1 - g\alpha\bar{T}_z N^{-2}. \qquad (3.10)$$

Depending on the relative contributions of the ambient T and S profiles to N^2, $\Delta'T$ may well be constrained by (3.10) to be positive. In particular, if the

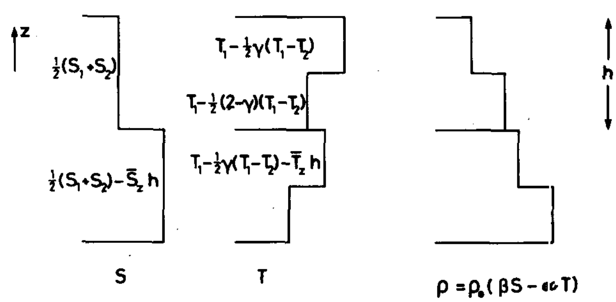

FIG. 4. Schematic of S, T and ρ profiles through two double layers after salt fingers have removed salinity differences within each double layer. The labels follow the notation of Fig. 2, with $\bar{S}_z = {}^1\!/_2(S_{1z} + S_{2z})$ and $\bar{T}_z = {}^1\!/_2(T_{1z} + T_{2z})$.

stratification arises equally from T and S, the lower limit in (3.10) is ¼. If $\Delta' T$ is positive, diffusive fluxes should occur across the interface between the two double layers. A quantitative assessment of the magnitude of this cannot be made in view of the range of permitted h, and the results will clearly also be sensitive to the relative importance of $\bar{S}_z$ and $\bar{T}_z$. However, we note that for $\bar{T}_z > 0$ and $\bar{S}_z < 0$ the diffusive fluxes will increase the potential energy associated with the salinity field, and decrease that associated with the temperature field, as for the original changes associated with the intrusions.

The scenario for an intrusive front thus involves a rapid run down, by salt fingers, of salinity inversions caused by the intrusions, as envisaged by RT, followed by a slower run down of temperature inversions. In practice, of course, both types of double diffusion will proceed simultaneously, but the typically greater fluxes associated with salt fingers will probably ensure that, for $\bar{T}_z > 0$ and $\bar{S}_z < 0$, the net result will show an increase in P.E. associated with salinity, and a decrease associated with temperature.

Incorporating these concepts into a parameterization of vertical mixing for use in models of larger scale processes will require estimates of the intermittency of frontal intrusions in time and space. This will be discussed in Section 4. Meanwhile, we note that the effective vertical diffusivity for salt, associated with the effects of the salt fingering, is $\frac{1}{12}h^2\tau^{-1}$, where τ is the time interval between successive involvement in frontal intrusions for a given water parcel. A passive scalar with a small diffusivity will tend to diffuse with the salt and so have an effective vertical eddy diffusivity equal to that for salt. We ignore the addition to the value $\frac{1}{12}h^2\tau^{-1}$ that might arise from the hypothesized diffusive fluxes that might run down temperature inversions after salt fingers have run down salinity inversions.

One final point is worth a brief discussion before moving on to an attempt at parameterizing frontal intermittency. The RT energy argument compares only the potential energy levels before and after mixing. In fact the cross-frontal density gradients that develop within each double layer during the intrusion process will tend to generate geostrophic currents, and hence kinetic energy that must come from the potential energy of the basic state. However, provided the intrusions spread more than an internal Rossby radius, of order Nh/f, this kinetic energy is small compared with the potential energy and may be ignored.

4. Parameterization in large-scale models

We envisage a field of mesoscale eddies stirring up the horizontal gradients of T and S on isopycnal surfaces, and steepening the epipycnal gradients to the point where intrusive instabilities can develop. We assume that a frontal region within which the intrusions occur maintains its width, against the further sharpening that would otherwise be produced by the eddy field, until the frontal convergence becomes a divergence.

A proper statistical treatment of this scenario will be difficult, but some guidance may be obtained from the simple deterministic eddy field sketched in Fig. 5, with the streamfunction $\psi = A \sin(\pi x/D) \sin(\pi y/D)$. We suppose that thermohaline fronts of width W form at the center of regions of convergence (the centers of the shaded areas in Fig. 5), and that the resulting mixed water is expelled, from the ends of the shaded regions, with the local speed $A(\pi/D)$. The rate of supply of mixed water is then $2WA(\pi/D)$ for each area D^2, and so the time τ for use in (3.3)–(3.5) is $D^3(2WA\pi)^{-1}$, which may be written approximately as $D(W\Omega)^{-1}$, where $\Omega = 2^{-1/2}A(\pi/D)^2$ is the rms strain rate of the assumed eddy field.

To estimate the thickness h of a double layer in the intrusive region, we need a value of ΔS, the cross-frontal salinity difference on an isopycnal. We take this to be approximately the eddy length scale D times the large-scale mean epipycnal salinity gradient, which we denote $\bar{\bar{S}}_x$ to distinguish it from some smaller scale mean gradient $\bar{S}_x$, across a front, that one might use in the theories of Stern (1967) and TG. We also use RT's laboratory result that h is in fact close to the calculated upper limit $\frac{3}{2}(1-\gamma)g\delta N^{-2}$, which is then approximately $\frac{1}{2}Dg\beta\bar{\bar{S}}_x N^{-2}$. We then have a diffusivity for salt, or a tracer, given by

$$K_S \approx 0.02DW\Omega(g\beta\bar{\bar{S}}_x/N^2)^2 \tag{4.1}$$

and it remains only to estimate W.

One way of estimating W is to assume that significant mixing will occur when a front becomes sharp enough that the growth rate of the instability studied by Stern (1967) and TG becomes comparable with the strain rate Ω of the eddy field. With $\sigma = 1$ and $\gamma = 0.7$ in (2.4) this gives

$$g\beta\bar{S}_x \approx 13N\Omega. \tag{4.2}$$

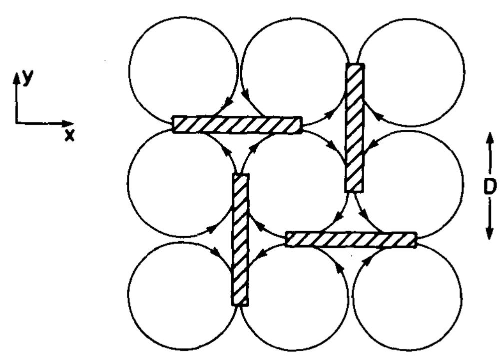

FIG. 5. Schematic of eddy field, with streamfunction $\psi = A \times \sin(\pi x/D) \sin(\pi y/D)$. The shaded areas are the regions where fronts are assumed to occur.

If we assume that this gradient is achieved by squeezing a salinity difference $D\bar{\bar{S}}_x$ into a width W, we have

$$W = 0.075(g\beta\bar{\bar{S}}_x/N^2)DN\Omega^{-1}. \quad (4.3)$$

We will check later, for typical values of the parameters, that the ratio W/D is, as assumed, much less than 1, and that W is comparable with observed frontal widths.

Using (4.3) in (4.1) leads to

$$K_S \approx 1.5 \times 10^{-3}D^2N(g\beta\bar{\bar{S}}_x/N^2)^3, \quad (4.4)$$

in which we notice that the strain rate Ω has disappeared; as Ω increases the along-front speed increases but the frontal width decreases.

A lower bound for W could be estimated from the assumption that the layers spread to at least a Rossby radius Nh/f, so that

$$W \approx 0.5(g\beta\bar{\bar{S}}_x/N^2)DNf^{-1}. \quad (4.5)$$

This is a fraction $7(\Omega/f)$ times the estimate in (4.3), or 0.07 if we take $\Omega = 10^{-6}$ s^{-1} and $f = 10^{-4}$ s^{-1}.

Alternatively, we could assume that intrusions of the sort described by RT propagate at a rate ϵNh, where ϵ is some number less than 1, and that the width achieved by the front is such that this speed is balanced by the speed of convergence, i.e., $\frac{1}{2}W\Omega = \epsilon Nh$. This leads to

$$W = \epsilon(g\beta\bar{\bar{S}}_x/N^2)DN\Omega^{-1}, \quad (4.6)$$

exactly similar in form to (4.3), suggesting that (4.4) does at least have the correct dependence on external parameters. For the purposes of making numerical estimates of K_S, we round off (4.4) to

$$K_S \approx 10^{-3}D^2N(g\beta\bar{\bar{S}}_x/N^2)^3. \quad (4.7)$$

Note that this is the prediction for the large-scale average vertical eddy diffusivity, averaged over frontal and non-frontal areas.

The form of (4.7) immediately suggests that diapycnal mixing due to double-diffusive intrusions will be most effective in regions of the ocean with weak stratification and substantial epipycnal gradients of S (and T). For the abyssal South Pacific at high latitudes we might take $N = 10^{-3}$ s^{-1} and $\bar{\bar{S}}_x = 10^{-7}$‰ m^{-1} (Reid, 1981). This gives $g\beta\bar{\bar{S}}_x/N^2 \approx 10^{-3}$ and, with $D \approx 100$ km, we have $K_S \approx 10^{-5}$ m^2 s^{-1}, which is neither big enough to confirm the importance of the process, nor small enough to dismiss it! For the core of the Mediterranean outflow we might take a range of 2×10^{-7}‰ m^{-1} to as high as 2×10^{-6}‰ m^{-1} for $\bar{\bar{S}}_x$ (see Fig. 1 of Needler and Heath, 1975). With $N^2 \approx 5 \times 10^{-6}$ s^{-2} we have $g\beta\bar{\bar{S}}_x/N^2 \approx 3 \times 10^{-4}$ to 3×10^{-3} and $K_S \approx 10^{-6}$ to 10^{-3} m^2 s^{-1}, the latter value certainly appearing to be important. To this value we must add a value appropriate to the double diffusion that could occur in the water column without interleaving. Below the Mediterranean tongue Schmitt (1981) suggests a value of about 10^{-3} m^2 s^{-1}, which would dominate.

With the South Pacific parameter values we note that the double-layer thickness, $h = \frac{1}{2}Dg\beta\bar{\bar{S}}_xN^{-2}$, is ~50 m, which seems reasonable, and, from (4.3), $W/D \approx 7.5$ km, which also seems reasonable.

In more strongly stratified parts of the ocean, with weaker epipycnal gradients, h, W and K_s will also be reduced, with the effective diffusivity becoming small enough to be negligible.

5. Consequences

In Section 4 we concentrated on deriving a formula for the effective diapycnal eddy diffusivity K_S, that should apply to salinity or some other soluble tracer. The final formula [(4.7)] is clearly rather uncertain. Corresponding formulas for heat and density would be even more uncertain as we do not know the ratio h/h_0. However, K_T and K_ρ will both have magnitudes comparable to that of K_S, and we note that K_ρ is always negative, and K_T is negative if $\bar{T}_z > 0$ and $\bar{S}_z < 0$.

If K_S is positive and K_T negative, one might expect to see the downstream development of a step, parallel to the T-axis, in the T–S diagram of a water mass. However, the evolution of the T–S structure is affected by lateral (epipycnal) diffusion as well as diapycnal diffusion. Denoting the diapycnal diffusivity K_v and the epipycnal diffusivity K_h, the ratio of their contributions to the fluxes down-gradient is given by $K_v(K_h\theta^2)$, where θ is the slope of an isoline relative to an isopycnal. For $K_h = 10^{-1}D^2\Omega$ (which gives 10^3 m^2 s^{-1} for the values $D = 100$ km and $\Omega = 10^{-6}$ s^{-1} which we have been using), we have, for salt

$$K_v/(K_h\theta^2) \approx 10^{-2}(N/\Omega)(g\beta\bar{\bar{S}}_x/N^2)(g\beta\bar{S}_z/N^2)^2 \quad (5.1)$$

as $\theta = -\bar{S}_z/S_x$. Assuming $g\beta\bar{S}_z/N^2$ is not much different from 1, and with the other parameters as before, this gives $K_v/(K_h\theta^2) \approx 10^{-2}$. A similar value would apply for T, suggesting that even though the diapycnal diffusion is up-gradient, the net *diathermal* diffusion of heat is down-gradient on account of the dominance of the epipycnal diffusion. The evolution of a T–S diagram is likely to be dominated by lateral diffusion, which is presumably equal for heat and salt and hence does not produce any startling consequences.

6. Discussion

A speculative scenario has been developed in an attempt to estimate the magnitude of diapycnal mixing that is presumably associated with the thermohaline fronts that form on isopycnal surfaces as a result of the stirring by mesoscale eddies of large-scale epipycnal gradients of T and S. The model relies heavily on the adoption of Ruddick and Turner's (1979) estimate of the thickness of intrusive layers, on a grossly oversimplified model of mesoscale eddies, and on a crude estimate of the width of a frontal zone within which intrusions develop.

The final prediction of the model [Eq. (4.7)] suggests that the process may lead to significant diapycnal mixing in regions of large epipycnal gradients of T and S and weak density stratification, but it is probably negligible elsewhere. Another key result, which is a direct consequence of Ruddick and Turner's (1979) model for double-diffusive intrusions, is that while the effective eddy diffusivity for salt is positive, that for heat will be negative if temperature increases upward. However, a comparison of diathermal mixing by epipycnal and diapycnal mixing suggests the dominance of the former, so that the negative diapycnal diffusivity is unlikely to lead to observable consequences in, say, the T–S diagram of a water mass.

More theoretical work on double-diffusive intrusions is clearly required to discriminate between, or combine, the approaches of Stern (1967) and Toole and Georgi (1981) on the one hand, and Ruddick and Turner (1979) on the other. In this paper I have basically assumed that the growth rates calculated by the former authors are appropriate, but that the layer thickness and final distribution of properties can be taken from the model of the latter. A fundamental difference between the two models is in the extent to which the intrusive layers are tilted in the along-front direction. Further field work related to this question would be particularly useful if it could be carried out at an epipycnal thermohaline front that is not, as in many previous studies, associated with a horizontal density front.

Further field work is also clearly required to establish the frequency and characteristics of thermohaline fronts, with associated double-diffusive intrusions, in regions, such as the abyssal South Pacific, where the associated diapycnal mixing may be significant. The occurrence of thermohaline fronts should be related to the local flow patterns associated with mesoscale eddies, both in the field and in numerical simulations of eddy fields, in an attempt to calibrate, or improve on, the formula given in Eq. (4.7).

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REFERENCES

Gargett, A. E., P. J. Hendricks, T. B. Sanford, T. R. Osborn and A. J. Williams, III, 1981: A composite spectrum of vertical shear in the upper ocean. *J. Phys. Oceanogr.*, **11**, 1258–1271.

Georgi, D. T., 1978: Finestructure in the Antarctic Polar Front zone: Its characteristics and possible relationship to internal waves. *J. Geophys. Res.*, **83**, 4579–4588.

Gordon, A. L., D. T. Georgi and H. W. Taylor, 1977: Antarctic Polar Front zone in the western Scotia Sea—Summer 1975. *J. Phys. Oceanogr.*, **7**, 309–328.

Gregg, M. C., 1975: Microstructure and intrusions in the California Current. *J. Phys. Oceanogr.*, **5**, 253–278.

Horne, E. P. W., 1978: Interleaving at the subsurface front in the Slope Water off Nova Scotia. *J. Geophys. Res.*, **83**, 3659–3671.

Huppert, H. E., 1971: On the stability of a series of double-diffusive layers. *Deep-Sea Res.*, **18**, 1005–1021.

Joyce, T. M., 1977: A note on the lateral mixing of water masses. *J. Phys. Oceanogr.*, **7**, 626–629.

——, W. Zenk and J. M. Toole, 1978: The anatomy of the Antarctic Polar Front in the Drake Passage. *J. Geophys. Res.*, **83**, 6093–6113.

Needler, G. T., and R. A. Heath, 1975: Diffusion coefficients calculated from the Mediterranean salinity anomaly in the North Atlantic Ocean. *J. Phys. Oceanogr.*, **5**, 173–182.

Reid, J. L., 1981: On the mid-depth circulation of the world ocean. *Evolution of Physical Oceanography*, B. A. Warren and C. Wunsch, Eds., MIT Press, 70–111.

Ruddick, B. R., and J. S. Turner, 1979: The vertical scale of double-diffusive intrusions. *Deep-Sea Res.*, **26A**, 903–913.

Schmitt, R. W., 1979: Flux measurements on salt fingers at an interface. *J. Mar. Res.*, **37**, 419–436.

——, 1981: Form of the temperature-salinity relationship in the Central Water: Evidence for double-diffusive mixing. *J. Phys. Oceanogr.*, **11**, 1015–1026.

——, and D. L. Evans, 1978: An estimate of the vertical mixing due to salt fingers based on observations in the North Atlantic Central Water. *J. Geophys. Res.*, **83**, 2913–2919.

Stern, M. E., 1967: Lateral mixing of water masses. *Deep-Sea Res.*, **14**, 747–753.

Stommel, H., and K. N. Fedorov, 1967: Small-scale structure in temperature and salinity near Timor and Mindanao. *Tellus*, **19**, 306–325.

Toole, J. M., and D. T. Georgi, 1981: On the dynamics and effects of double-diffusively driven intrusions. *Prog. Oceanogr.*, **10**, 123–145.

Turner, J. S., 1967: Salt fingers across a density interface. *Deep-Sea Res.*, **14**, 599–611.

Voorhis, A. D., D. C. Webb and R. C. Millard, 1976: Current structure and mixing in the shelf/slope water front south of New England. *J. Geophys. Res.*, **81**, 3695–3708.