Estimates of Mass, Momentum and Kinetic Energy Fluxes of the Gulf Stream

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(Manuscript received 24 November 1982, in final form 9 June 1983)

ABSTRACT

Mass, momentum and kinetic-energy fluxes in the Gulf Stream have been estimated from hydrographic data taken by Fuglister in the Gulf Stream '60 project; the data cover the Stream as it flows eastward, from south of Georges Bank to the Grand Banks. The results are compared to a two-layer, constant potential-vorticity inertial-jet model and reasonable agreement is found. Error estimates based on the model and the data indicate errors of up to about 30% for mass and momentum and 50% for kinetic energy fluxes. All three fluxes exhibit considerable downstream divergence; the dynamical implications of these divergences for the region are assessed, and the importance of nonlinear effects in the Stream is discussed. It is suggested that there may be a significant conversion of kinetic to potential energy and that this mechanism ought not be excluded *a priori* by examining primarily linear models of the Stream.

1. Introduction

Understanding of the energetics of the circulation in the North Atlantic requires an accounting of the large fluxes of momentum and kinetic energy in the Gulf Stream. Since the flux of kinetic energy, for example, must be at least comparable to the energy input by wind stress over the entire ocean basin, significant nongeostrophic effects can be expected in the Gulf Stream region. These effects need to be evaluated to assure a consistent interpretation of the circulation and associated heat and salt fluxes in the region. The present study was undertaken to estimate the momentum and kinetic energy fluxes of the Gulf Stream in the region between Cape Cod and the Grand Banks of Newfoundland, using hydrographic stations from the meridional sections taken in Gulf Stream '60 (Fuglister, 1963). The study was motivated in part by the observation by Stommel *et al.* (1978) that the geopotential anomaly is higher on the eastern side of the North Atlantic than on the western side. They interpreted this difference as an indication of barotropic recirculation. The possibility that the differences may be due to momentum flux divergence is explored in the present paper.

Mass or volume transports of the Gulf Stream have been estimated by many authors. Fuglister (1963) computed volume transports relative to 2000 m and to the bottom for each of the nine meridional sections taken in the Gulf Stream '60 survey. Worthington (1976) has tabulated volume transport estimates for a 35-year period from 1932 to 1968 to search for seasonal variability. Comparable studies of the momentum and kinetic energy fluxes have apparently not been made.

The circulation in the North Atlantic is driven by a combination of energy sources. Potential energy is accumulated in the ocean interior by the action of wind stress and heating. The potential energy is converted to kinetic energy in the Gulf Stream as the flow accelerates to its maximum between Cape Hatteras and Cape Cod. At the maximum, the energy flux must occur entirely as a kinetic energy flux. As the flow decelerates further downstream, kinetic energy is removed from the Stream and transferred to other forms. Presumably, internal dissipation acts throughout the Stream to extract kinetic energy. The downstream pressure gradient reverses to act against the flow and convert kinetic to potential energy. The Stream forms and ejects eddies to the north and south diminishing the fluxes of mass, momentum and kinetic energies. It loses momentum and kinetic energy to the deeper layers to create a vigorous eddy field extending to the ocean bottom. All of these processes combine to reduce the momentum and kinetic energy fluxes to insignificant levels east of the Grand Banks.

The Gulf Stream '60 sections (Fuglister, 1963) were occupied over a 2½-month period from the beginning of April to mid-June 1960. Nine sections spaced at two-degree intervals in longitude extended from

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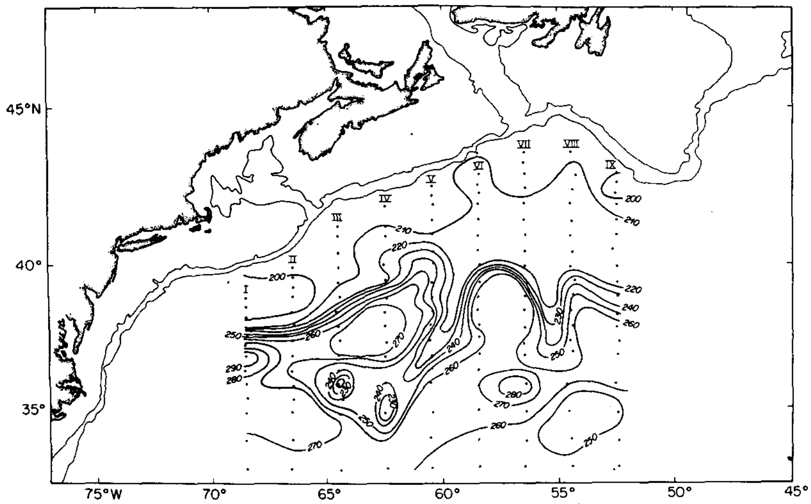

FIG. 1. Transport potential (anomaly of potential energy) χ relative to 3000 db in Gulf Stream '60 survey area. Units are 10^5 J m^{-2}.

68°30′W to 52°30′W and from 33°N to the continental shelf (Fig. 1). Station spacing varied from 1° in the southern part to ½° (50–60 km) across the Gulf Stream and slope water. In some sections, stations were taken at 30–35 km intervals. The Gulf Stream '60 sections, organized by Fuglister, remain the most comprehensive hydrographic description available to date.

The mass, momentum and kinetic energy fluxes computed in the present study are subject to a variety of errors, only some of which can be estimated. Momentum and kinetic energy fluxes are essentially averages of the second and third moments of the geostrophic velocity components normal to the section. Uncertainties in estimating geostrophic velocities are magnified in the higher moments, but bounds can be placed on such errors by consideration of the data. The effect of discrete sampling on flux estimates can be examined using a simple two-layer jet model of approximately the same mass transport as the Stream. The large space and time variability of the Gulf Stream may introduce errors which are difficult to evaluate and are only mentioned in the present discussion. Thus, although there appear to be clear trends in the computed fluxes, interpretations based on the estimates must be subject to reservations. Nevertheless, it is useful to examine the sections in some detail to assess the magnitude of the nongeostrophic effects and to extend qualitative comparisons with numerical models.

In the following sections, first the two-layer model is reviewed. Flux magnitudes based on the two-layer jet are estimated for comparison with the geostrophic calculations. The model is also used to estimate discrete sampling errors. In the next section, the choice of reference level is discussed, and associated errors are indicated. Next, the results of the geostrophic computations are presented, with some discussion on the relevance of the previous error estimates. Assuming the downstream flux divergences are real, we assess dynamical balances in the Stream and discuss the implications for the rest of the North Atlantic circulation.

2. The two-layer jet

A particularly simple free inertial-jet solution can be obtained for a two-layer model using the β-plane approximation and assuming constant potential vorticity (Fofonoff, 1962). This solution is used to obtain order-of-magnitude estimates for the momentum and kinetic energy fluxes by adjusting the transport and scale width of the jet to approximate those of the Gulf Stream. The model can also be used to estimate the effects of discrete sampling across a strongly sheared current. The eastward flowing jet is described by an interfacial depth h of the form

$$h = h_0(1 - e^{-y/R}), \qquad (1)$$

where h_0 is the maximum depth of the upper layer in the interior and $R = (U_0/\beta)^{1/2}$, the scale width of the jet. The coordinate y is positive southward measured from the northern edge of the jet.

For the constant potential vorticity case, the scale width R coincides with the Rossby radius of deformation $(g'h_0/f)^{1/2}$, where $g' = g\Delta\rho/\rho$ is reduced gravity, $\Delta\rho/\rho$ the fractional step in density across the interface and f is the Coriolis parameter. The velocity in the jet is given by the geostrophic equation

$$u = \frac{g'}{f}\frac{\partial h}{\partial y} = \frac{g'h_0}{fR}\, e^{-y/R} = (g'h_0)^{1/2} e^{-y/R}. \qquad (2)$$

The corresponding fluxes are

$$\left.\begin{aligned} T &= \int_0^y \rho u h dy \\ &= \frac{1}{2}\rho g' \frac{h_0{}^2}{f}(1 - 2e^{-y/R} + e^{-2y/R}) \\ TM &= \int_0^y \rho u^2 h dy \\ &= \frac{1}{6}\rho g'^2 \frac{h_0{}^3}{f^2 R}(1 - 3e^{-2y/R} + 2e^{-3y/R}) \\ TKE &= \frac{1}{2}\int_0^y \rho u^3 h dy \\ &= \frac{1}{24}\rho g'^3 \frac{h_0{}^4}{f^3 R^2}(1 - 4e^{-3y/R} + 3e^{-4y/R}) \end{aligned}\right\}, (3)$$

where T is mass flux (kg s^{-1}), TM momentum flux (J m^{-1} or N) and TKE kinetic energy flux (J s^{-1}). These fluxes are shown as a function of distance from the edge of the jet in Fig. 2. Note that most of the kinetic energy flux (about 90%) is associated with only 40% of the transport. Taking representative values:

$$\left.\begin{aligned} h_0 &= 800 \text{ m} \\ \Delta\rho/\rho &= 2 \times 10^{-3} \\ g' &= 0.02 \text{ m s}^{-2} \\ f &= 10^{-4} \text{ s}^{-1} \end{aligned}\right\}$$

yield

$$\left.\begin{aligned} R &= (g'h_0)^{1/2}/f = 40 \text{ km} \\ U_{\max} &= (g'h_0)^{1/2} = 4 \text{ m s}^{-1} \\ T_{\max} &= {}^1\!/_2\rho g' h_0{}^2/f = 64 \times 10^9 \text{ kg s}^{-1} \\ TM_{\max} &= {}^1\!/_6\rho g'^2 h_0{}^3/f^2 R = 85 \times 10^9 \text{ N} \\ TKE_{\max} &= {}^1\!/_{24}\rho g'^2 h_0{}^4/f^3 R^2 = 85 \times 10^9 \text{ J s}^{-1} \end{aligned}\right\}.$$

The parameters have been chosen so that the mass flux coincides approximately with the geostrophic mass flux referenced to 3000 db, computed from the Gulf Stream '60 sections. Fofonoff (1981) estimated wind work over the subtropical North Atlantic to be of the order of 20 × 10^9 J s^{-1}. With recirculation included, his estimate of kinetic energy flux in the Gulf Stream was 60 × 10^9 J s^{-1}. These estimates, although crude, are within the range obtained from the Gulf Stream '60 data.

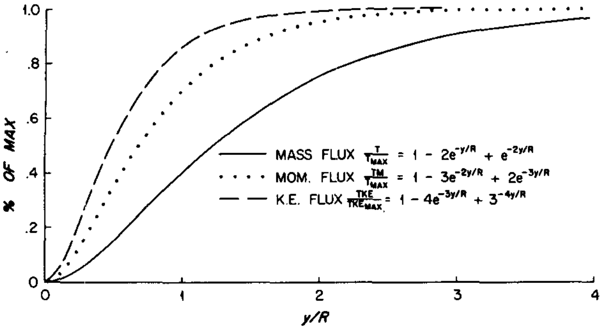

FIG. 2. From two-layer model, percentage of maximum (= total) flux attained as a function of distance from the edge of the jet. R is the Rossby deformation radius: Mass flux (solid line); momentum flux (dotted line); kinetic energy flux (dashed line).

As there are no concentrated fluxes of momentum or kinetic energy in the eastern North Atlantic, the fluxes presumably decay within the Gulf Stream region itself. A momentum flux 85 × 10^9 N entering the ocean interior and diminishing to zero downstream represents a force acting over the scale width of the jet that is equivalent to a transport potential difference of 21 × 10^5 J m^{-2} or the total Coriolis force acting on a mass transport of 24 × 10^9 kg s^{-1}. As the difference is comparable to the difference in transport potential observed between the slope water region inshore of the Gulf Stream and the eastern side of the North Atlantic, at the same latitude, it appears reasonable to postulate that the observed pressure difference across the ocean is maintained by the momentum-flux divergence rather than a geostrophically balanced meridional transport. This point is discussed further in Section 5.

Discrete sampling errors. The spacing of hydrographic stations and their location relative to the edge of the Gulf Stream can have significant effects on estimates of the momentum and kinetic energy fluxes. Close spacing is necessary to resolve the high-speed core of the Stream in which the major contribution to the fluxes occurs. The effects of location and spacing are examined for the two-layer jet and are assumed to be representative, at least qualitatively, for the Gulf Stream sections.

Let the station spacing be given by $\Delta Y = \lambda R$, with the first station within the jet located at $Y = \alpha R$, $0 < \alpha < \lambda$. Using finite difference approximations, the mass flux estimate is

$$T = \int_{y_0}^{y_N} \rho u h dy = \frac{1}{2}\rho g' \frac{h_1{}^2}{f} + \sum_{n=1}^{N-1} \frac{1}{2}(h_{n+1} + h_n)$$

$$\times \left[\frac{\rho g'}{f}\left(\frac{h_{n+1} - h_n}{\lambda R}\right)\lambda R\right] = \frac{1}{2}\rho g' \frac{h_N{}^2}{f} \approx \frac{1}{2}\rho g' \frac{h_0{}^2}{f}.$$

No cumulative error is incurred by the finite spacing provided variations in f are neglected over the jet width.

The momentum flux estimate is

$$TM = \int_{Y_0}^{Y_N} \rho u^2 h dy \approx \frac{1}{2}\rho g'^2 \frac{h_1{}^3}{f^2\lambda R} + \sum_{n=1}^{N-1} \frac{1}{2}(h_{n+1} - h_n)\,\rho\,\frac{g'^2}{f^2}\left(\frac{h_{n+1} - h_n}{\lambda R}\right)^2 \lambda R$$

$$\approx \frac{1}{6}\frac{\rho g'^2 h_0{}^3}{f^2 R}\left[\frac{3e^{-2\alpha}}{\lambda}\left(\frac{2(1 - e^{-\lambda})}{1 + e^{\lambda}} - \frac{e^{-\alpha}(1 - e^{-2\lambda})}{1 + e^{-\lambda} + e^{-2\lambda}}\right) + (1 - e^{-\alpha})^3\right].$$

Similarly, the kinetic energy flux estimate is

$$TKE = \frac{1}{24}\frac{\rho g'^3 h_0{}^4}{f^3 R^2}\left[\frac{6e^{-3\alpha}(1 - e^{-\lambda})^2}{\lambda^2} \times \left(\frac{2}{1 + e^{-\lambda} + e^{-2\lambda}} - \frac{e^{-\alpha}}{1 + e^{-2\lambda}}\right) + (1 - e^{-\alpha})^4\right].$$

The errors incurred by discrete sampling can be calculated for various α, λ from the above formulae. The estimates are more sensitive to location relative to the edge or high-speed core of the jet than to station spacing. For station spacing of 40 km ($\lambda = 1$), the momentum flux can be underestimated by 20% and the kinetic energy flux by 40% if the high speed core falls between stations.

If a station is taken at the edge of the jet, the estimates are insensitive (at the 10% level) to station spacing up to twice the radius of deformation (80 km). As the station spacing within the Gulf Stream varied from about 30 to 50 km, errors from discrete sampling can be expected to be up to 25% for momentum flux and up to 50% for kinetic energy flux.

3. Reference level errors

A reference level of no motion has been chosen at 3000 db. This pressure surface is the deepest level at which observations are available at all Gulf Stream stations in the survey. The estimates of momentum and kinetic energy flux are less strongly affected by choice of reference level than those of mass flux because of the relatively weak shears in the deep water. The velocity difference between 2000 and 3000 decibars is less than 0.05 m s^{-1}.

Assume that a velocity u_r exists at the reference level, so that

$$u = u_g + u_r,$$

where u_g is the geostrophic velocity relative to the reference surface. The flux estimate between station pairs for the mass flux is

$$\Delta T = \iint \rho(u_g + u_r)dzdy = \Delta T_g + Mu_r,$$

where $M = \iint \rho dzdy$ is the mass per unit width of the section segment and ΔT_g the mass flux relative to the reference surface. The momentum and kinetic energy fluxes become

$$\left.\begin{aligned} \Delta TM &= \iint \rho(u_g + u_r)^2 dzdy \\ &= \Delta TM_g + 2\Delta T_g u_r + Mu_r{}^2 \\ \Delta TKE &= \iint {}^1\!/_2\rho(u_g + u_r)^3 dzdy \\ &= TKE_g + {}^3\!/_2\Delta TM_g u_r + {}^3\!/_2\Delta T_g u_r{}^2 + {}^1\!/_2 Mu_r{}^3 \end{aligned}\right\}.$$

Using flux values for the model, for the entire width of the jet, the percentage changes for a range of reference velocities are given in Table 1. Changes of reference level in the deep water can produce changes of the order of 15 to 20% in the estimated fluxes. A barotropic flow added to the high-speed core of the Stream can contribute significantly to the fluxes. Doubling the mass transport would increase the momentum flux by 80% and the kinetic energy flux by 160%.

The effects of reference level choice can be examined using the Gulf Stream '60 data. These are described later.

4. Computational results

While the two-layer model is useful for making order of magnitude flux estimates, as well as for estimating some errors, it depends only on y, i.e., latitude, and is incapable of predicting any downstream variability of the fluxes. Mass, momentum and kinetic energy fluxes through each of the nine Gulf Stream sections have thus been calculated to search for such variability. If the sections are considered to be approximately synoptic, downstream changes in any of these fluxes must be compensated by other terms in the equations of motion. Before considering such balances, however, it is necessary to examine the results of the geostrophic calculations for the Gulf Stream '60 sections.

Figure 1 shows contours of the transport potential χ (anomaly of potential energy) relative to $P_{\text{ref}} = 3000$ db, the greatest common depth for which observations exist in this survey. Here χ is defined by $\chi = g^{-1}\int_0^{3000\text{ db}} P\delta dP$ and satisfies the geostrophic relations $\int_0^{z_{\text{ref}}} \rho f v dz = \partial\chi/\partial x$, $\int_0^{z_{\text{ref}}} \rho f u dz = -\partial\chi/\partial y$ (see Fofonoff, 1962) so that the geostrophic transport between any two contours χ_1 and χ_2 is proportional to their difference. The interpretation of the data in Fig. 1 is based on Fuglister's contours of the 10°C isotherm depth in the survey region (Fuglister, 1963; his Fig. 4). Using hydrographic and surface current data besides those collected in Sections I to IX, Fuglister

TABLE 1. Reference level errors: geostrophic flux estimates for the two-layer model are at the top. Upper grid: change in flux estimates for an additional barotropic velocity u_r at a reference level of 3000 m. Lower grid: corresponding percentage changes to the geostrophic flux estimates.

$\Delta T_g = 64 \times 10^9$ kg s^{-1}
$\Delta TM_g = 85 \times 10^9$ N
$\Delta TKE_g = 85 \times 10^9$ J s^{-1}
$M = 120 \times 10^9$ kg m^{-1}

$\rho = 10^3$ kg m^{-3}
$\Delta y = 40$ km
$\Delta z = 3000$ m

u_r (m s^{-1})	Mu_r (10^9 kg s^{-1})	$2\Delta T_g u_r$ (10^9 N)	Mu_r^2 (10^9 N)	$\frac{3}{2}\Delta TM_g u_r$ (10^9 J s^{-1})	$\frac{3}{2}\Delta T_g u_r^2$ (10^9 J s^{-1})	$\frac{1}{2}Mu_r^3$ (10^9 J s^{-1})
			Fluxes			
0.01	1.2	1.3	0.01	1.3	0.01	—
0.05	6.0	6.4	0.3	6.4	0.24	0.01
0.10	12.0	12.8	1.2	12.8	0.96	0.06
			% Changes			
0.01	1.9	1.5	0.01	1.5	0.01	—
0.05	9.4	7.5	0.35	7.5	0.28	0.01
0.10	18.8	15.1	1.4	15.0	1.1	0.07

was able to deduce the meandering character of the Stream from Sections V to VIII. We have relied on his interpretation and included such meanders where we lack actual values of χ, as between Sections VII and VIII.

In Fig. 1 the Gulf Stream is present in all meridional sections and has been contoured as a continuous current. The transport is strongest in Section I, having a maximum difference of about 100×10^5 J m^{-2} in transport potential. The corresponding mass flux is about 110×10^9 kg s^{-1}. Part of the flux in Section I is associated with a cyclonic eddy north of the Stream and part with an anticyclonic eddy to the south of the Stream. These eddies contribute about 30–40 ($\times 10^9$ kg s^{-1}) to the apparent total transport. The core or central portion of the Gulf Stream appears to retain its intensity over all sections. The contours of 200 and 210 ($\times 10^5$ J m^{-2}) separate gradually into the Slope Water indicating a northward flux of about 20×10^9 kg s^{-1}. There may also be a decrease in transport potential eastward, south of the Gulf Stream, although the intense eddy activity obscures any trend.

Estimates of momentum and kinetic energy fluxes should be approached more cautiously. It is evident from Fig. 1 that the core of the Gulf Stream is spanned by only a few hydrographic stations. Since error discussion is to be based on the assumption that the Gulf Stream's meridional structure is roughly described by the two-layer model, the estimates of mass, momentum, and kinetic energy fluxes might be made according to criteria which take the discretely sampled Gulf Stream and fit it to the model in the best way possible. However, locating the northern and southern limits of the Stream, for the purpose of calculating fluxes due only to the Stream, would necessarily be a somewhat subjective exercise. The feature characterizing the northern edge of the two-layer jet is the intersection of the upper layer with the surface, which corresponds to a discontinuity in velocity there. The analogous feature in the Gulf Stream is the sudden change in slope of the transport potential χ (see Fig. 3), which may be smeared out as a result of discrete sampling; in this case momentum and kinetic energy fluxes for the pair of stations bracketing the edge may be underestimated, though mass flux remains unaffected. If the ocean interior south of the Stream is reasonably quiet, defining the southern boundary is a less crucial task. According to the 2-layer model, the southward integrated momentum and kinetic energy fluxes reach 95 and 99%, respectively, of their total values within two Rossby radii of the northern edge of the jet (see Fig. 2). Integrating farther southward does not affect the estimates significantly. However, the Gulf Stream does not merge smoothly into a placid interior, as can be seen from the jaggedness in χ (Fig. 3) south of the core for most of the sections.

It seems safest to estimate mass, momentum, and kinetic energy fluxes, to lowest order, only from those stations which evidently bracket the core of the Stream. These are the stations at which, according to Fig. 3, the break in the slope of χ occurs. Table 2 shows specifically which stations we have chosen at each section. Such calculations merely quantify what is evident from inspection of Fig. 1, that all the fluxes decrease downstream: at the western edge, excluding the eddies north and south of the Stream, the core is spanned by isolines of $\chi = 260$ to 210 ($\times 10^5$ J m^{-2}). Since transport is proportional to $\Delta\chi$, the transport decreases by roughly 20% downstream. Moreover, closer spacing of isolines of χ indicates higher velocities, and correspondingly higher momentum and kinetic energy fluxes. The isolines are much more

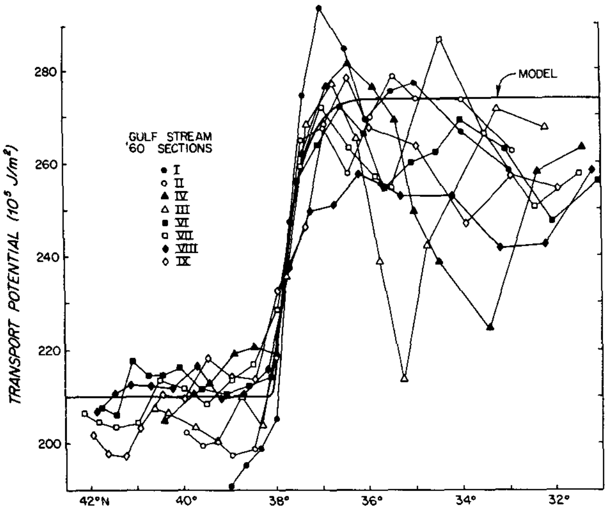

FIG. 3. Transport potential χ relative to 3000 db for each section. Latitude axis is correct for Section I; other profiles have been shifted so that core is at approximately the same latitude for each. Heavy line is for two-layer model. Notice that Sections VIII and IX have somewhat different slopes in the core than do Sections I through VII. Section V has been omitted, for reasons discussed in text.

closely spaced at the western edge than the eastern, and so we anticipate decreases in momentum and kinetic energy fluxes downstream.

In terms of the flux calculations, a major concern is the strong meandering of the Stream in Sections V to VIII. Only velocities normal to the sections have been calculated and used to estimate the streamwise momentum and kinetic energy fluxes as ρu^2 and $\frac{1}{2}\rho u^3$, respectively. If the Stream crosses a section obliquely, there may in fact be substantial meridional velocities, and the fluxes could be seriously underestimated. The errors induced by oblique crossings will be assessed later.

It should be noted at this point that Section V is omitted from Figs. 3 and 5, both for the reasons given in the previous paragraph and because the break in the slope of χ is not at all well-defined for this section. However for the purpose of Fig. 4 and Table 3, it makes little difference to the actual numbers (<5%) whether fluxes are totaled for the section (as they are) or only for a subjectively chosen "core".

TABLE 2. Stations which have been selected as representing "core" of Gulf Stream according to criteria discussed in text. Latitudes are also given, as stations in Fig. 1 are unnumbered.

Section	Stations	Latitudes (deg North)
I	7–10	37–38
II	7–10	37–38.5
III	8–11	37.5–39
IV	8–11	38–39.6
V	all	33–42.6
VI	9–12	38.5–40
VII	8–11	38.5–40
VIII	8–11	38.5–40
IX	6–10	37.5–39.5

The values of momentum and kinetic energy fluxes for each of the nine sections are shown in Fig. 4, along with the constant value for the two-layer inertial jet, and are listed in Table 3. Momentum flux decreases from 89 × 10^9 at Section I to 25 × 10^9 N across Section IX, attaining a minimum value of 14 × 10^9 at Section VIII. Kinetic energy flux decreases from 121 × 10^9 at Section I to values of 7–12 × 10^9 J s^{-1} in the eastern sections. The anticipated reduction in these fluxes downstream is evident, but the values at Section I should be noted with caution. In this section, it was impossible to isolate the contributions of the eddies north and south of the Stream from those due to the core alone. All fluxes at this section are likely to be somewhat augmented by the inclusion of part of the eddies in the calculation.

Errors associated with discrete sampling and choice of reference level have been estimated within the

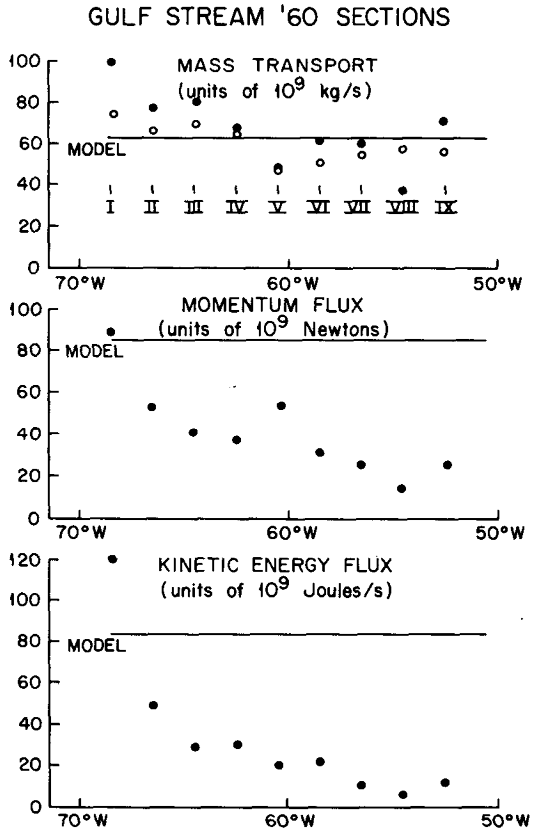

FIG. 4. Geostrophic mass, momentum and kinetic energy fluxes relative to 3000 db calculated for core of Stream (see text). Mass transport for entire section also shown as open circles.

framework of the inertial jet model. To investigate the extent to which reference level errors in the Stream may be described by the model results, mass, momentum, and kinetic energy fluxes have been calculated and plotted as a function of P_{ref}, the reference level pressure (see Fig. 5). At individual hydrographic stations, the velocity difference between 1500 and 3000 db is generally less than 5 cm s^{-1}, even in the core of the Stream, except for a few stations where it is as high as 10 cm s^{-1}. The relative errors incurred by shifting reference levels from P_{ref} = 3000 db to other values are approximately the same for all of the sections. As a typical example, the values for Section VI are given in Table 4; relative errors are estimated from

$$R.E. = \frac{T_{3000} - T_{\text{ref}}}{T_{3000}} \times 100\%$$

(and similarly for TM and TKE, the momentum and kinetic-energy fluxes), where T_{ref} is the transport relative to the new reference level. Note that the errors are somewhat larger than predicted by the two-layer model even for the addition of a barotropic velocity u_r = 10 cm s^{-1} at 3000 m. The choice of a reference level at 3000 db stems rather from its convenience than any conviction on the authors' part that this is the true reference level. However, we emphasize that shifting the reference level much deeper, or adding a barotropic velocity at 3000 db, affects mass flux estimates more strongly than those of momentum and kinetic energy (see Tables 1, 4). Furthermore, the simple trend in Fig. 5, if extrapolated to deeper levels, indicates that a change of reference level to a deep value would increase the fluxes only weakly, and rather uniformly (except perhaps at Section I), preserving the major result that momentum and kinetic energy fluxes are largely convergent downstream.

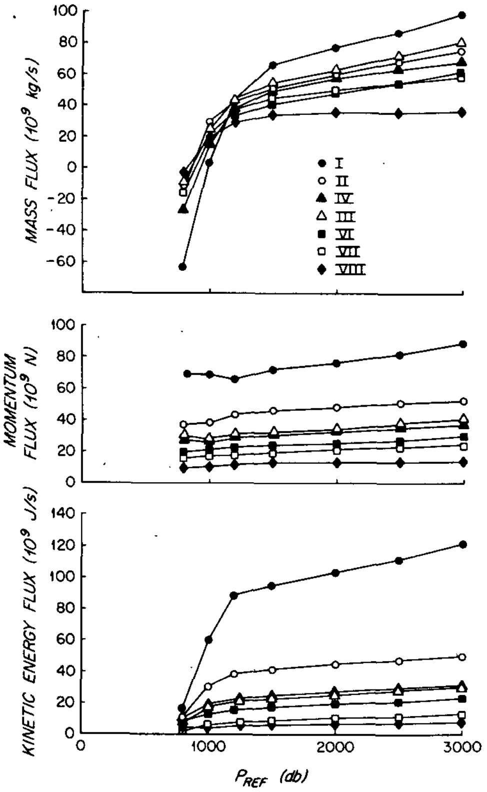

FIG. 5. Geostrophic mass, momentum and kinetic energy fluxes, calculated from Gulf Stream '60 data relative to various reference pressures. Section V is omitted because it cuts the Stream at such an angle; Section IX is omitted to avoid cluttering the figure (values are repetitive).

It has been noted that in the two-layer model, estimates of momentum and kinetic energy fluxes were more sensitive to the location of hydrographic stations relative to the edge of the jet than to the actual station spacing. The real Stream does not have quite so abrupt an edge as the model jet, nor is it likely that the high-velocity core of the Stream falls precisely halfway between stations at every section. Thus, the sensitivity of the fluxes to edge location may be weakened. The model results suggest that allowances should be made for underestimation of momen-

TABLE 3. Mass, momentum and kinetic energy fluxes relative to 3000 db for core of Gulf Stream (see text for discussion) in Gulf Stream '60 survey. Mass fluxes for the entire width of each section are also included.

	Mass (10^9 kg s^{-1})			
Section	Core	Section	Momentum (10^9 N)	Kinetic energy (10^9 J s^{-1})
1	98.2	74.00	89.35	121.36
2	77.14	66.17	53.28	49.51
3	81.20	69.65	41.60	29.52
4	68.09	64.89	37.32	30.30
5	48.10	47.84	53.39	20.73
6	62.60	51.62	30.12	22.49
7	59.99	54.56	24.12	12.75
8	37.91	56.99	14.00	6.79
9	71.86	55.94	24.88	12.15

TABLE 4. Reference level errors from data. Geostrophic mass, momentum and kinetic energy fluxes for Section VI of Gulf Stream '60 computed relative to five different reference levels. R.E. is relative error (see text).

	Mass		Momentum		Kinetic energy	
P_{ref} (db)	Flux (10^9 kg s^{-1})	R.E. (%)	Flux (10^9 N)	R.E. (%)	Flux (10^9 J s^{-1})	R.E. (%)
800	−16.45	126	19.83	34	7.34	67
1500	41.68	33	24.09	20	16.94	25
2000	48.40	23	25.63	15	18.40	18
2500	55.04	12	27.64	8	20.32	10
3000	62.60	—	30.12	—	22.49	—

tum flux by up to 25% and kinetic energy flux by up to 50%.

The problem of oblique crossings of the Stream has been mentioned and can be quantified as follows. Suppose the normal to the true path of the Stream is an angle α from the section line. If the actual velocity along the Stream is u_s, then $u = u_s \cos\alpha$, where u is velocity normal to the section (i.e., eastward). The apparent width of the Stream $y_N - y_S$ is greater than the actual width $W = (y_N - y_S)\cos\alpha$. If ds is the incremental distance normal to the Stream, the true transport is calculated correctly, $\int \rho u dy = \int \rho u_s \, ds$ while the momentum flux $\int \rho u^2 dy = \cos\alpha \int \rho u_s^2 ds$ and kinetic-energy flux $\int \tfrac{1}{2}\rho u^3 dy = \cos^2\alpha \int \tfrac{1}{2}\rho u_s^3 ds$ are underestimated by factors of $\cos\alpha$ and $\cos^2\alpha$ respectively. For $\alpha = 45°$, momentum flux is about 30% too low and kinetic energy flux 50% too low. Notice, however, that at Sections VII and VIII, where oblique crossings are suspected, doubling these fluxes would actually bring them more in line with values and trends in Fig. 4.

The most glaring deficiency of the two-layer model for purposes of comparison is its lack of time dependence. The Gulf Stream experiences variations on time scales on the order of weeks or months, while the "synoptic" Gulf Stream '60 data were actually collected over a period of 2½ months. Thus, there are two major problems associated with this time dependence. The Gulf Stream '60 data set is only one realization of an ever changing feature, and repeated realizations would be required to attach some statistical significance to the results obtained from these data. More problematic for the present investigation is that the structure of the Stream may have altered considerably during the course of the survey. In particular, while gross features of the Stream such as the meander at 60°W may have evolved only slightly, it is conceivable that the reference level of no motion (if one exists) could be changing in time. Although surface expressions of the Stream (features in χ, for example) could remain largely unaffected by this time dependence, the integrated mass, momentum, and kinetic energy fluxes depend on the choice of reference level.

With these caveats in mind, we offer an analysis of this particular data set which seeks at least to present a self-consistent picture of the kinematics and dynamics of the Gulf Stream at the time of the survey and at best to suggest a more generalized picture of the gross-scale dynamical and energetic balances in this region.

5. Discussion

Notwithstanding possible errors, it appears from the above results that momentum flux in the Gulf Stream decreases downstream. The decrease can be interpreted as a force acting in the downstream direction that must be balanced by another term in the momentum conservation equation from the eastward components of momentum flux,

$$\frac{\partial \overline{\rho u^2}}{\partial x} + \frac{\partial \overline{\rho uv}}{\partial y} + \frac{\partial \overline{\rho uw}}{\partial z} - \rho f \bar{v} = -\frac{\partial \bar{P}}{\partial x} + \bar{F}_x, \quad (4)$$

where x is eastward, y northward and z upward, with corresponding velocity components u, v, w. The density is given by ρ, Coriolis parameter f, pressure P and frictional force F_x. The equation is time-averaged, indicated by the overbars, and the flow is assumed to be primarily eastward. [It is important to realize that the flow is semi-geostrophic: i.e., while the Rossby number associated with the along-stream momentum balance may be O(1), that associated with the cross-stream balance is $\ll 1$ so that this balance is geostrophic. Thus, it is not inconsistent to compute eastward velocities geostrophically, and then consider non-geostrophhic attributes of the flow.] Integrating (4) in x and z yields

$$\int_{z_{\mathrm{ref}}}^{0} [(\rho u^2)_9 - (\rho u^2)_1]dz + \frac{\partial}{\partial y}\left[\iint \tau_H dxdz\right] + \int (\tau_T - \tau_B)dx - fT_y = -(\chi_9 - \chi_1) + \iint F_x dxdz, \quad (5)$$

where subscripts 1, 9 indicate upstream, downstream positions, respectively; τ_H is horizontal Reynolds stress; τ_T, τ_B are top and bottom Reynolds stresses; and T_y is the northward mass flux (relative to z_{ref}). Somewhere in the Stream, $T_y = 0$ (because it has opposite signs on opposite sides of the Stream). Along such a line, ignoring $\tau_{T,B}$ and friction, the balance which must obtain is

$$\frac{\partial}{\partial y}[TM_9 - TM_1] + \frac{\partial}{\partial y}\left[\iint \tau_H dxdz\right] = -(\chi_9 - \chi_1), \quad (6)$$

where $TM = \iint dydz(\rho u^2)$. We expect (6) to hold roughly throughout the core of the Stream. Our data set permits no estimates of the Reynolds stress term, but deep measurements (Schmitz, 1977; Thompson, 1978) near the Gulf Stream indicate that this term tends to drive the mean flow, i.e., it has the same sign as the momentum flux convergence and therefore does not act to balance it. In subsequent calculations, although we drop the term $\partial\tau_H/\partial y$ for succinctness, it should be thought of as augmenting the momentum-flux convergence. The remaining terms estimated in the strongest part of the jet, near the northern edge, have values

$$\int_{z_{\mathrm{ref}}}^{0}[(\rho u^2)_2-(\rho u^2)_1]dz = -17.6\times10^5\ \mathrm{J\,m^{-2}}$$

$$-(\chi_9-\chi_1) = -8.4\times10^5\ \mathrm{J\,m^{-2}}$$

which are of comparable magnitude. The first term averaged over the core as defined by Table 2 has a value of -8.3×10^5 J m^{-2}. Estimates have also been made to determine whether bottom drag (i.e., form drag) should be included in (5); the conclusion was that unrealistically large dynamic height differences would be required in the lower part of the water column to produce effects of order 10^6 J m^{-2}.

Beyond the northern edge of the Stream the momentum flux and its convergence drop abruptly to negligible values. However, the rise of pressure along the northern edge of the Stream must be matched by a pressure rise north of the Stream, to maintain continuity of sea surface height. In this region, where we expect the dynamics to be linear, this pressure gradient implies a geostrophic mass transport into the Slope Water of

$$T_y = (\chi_{9_N}-\chi_{1_N})/f = 9.4\times10^9\ \mathrm{kg\,s^{-1}}.$$

It is interesting to reconsider Eq. (4) in the context of basin-wide dynamical balances. In anticipation of the wind stress being important, we now include the term $\partial\tau^x/\partial z$ on the rhs, where τ^x is the eastward component of stress and incorporates the effect of vertical Reynolds stresses. We are motivated by inspection of Fig. 6, which shows χ relative to 3000 db plotted versus longitude across the ocean at latitude 36°N. At the western edge, in the Slope Water, χ is around 192×10^5 J m^{-2}. The sharp increase at longitude 70°W is due to the Gulf Stream. The remainder of the plot is somewhat noisy, but there is evidently a systematic decay to eastern boundary values of 223–232 ($\times10^5$ J m^{-2}). There is a net difference in the transport potential relative to 3000 db from the western to the eastern boundary of the North Atlantic at this latitude. Stommel *et al.* (1978) note the increase in the dynamic topography across the Atlantic from inshore of the Gulf Stream to Gibraltar. By specifying a small net northward flow of water warmer than 10°C, which is converted to cold water at northern latitudes, they estimate the size of a tight barotropic recirculation southeast of the Gulf Stream necessary to conserve mass in each of their 2 layers ($\lessgtr$10°C). This model is invoked to avoid unreasonably large net northward flows of warm water to balance the basin-wide pressure gradient, which would in turn imply similarly large net sothward flows of cold water.

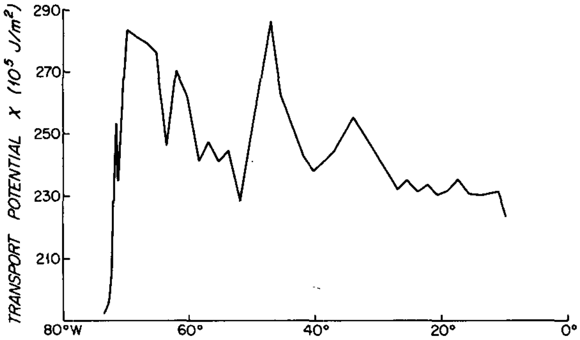

FIG. 6. Anomaly of potential energy χ relative to 3000 db at 36°N (from CHAIN 7 cruise).

From Fig. 6, if the net difference in χ relative to 3000 db were to be balanced geostrophically the required northward transport relative to 3000 db would be

$$T_y = \frac{\Delta\chi}{f} = \frac{(227-192)\times10^5\ \mathrm{J/m^2}}{(.8797\times10^{-4}\ \mathrm{s^{-1}})} = 40\times10^9\ \mathrm{kg\,s^{-1}}.$$

This would require a southward transport of the same magnitude below 3000 db, yet there is no indication that such a large southward flow exists in the deep water. Recently, Hall and Bryden (1982) estimated the total southward flow of water of potential temperature less than 4°C at 25°N to be 15.6×10^6 m^3 s^{-1} (Sv) while Worthington's estimate of the same is only 7 Sv. Since the 4°C isotherm generally lies well above 3000 db, these are actually upper limits on the magnitude of such a meridional cell. From Table 5 of Hall and Bryden (1982) it can be estimated that about 8 Sv flows southward below 3000 m. A transport $T_y = 8\times10^9$ kg s^{-1} at latitude 35°N could account for a difference in χ of

$$\Delta\chi = T_y\cdot f = (8\times10^9\ \mathrm{kg\,s^{-1}})(.8367\times10^{-4}\ \mathrm{s^{-1}}) = 6.7\times10^5\ \mathrm{J\,m^{-2}}.$$

From Fig. 6, the total difference in χ is approximately 38×10^5 J m^{-2}. Thus, most of the total difference in χ, approximately 31×10^5 J m^{-2}, must be balanced some other way.

Integration of Eq. (4) from the western to eastern boundary, and from 3000 db to 0, gives

$$\underset{\text{small}}{\int_{3000\ \mathrm{db}}^{0}[(\rho u^2)_E-(\rho u^2)_W]dz} + \underset{\text{unknown}}{\frac{\partial}{\partial y}\int_{3000\ \mathrm{db}}^{0}\int_{x_W}^{x_E}\tau_H\,dx\,dz}$$

$$\underset{-7\times10^5\ \mathrm{J\,m^{-2}}}{-fT_y} = \underset{-38\times10^5\ \mathrm{J\,m^{-2}}}{-(\chi_E-\chi_W)} + \underset{+10\times10^5\ \mathrm{J\,m^{-2}}}{\int_{x_w}^{x_E}\tau^x dx}\quad, \quad (7)$$

where τ^x is the eastward wind stress (taken to be 2 dyn cm^{-2} acting over a distance of 5000 km), and we have assumed the stress negligible at depth. We have ignored friction as well. This is essentially Eq. (5) with subscripts 1, 9 replaced by W, E, respectively. In Eq. (7), however, the first term no longer yields a large contribution, since $u \equiv 0$ (strictly speaking) if we integrate exactly to the boundaries; if we integrate to the last deep station on either side of the basin, u will still be very small. Of the remaining terms, the three for which reasonable estimates exist clearly do not balance. While the details of the situation remain obscure to us at present, we speculate that in some sense the convergence of momentum in the Gulf Stream helps to balance the basin-wide pressure gradient. An estimate of this term from the northern edge of the Stream at Section I is

$$-\int \rho u^2 dz = -20 \times 10^5 \text{ J m}^{-2},$$

where the sign indicates that momentum converges as we go from west to east. This term would suffice to bring Eq. (7) into balance, but whether that result is fortuitous or physically meaningful, we have been unable to determine. The "unknown" term may also be important but is presently difficult to evaluate. Thus, for the time being, the integrated balance of forces at this latitude remains a rather intriguing mystery.

6. Summary

The intention of this discussion has been to present the mass, momentum and kinetic energy fluxes computed from the Gulf Stream '60 data; to assess the errors involved; and to suggest what gross balances exist in the survey region. Without formally placing error bars on the results, it has been demonstrated that all three fluxes evidently decay downstream. In fact, error estimates from a two-layer, constant potential-vorticity, inertial-jet model approximate similar errors rather well as estimated from the data. Momentum fluxes are probably correct to within 25% while kinetic energy fluxes may be in error by up to 50%.

It has been suggested that the downstream momentum-flux divergence is balanced by a rise in pressure along the Stream, accompanied by conversions from kinetic to available potential energy. The complete dynamical and energetical picture must include a variety of other effects, such as the radiation of energy via rings. However, Fofonoff (1981) has discussed the possible role of Gulf Stream rings as a decay mechanism and concluded that ring production alone cannot be invoked to explain all of the energy loss in the Stream. An alternative interpretation of the data in this region is that the barotropic velocity component grows downstream, or that the reference level continually deepens downstream, in a way that maintains approximately constant momentum and kinetic energy fluxes; however, that would require an intensifying mass transport downstream. Direct measurements would be necessary to test the validity of this interpretation.

Both analytical and numerical quasi-geostrophic theories and models of the Gulf Stream abound in current literature. For the theory to be valid, however, the Rossby number must be $\ll O(1)$. That requirement is not necessarily satisfied, particularly in the core of the Gulf Stream, whereby nonlinear terms and pressure work done along a streamline may enter the momentum equations at lowest order. This analysis suggests that there are areas beyond quasi-geostrophy that ought to be explored, and which might shed new light on the dynamics and energetics governing this complicated region.

Acknowledgments. Support for this work was provided by Contract N00014-76-C-0197, NR 083-400, from the Office of Naval Research. We would like to thank A. Williams for her patience in typing and retyping this manuscript.

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