On the Relative Importance of Ventilation and Mixing of Potential Vorticity in Mid-Ocean Gyres[1]

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ABSTRACT

An *a priori* estimate is given of the effect of ventilation on the process of potential vorticity homogenization. Since the homogenization process depends on the presence of weak mixing, it is shown that only a small exposure to a zone of ventilation is required to lead to an arrest of the homogenization.

In a recent paper, Luyten *et al.* (1983) attribute the structure of the oceanic thermocline to the advection of properties, specifically potential vorticity, which the fluid picks up in regions of ventilation at the sea surface in the subtropical gyre. After the fluid plunges beneath the surface, potential vorticity is imagined to be conserved in the model.

On the other hand, the role of horizontal mixing of potential vorticity has been invoked by Rhines and Young (1982) to describe an alternative view in which subsurface motion is limited essentially to pools of constant potential vorticity produced by relatively weak horizontal diffusion. The effective absence of ventilation is a pre-requisite for the cross-gyre diffusion of potential vorticity to reach completion. In addition, the dissipation must be weak to lead to homogenization of potential vorticity.

It is natural to ask how effective ventilation must be to arrest the fairly slow process of homogenization. That estimate is the purpose of the present note. It is worth pointing out that the rapidity with which fluid circulates around the gyre is largely irrelevant. The vital estimate is the relative potency of ventilation to cross-gyre diffusion.

Consider the circuit shown in Fig. 1. The curves C_1 and C_2 are two nearly adjacent closed streamlines of a quasi-horizontal flow.

The equation for q, any quasi-conservative property, (but especially potential vorticity), is supposed to satisfy the following equation in steady flow

$$\mathbf{u}\cdot\nabla q = \nabla\cdot(\kappa\nabla q) + S, \qquad (1)$$

where S is a source function for q and κ is the, perhaps spatially variable, diffusivity for q. The function S represents the ventilation process. We take S to be different from zero only in the small interval δ between C_1 and C_2 and only for a relatively short distance l along the streamlines. Away from the zone where $S \neq 0$, the circuits C_1 and C_2 may gently plunge downward but at a small enough slope so that the circuits remain nearly horizontal.

If (1) is integrated over the area between C_1 and C_2, then under the assumption that $\mathbf{u}$ is non-divergent, the resulting integral becomes:

$$0 = \oint_{C_1} \kappa\nabla q\cdot\mathbf{n}dl - \oint_{C_2} \kappa\nabla q\cdot\mathbf{n}dl + \iint SdA, \qquad (2)$$

where the area integral of S is restricted to the stippled region shown in the figure.

If κ is nearly zero and S vanishes, q must be nearly constant on streamlines. Then if C_2 is shrunk to a point, it follows by considering the integral around C_1 that q must be independent of streamline, i.e., homogeneous within C_1 (Rhines and Young, 1982). The resulting constraint leading to that result is due to the small residual term of $O(\kappa)$ in (2), hence the presence of even a small source term can be significant.

To estimate the size of the terms in (2), I will assume that horizontal gradients of q are $O(\Delta q/L)$ where Δq is a typical value of the potential vorticity anomaly and L is a characteristic, gyre-scale, length. This assumes that the circuits C_1 and C_2 close in the gyre's interior, or if they require boundary current regions, that these do not alter the gross mid-gyre balances when integrated. These last assumptions are surely questionable, but they are identical to those made in deriving the standard conditions for homogenization of q.

The first two terms in (2) combine to yield the estimate of their net contribution as

$$D \equiv \oint_{C_1} \kappa\nabla q\cdot\mathbf{n}dl - \oint_{C_2} \kappa\nabla q\cdot\mathbf{n}dl = O(\kappa\delta\nabla(\nabla q\cdot\mathbf{n})\cdot L)$$

$$= O\left(\kappa\frac{\Delta q}{L}\delta\right) \qquad (3)$$

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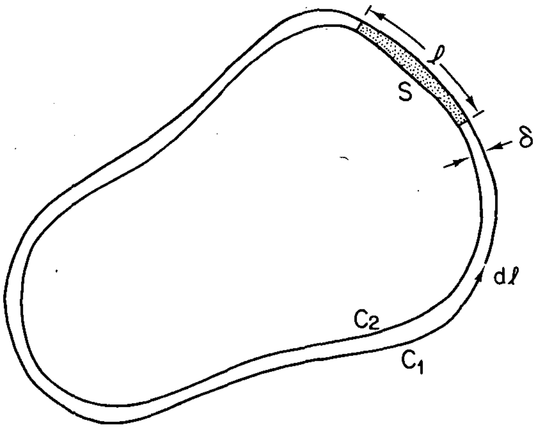

FIG. 1. The circuits C_1 and C_2 represent streamlines of steady flow separated by a characteristic distance δ. The ventilation, or potential vorticity source is limited to the stippled area whose length along the streamline is l.

while the last term is estimated as

$$V = S\delta l. \tag{4}$$

If Δq is the potential vorticity anomaly,

$$\Delta q = O\left(f\frac{\Delta H}{H^2}\right), \tag{5}$$

where ΔH is the thickness anomaly between neighboring isopycnal surfaces. Geostrophy and hydrostatic balance imply that

$$f\frac{\Delta H}{H^2} = O\left(\frac{f^2UL}{g'H^2}\right), \tag{6}$$

where g' is the reduced gravity and U is a characteristic horizontal velocity. Thus

$$D = O\left(\kappa\frac{f^2U}{g'H^2}\delta\right). \tag{7}$$

The source of potential vorticity is Ekman pumping w_E acting over the layer depth H: thus

$$S\delta l = O\left(fw_E\frac{\delta l}{H^2}\right). \tag{8}$$

From the Sverdrup relation

$$w_E = O\left(\frac{\beta UH}{f}\right), \tag{9}$$

so that

$$S\delta l = O\left(\frac{\beta U}{H}\delta l\right) \tag{10}$$

or

$$\frac{D}{S} = O\left(\frac{\kappa}{\beta}\Big/ lL_R{}^2\right), \tag{11}$$

where L_R is the deformation radius, $(g'H)^{1/2}/f$.

If we choose values of κ, β and L_R which are 10^7 cm^2 s^{-1}, 10^{-13} s^{-1} cm^{-1} and 50 km, respectively, then

$$\frac{D}{S} = 40 \text{ km}/l, \tag{12}$$

where l is measured in kilometers. Thus, if l exceeds 40 km, the presence of the potential vorticity source will effectively disrupt the process of lateral homogenization. Larger values of κ will, of course, increase D and require larger values of l to have the ventilation dominate. However, even a five-fold increase in κ would still only require l to be O(200 km) which is still small compared to the gyre scale. This only requires that each fluid filament be exposed, along its path, to ventilation for an order of an internal deformation radius which is a small fraction of its gyre-scale path length. The rate at which a fluid element circulates around the gyre is irrelevant as long as it is fast enough to make the diffusion term relatively small in (1). Rapid recirculation only means that the element is more frequently exposed (for shorter intervals) to potential vorticity ventilation.

Of course, the estimates given above are crude ones. If the zone of ventilation lies in an area of particularly weak Ekman pumping, the process of potential vorticity mixing would tend to be more effective so that density surfaces that outcrop near the latitude of vanishing Ekman pumping are more likely to contain substantial pools of homogenized potential vorticity.

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