Geostrophic Control of Fluctuating Barotropic Flow through Straits

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ABSTRACT

A simple model in which the cross-strait sea surface slope is geostrophically balanced and the along-strait slope is balanced by acceleration and friction, is shown to be supported by the results of Buchwald and Miles for fluctuating flow through a gap between two semi-infinite oceans. For a narrow gap (compared with the Rossby radius and the scale of the motion in the far field), the transport through it is exactly the same as that predicted by the model, provided that the gap is regarded as having an effective length as determined in this paper. The importance of the models is that they demonstrate that, at low frequency, the flow may be "geostrophically controlled" and the transport limited to a value much less than that which would arise in a nonrotating system. The neglect of nonlinear advective terms in the models is justified by a comparison of the Bernoulli set-down in the strait with the driving head and the mean water depth. The formula for the flux through a strait may be applied in studies of the forced response of ocean basins connected by straits. In particular, we draw attention to the existence of damped low-frequency normal modes for two connected (but frictionless) ocean basins.

1. Introduction

The understanding and prediction of flow through straits is important because of the significant effect of the flow on the properties of the bodies of water connected by the straits, as well as on the oceanographic conditions in the straits themselves. In many locations, such as the Strait of Gibraltar (Lacombe and Richez, 1961) and the Strait of Belle Isle (Garrett and Petrie, 1981) the low-frequency fluctuations in the flow are comparable in magnitude to the mean flow or tidal flow. The main driving force for these fluctuations is the sea level difference between opposite ends of the strait (e.g., Bowden, 1956; Garrett and Petrie, 1981; Schumacher *et al.*, 1982) that may be largely associated with meteorological forcing in the bodies of water separated by the strait.

Garrett and Toulany (1982) proposed a simple theoretical model for fluctuating barotropic flow through a strait, driven by differences in sea level between the connected bodies of water. The model was based on the assumptions of a cross-strait geostrophic balance and an along-strait balance between the pressure gradient (due to a sea surface slope), acceleration and friction. It led to a simple formula for the volume flux through the strait in terms of the sea level difference between the connected water bodies, showing in particular that at low frequency the flux can be much less than in a nonrotating system. This limit, which we term "geostrophic control," arises from the simple physical effect that the sea level difference across the strait due to geostrophic setup cannot be greater than the sea level difference between the connected water bodies.

The Garrett and Toulany (1982) model was based on simple physical reasoning and assumptions that might not be universally accepted. The purpose of the present paper is to provide some justification of, and further insight into, the results of the simple model by comparing its predictions with the results of a precisely formulated linearized diffraction theory (Buchwald and Miles, 1974) for fluctuating flow through a narrow gap connecting two semi-infinite oceans.

The two models are described in Sections 2 and 3 of this paper and compared in Section 4. The discussion in Section 5 includes a discussion of the validity of the linearization that is basic to both models. The application of the flux formula, in particular to the normal modes of two basins connected by a strait, is discussed in Section 6.

2. Simple model

Consider a strait of uniform depth h, length L and uniform width W connecting two large basins of homogeneous water (Fig. 1). The surface elevations in each basin vary slowly with position. We denote their values, away from the perturbing influence of the strait, by $\mathrm{Re}[\zeta_1 e^{i\omega t}]$ and $\mathrm{Re}[\zeta_2 e^{i\omega t}]$. A more precise definition of ζ_1, ζ_2 will be discussed in Section 4. Let $\mathrm{Re}[ue^{i\omega t}]$ be the surface current, averaged across the strait, that is driven by the difference in ζ_1 and ζ_2, and let ζ_3, ζ_4, ζ_5, ζ_6 (all times $e^{i\omega t}$) be the surface elevations on opposite sides of each end of the strait (Fig. 1).

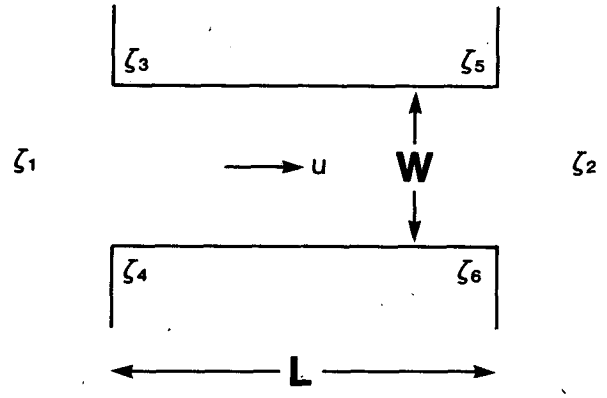

FIG. 1. Schematic diagram of sea levels and current (all periodic with frequency ω) associated with flow through a strait of depth h, length L, and width W connecting two large water bodies.

The assumption of cross-strait geostrophy (easily justified by scale analysis even if ω is not much less than the Coriolis frequency f) leads to

$$\zeta_4 - \zeta_3 = \zeta_6 - \zeta_5 = \frac{f}{g} Wu. \tag{2.1}$$

Along the strait, assumed to be in the x-direction, we assume a balance between acceleration, given by $i\omega u$, the sea surface slope and bottom friction, which we represent in a linearized form by λu. The neglect of the advective terms will be discussed further in Section 5. Hence

$$\zeta_5 - \zeta_3 = \zeta_6 - \zeta_4 = -\frac{L}{g}(i\omega + \lambda)u. \tag{2.2}$$

To close the problem we must relate ζ_3, ζ_4, ζ_5, and ζ_6 to the basin elevations ζ_1 and ζ_2. In the absence of rotation we would have $\zeta_4 = \zeta_3$ and $\zeta_6 = \zeta_5$, and might argue that ζ_3 and ζ_4 are approximately equal to ζ_1 and ζ_5 and ζ_6 are approximately equal to ζ_2. We would then obtain simply, from (2.2),

$$u = \frac{g}{L}(\zeta_1 - \zeta_2)(i\omega + \lambda)^{-1}. \tag{2.3}$$

If $f \neq 0$, we clearly cannot have $\zeta_4 = \zeta_3$; as the flow starts to accelerate through the strait (in the positive x-direction, say), either ζ_4 must rise or ζ_3 must fall in order to satisfy the geostrophic balance (2.1). The latter seems more likely if ζ_1 is already the greatest elevation in the system, so that we abandon the assumption, $\zeta_3 = \zeta_1$, made for the nonrotating case, but retain the approximation $\zeta_4 = \zeta_1$. Similarly, at the other end of the strait, we no longer have $\zeta_6 = \zeta_2$ but do retain $\zeta_5 = \zeta_2$, noting that these matching conditions are consistent with Kelvin wave propagation in the two basins imposing upstream values on ζ_4 and ζ_5. Subtracting (2.2) from (2.1), we get

$$u = \frac{g}{L}(\zeta_1 - \zeta_2)\left[i\omega + \lambda + f\frac{W}{L}\right]^{-1}, \tag{2.4}$$

which clearly reduces to (2.3) if $f = 0$.

This simple result, derived by Garrett and Toulany (1982), has a number of interesting features. First, we note that the Coriolis frequency, multiplied by the width-to-length ratio for the strait, appears in (2.4) as a term in direct addition to the friction coefficient! Second, for low frequency and weak friction such that $\omega, \lambda \ll f(W/L)$, (2.4) reduces to

$$u = g(\zeta_1 - \zeta_2)(fW)^{-1}. \tag{2.5}$$

In this limit of "geostrophic control" the sea level difference across the strait is equal to the sea level difference $\zeta_1 - \zeta_2$ between the two basins; we argue that it cannot be more.

This concept of geostrophic control has a simple interpretation, for $\omega < f$, in terms of Kelvin wave propagation. If the flow through the strait is carried away by a Kelvin wave, of amplitude $\zeta_1 - \zeta_2$ and effective width given by the Rossby radius $(gh)^{1/2}f^{-1}$, traveling at speed $(gh)^{1/2}$, the volume flux is given by $ghf^{-1}(\zeta_1 - \zeta_2)$. This is exactly as given by Wh times u from (2.5). This argument, of course, assumes a water depth in the second basin equal to the depth in the strait. In practice, it may be considerably greater, in which case the Kelvin wave amplitude is reduced so that the volume flux is still $ghf^{-1}(\zeta_1 - \zeta_2)$.

The general formula for the volume flux $Q = Whu$, with u from (2.4), is

$$Q = gh(\zeta_1 - \zeta_2)\left[f + (i\omega + \lambda)\left(\frac{L}{W}\right)\right]^{-1}. \tag{2.6}$$

and it is this formula that one would use in considering the response of sea level in a semi-enclosed sea, such as the Mediterranean or Baltic, to exterior changes. However, for low-frequency changes (corresponding to meteorological forcing), for reasonable estimates of the bottom friction coefficient λ, and for straits that are not too long in relation to their width, f tends to dominate the denominator of (2.6), so that the volume flux is primarily limited by geostrophic control.

The above model has a number of imprecise definitions and weak assumptions. In particular, ζ_1 and ζ_2 are not clearly defined, and it is not valid to take ζ_4 and ζ_5 precisely equal to the far-field elevations ζ_1 and ζ_2, respectively. These problems can be clarified and support provided for this simple model by considering a rather precisely defined problem for the flow through a narrow gap connecting two semi-infinite oceans. A discussion of this follows in the next section.

3. Oscillatory flow through a narrow gap

Buchwald and Miles (1974) considered the well-defined problem of Kelvin wave diffraction by a narrow gap in a straight wall separating two semi-infinite oceans of constant depth h (Fig. 2). In other words, they assumed that the elevation field, in the absence of the gap, was given by

$$\left.\begin{aligned}\zeta(x, y) &= \zeta_0(x, y) = A \exp[(i\omega y - fx)/c], \quad x > 0 \\ &= 0, \quad x < 0\end{aligned}\right\} \tag{3.1}$$

with $c = (gh)^{1/2}$ the shallow water wave speed and, as before, all motions being proportional to $e^{i\omega t}$. The formula in (3.1) thus describes a Kelvin wave propagating in the negative y-direction in the half-space $x > 0$. In fact, $\zeta_0(x, y)$ could refer to any permissible motion in $x > 0$, such as Poincaré waves if $\omega > f$; the important point is that ζ_0 refers precisely to the elevation field with the gap closed.

If the gap is now opened a current $u(0, y)$ will flow through the gap, its value determined by the need to equalize the elevation field across $x = 0$ for $-W/2 \leqslant y \leqslant W/2$. The resulting elevation field is (Buchwald and Miles, 1974).

$$\zeta(x, y) = \zeta_0(x, y)H(x) + \operatorname{sgn}x \int_{-W/2}^{W/2} G[|x|, (y-\eta)\operatorname{sgn}x]u(0, \eta)d\eta, \tag{3.2}$$

where $H(x)$ is the Heaviside step function and $G(x, y)$ the Green's function, described and determined by Buchwald (1971), giving the elevation field in a semi-infinite ocean, $x \geqslant 0$, in response to a delta-function source $u(0, y) = \delta(y)$ on the boundary. Matching ζ across $x = 0$ for $|y| \leqslant W/2$ leads to

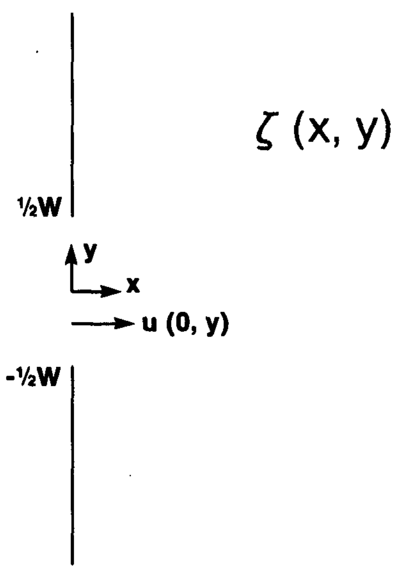

FIG. 2. Schematic diagram of a gap of width W between two semi-infinite oceans of constant depth h. The elevation field is $\mathrm{Re}[\zeta(x, y)e^{i\omega t}]$; the current through the gap is $\mathrm{Re}[u(0, y)e^{i\omega t}]$.

$$\int_{-W/2}^{W/2} [G(0, y-\eta) + G(0, \eta - y)]u(0, \eta)d\eta = -\zeta_0(0, y), \quad \text{for } |y| < W/2 \tag{3.3}$$

as an integral equation from which $u(0, \eta)$ may be determined.

For a gap that is narrow compared with the scale of $\zeta_0(x, y)$, i.e., for $\epsilon = \frac{1}{2}|\omega^2 - f^2|^{1/2}W/c \ll 1$, we may use the near-field approximation for G (Buchwald, 1971):

$$G(0, y) = \omega(2g)^{-1}\left\{Z - \frac{f}{\omega}\operatorname{sgn}y - \frac{i}{\pi}\left[2\ln(4|y|/W) - \pi\left(\frac{f}{c}\right)y\right] + O(\epsilon^2 \ln\epsilon)\right\}, \tag{3.4}$$

where

$$Z = \begin{cases} \dfrac{f}{\omega} - \dfrac{i}{\pi}\left[2\ln\left(\dfrac{\gamma k' W}{8}\right) + \dfrac{f}{\omega}\ln\left(\dfrac{f+\omega}{f-\omega}\right)\right], & 0 < \omega < f \\ 1 - 2\dfrac{i}{\pi}\ln\left(\dfrac{\gamma f W}{4c}\right), & \omega = f \\ 1 - \dfrac{i}{\pi}\left[2\ln\left(\dfrac{\gamma k W}{8}\right) + \dfrac{f}{\omega}\ln\left(\dfrac{\omega+f}{\omega-f}\right)\right], & f < \omega \end{cases} \tag{3.5}$$

with $k' = (f^2 - \omega^2)^{1/2}c^{-1}$, $k = (\omega^2 - f^2)^{1/2}c^{-1}$, and $\ln\gamma = 0.577$ is Euler's constant, so that $\gamma = 1.781$. Note that these expressions for Z are continuous at $\omega = f$.

The integral equation (3.3) is then

$$\frac{\omega}{g}\int_{-W/2}^{W/2}\left[Z - i\frac{2}{\pi}\ln(4|y-\eta|W^{-1}) + O(\epsilon^2\ln\epsilon)\right] \times u(0, \eta)d\eta = -A\left[1 + i\frac{\omega}{c}y + O(\epsilon^2)\right], \quad |y| < W/2, \tag{3.6}$$

after expanding $\zeta_0(0, y)$ from (3.1). As pointed out by Buchwald and Miles (1974) the solution of this, correct to the order $\epsilon^2 \ln\epsilon$ of the neglected terms, is

$$u(0, y) = -(gA)\left[\frac{2}{\pi}(\omega Z)^{-1} + \frac{y}{c}\right](W^2 - 4y^2)^{-1/2}. \tag{3.7}$$

The average current through the gap is then

$$\bar{u} = W^{-1}\int_{-W/2}^{W/2} u(0, y)dy = -\frac{gA}{W}(\omega Z)^{-1} \tag{3.8}$$

with a corresponding volume flux

$$Q = Wh\bar{u} = gh(\zeta_1 - \zeta_2)(\omega Z)^{-1}, \quad (3.9)$$

writing $A = -(\zeta_1 - \zeta_2)$ to correspond to the simple model of Section 2.

We note that the term proportional to y in (3.7) has dropped out of the formula for Q, which is thus valid to $O(\epsilon^2 \ln\epsilon)$ independently of the assumption that the basic motion $\zeta_0(x, y)$ is a Kelvin wave.

4. Model comparison

We are now in a position to compare formulas (2.6) and (3.9) for the volume flux. We examine first the low-frequency case ($\omega < f$) for which Z from (3.5) may be written

$$Z = \frac{f}{\omega} - 2\frac{i}{\pi}\left[\ln(0.60\delta) - \sum_{n=1}^{\infty}\left(\frac{\omega}{f}\right)^{2n}(2n)^{-1}(2n+1)^{-1}\right], \quad (4.1)$$

where $\delta = Wf(gh)^{-1/2}$ is the ratio of the width of the gap to the Rossby radius, already assumed small. The volume flux through the gap, from (3.9), is then

$$Q = gh(\zeta_1 - \zeta_2)\left[f + i\omega\frac{L_e}{W}\right]^{-1} \quad (4.2)$$

with

$$L_e = -\frac{2}{\pi}W\left[\ln(0.60\delta) - \sum_{n=1}^{\infty}\left(\frac{\omega}{f}\right)^{2n}(2n)^{-1}(2n+1)^{-1}\right]. \quad (4.3)$$

This is exactly the same in form as (2.6) with the frictional parameter omitted. The flux through a narrow gap is thus exactly as given by the simple model, subject to allowance for an "effective length" L_e given by (4.3). It is as if there is a distance $L_e/2$ on each side of the gap within which the flow accelerates.

The error in (4.3) is $O(\epsilon^2 \ln\epsilon)$, which is $O(\delta^2 \ln\delta)$ at low frequency, and thus negligible compared with the terms retained, provided that δ is small. The infinite series in (4.3) does not contribute much; at its maximum for $\omega = f$, the appropriate formula for the effective length is

$$L_e = -\frac{2}{\pi}W\ln(0.44\delta), \quad (4.4)$$

so that L_e/W only increases by 0.20 from $\omega = 0$ to $\omega = f$. For $\omega > f$ (but $\omega \ll f\delta^{-1}$ in order to have $\epsilon \ll 1$) Z may be written

$$Z = 1 - i\frac{2}{\pi}\left[\ln\left(0.22\frac{\delta\omega}{f}\right) + \sum_{n=1}^{\infty}\left(\frac{\omega}{f}\right)^{-2n}(2n)^{-1}(2n-1)^{-1}\right], \quad (4.5)$$

so that

$$Q = gh(\zeta_1 - \zeta_2)\left[\omega + i\omega\frac{L_e}{W}\right]^{-1} \quad (4.6)$$

with

$$L_e = -\frac{2}{\pi}W\left[\ln\left(0.22\frac{\delta\omega}{f}\right) + \sum_{n=1}^{\infty}\left(\frac{\omega}{f}\right)^{-2n}(2n)^{-1}(2n-1)^{-1}\right]. \quad (4.7)$$

This form for Q for $\omega > f$ is not the same as derived in (2.6) from a model which is clearly more appropriate for $\omega < f$. However, as ω is typically rather less than the imaginary term $i\omega L_e/W$ in the denominator of (4.6), the difference in form between (4.6) and (2.6) is not very significant; the flow is primarily limited by acceleration in the neighbourhood of the gap.

The actual magnitude of the effective length L_e, based on (4.3, 4, 7), is shown in Fig. 3 for various values of $\delta = Wf(gh)^{-1/2}$ as a function of frequency ω. We note that at low frequencies, which are our main interest, L_e/W is not much more than 3 even for very narrow straits. In this case, L_e/W increases as δ de-

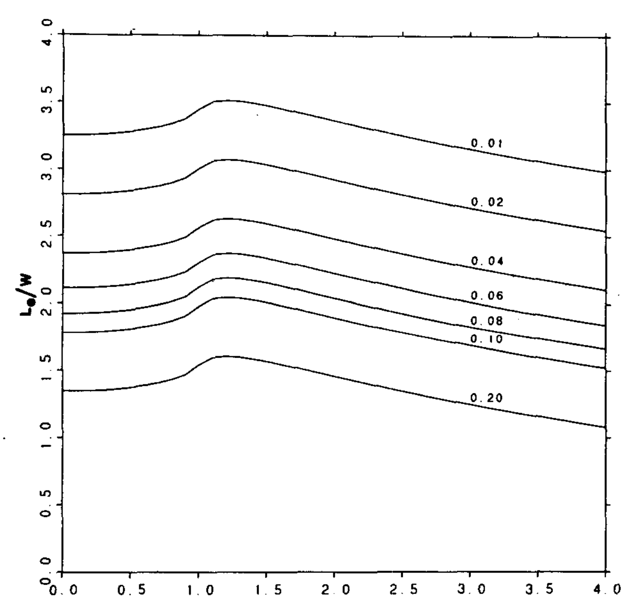

FIG. 3. The effective length L_e, as a multiple of strait width W, as a function of frequency ω for various values of $\delta = Wf/c$, which is the ratio of the strait width to the Rossby radius of deformation.

creases; the ratio of L_e to the Rossby radius is $(L_e/W)\delta$ which decreases with decreasing δ.

As already mentioned, L_e allows for acceleration of the water into the gap. Even for a strait of finite length L there is presumably an "end correction" which should be added to the actual length L in (2.6). We have not solved the appropriate problem, but might expect the end correction to be comparable in magnitude to the effective length L_e of a gap.

The key result of this section is that, particularly at low frequencies, there is a very close correspondence between the predictions of our simple model and the precise diffraction theory for a narrow gap, subject only to regarding the gap as having an effective length. This agreement thus lends support to the idea of geostrophic control that arose from the simple model.

The results for the narrow gap also permit one to be more precise about the definition of ζ_1 and ζ_2 in the simple model. For semi-infinite oceans they would be the values of the elevation in the two oceans if the gap were closed; for $\omega < f$ this would mean the amplitudes of the Kelvin waves incident on the gap from the upstream sides of the gap (to the left of the strait, facing the ocean, $f > 0$). The motion on the downstream side of the strait will be a Kelvin wave modified by the flow through the strait.

5. Discussion

Our analysis of the Buchwald and Miles (1974) solution for the flow through a narrow gap separating two semi-infinite oceans has provided justification for the formula (2.6) for the flow through a strait of finite length, subject to adding to its length L in the formula an end correction comparable to the effective length L_e for a narrow gap. In particular, for a strait that is not too long or frictional and at low frequencies, (2.6) reduces to the limit of geostrophic control for which the flux is given by $ghf^{-1}(\zeta_1 - \zeta_2)$.

a. Orders of magnitude

For an example, consider a strait of depth 250 m. The Rossby radius based on this, if $f = 10^{-4}$ s^{-1}, is 500 km. If the strait width is only 10 km, then δ, the ratio of width to Rossby radius, is 0.02, and at low frequency the effective length L_e for a narrow gap is about $2.8W$. If we use this as an end correction for a strait of finite length, say $3W$, then the total length to use in (2.6) is about $6W$. Hence f dominates $i\omega(L/W)$ in (2.6) provided that ω is less than about $f/6$, corresponding to a period longer than about 4 days. Motions of much longer period will be geostrophically controlled, unless the frictional term λ is important. We also note that the end correction may be rather less than the $2.8W$ used here if the water depth in the two ocean basins is much greater than in the strait.

We may make a rough estimate of λ by equating it to $C_D u_0 h^{-1}$, with C_D a quadratic drag coefficient of about 2.5×10^{-3} and u_0 a reference speed which might be that of the tidal currents. Taking $u_0 = 0.5$ m s^{-1} and $h = 250$ m as before, we have $\lambda = 5 \times 10^{-6}$, so that $\lambda(L/W) = 3 \times 10^{-5}$ if we take $L/W = 6$. The frictional term is thus a factor of 3 less important than f.

In practice, rather than using (2.6) in the limit where f dominates or making rough estimates of the appropriate value of L/W, one might want to use a numerical model for the detailed geometry of a particular strait and its vicinity, fitting results at various frequencies to a formula like (2.6).

b. Linearization

Both the simple model of Section 2 and the diffraction theory for a gap, described in Section 3, neglect the advective terms in the equation of motion. As these are vital to most discussions of the hydraulic control of flow over sills or through straits (e.g., Gill, 1977), their neglect in the present problem requires discussion.

An effective way to evaluate the importance of the advective terms is by considering the equivalent Bernoulli set-down, given by

$$B = \frac{1}{2}\frac{u^2}{g} \tag{5.1}$$

compared with the driving elevation difference $\zeta_1 - \zeta_2$, denoted $\Delta\zeta$ here. If we use in (5.1) the average surface current in the strait, which has a maximum given by (2.5), then

$$\frac{B}{\Delta\zeta} = \frac{1}{2}\delta^{-2}\left(\frac{\Delta\zeta}{h}\right), \tag{5.2}$$

with $\delta = Wf(gh)^{-1/2}$, the ratio of the strait width to the Rossby radius based on the strait depth.

For the parameter values used in the earlier discussion, $\delta = 0.02$ and is unlikely to be much smaller. A typical elevation difference at low frequencies between two basins is unlikely to be more than a few tenths of a meter, so $\Delta\zeta/h$ might be of the order of 10^{-3} and hence $B/\Delta\zeta$ can be about one.

However, B merely represents a setdown in the high-current regime of the strait itself; sea level recovers this set-down again as the current emerges from the strait, spreads out and slows down. The Bernoulli set-down is unlikely to be dynamically important unless it becomes a significant fraction of the water depth. For the above parameter values $B/h \approx 10^{-3}$ and is negligible.

These estimates are based on the average current in the strait; larger values near the sides are implied by (3.7), but these are associated with the sharp corners of the model and unlikely to be achieved in practice.

In summary, for flows through straits driven by a small head difference, the nonlinear terms are dynamically negligible, though they may lead to a setdown, in the strait itself, comparable in magnitude with the driving head.

c. *Baroclinic effects*

The theory of this paper has been for barotropic flows. The results can be adapted for baroclinic flows driven, for example, by small differences in stratification between two basins. In this case it is likely that the internal Rossby radius of deformation is less than the strait width, so that motions propagate from one basin to another as internal Kelvin waves following the appropriate coastline and without regard to the existence of an opposite side of the strait.

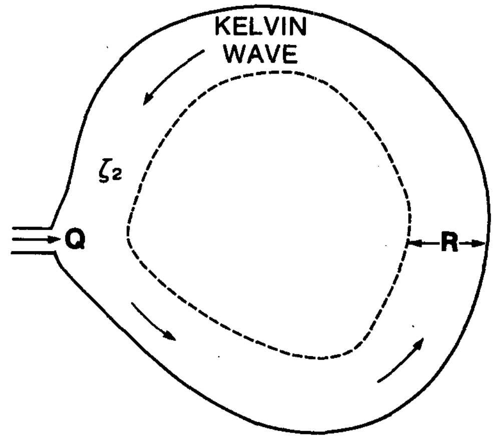

FIG. 4. Schematic of the response of a finite ocean basin to flux through the strait.

6. Application

The main applications of the theory of this paper, and particularly of the simple formula (2.6) and its limit of geostrophic control, are in understanding the response of the local flow in a strait, and the levels of connected basins, to sea level differences between the two basins induced by meteorological or other forcing. In particular, the reduced flux for a rotating system compared with a nonrotating system is important.

Care is needed in choosing the values of ζ_1 and ζ_2 to use in the flux formula. As discussed in Section 4, ζ_1, ζ_2 should be taken as the amplitudes of the Kelvin waves incident on the strait from the upstream side, with the motion on the downstream sides being Kelvin waves modified by the flow through the strait.

If the oceans are finite in size, these modified Kelvin waves will travel around the oceans and return as the new incident Kelvin waves. A simple case to consider is one where the travel time of the Kelvin wave around one ocean basin is short compared with the period of the fluctuating flow through the strait, so that continuity for the second basin requires

$$RP\partial\zeta_2/\partial t = Q, \qquad (6.1)$$

with Q from (2.6), P the perimeter of the basin and R the Rossby radius (Fig. 4). If R is greater than the radius of the basin, RP should be replaced by A_2, the area of the basin. The resulting formula, $A_2\partial\zeta_2/\partial t = Q$, is that used by Garrett (1983) and Garrett and Majaess (1984) in their studies of Mediterranean sea level.

One could clearly allow for finite travel time of the Kelvin wave around the ocean basin. However, it is of rather more interest to consider two connected basins, of depths H_1, H_2 and areas A_1, A_2 (where A_1, A_2 should be replaced by perimeter × Rossby radius if this is less than the actual area) which are both such that the time scale of the motion is much longer than the travel time of a Kelvin wave around either ocean basin.

If h is the depth of the strait (possibly much less than H_1, H_2), L the total length of the strait including an end correction, and W its width, we define $a_i = gh(f^2A_i)^{-1}$ for $i = 1, 2$ and $b = L/W$. Continuity then gives

$$-A_1\frac{\partial\delta_1}{\partial t} = A_2\frac{\partial\zeta_2}{\partial t} = gh\frac{\zeta_1 - \zeta_2}{f + i\omega b}. \qquad (6.2)$$

Normal-mode solutions, proportional to $e^{i\omega t}$, satisfy

$$\frac{\omega}{f} = \{-1 \pm [1 - 4b(a_1 + a_2)]^{1/2}\}(2ib)^{-1}. \qquad (6.3)$$

For $4b(a_1 + a_2) < 1$ both roots are imaginary and represent a decay of ζ_1 and ζ_2 as the levels of the two basins equilibrate. In particular, for $4b(a_1 + a_2) \ll 1$, the important solution behaves like $\exp[-(a_1 + a_2)ft]$. The decaying nature of the solution in the absence of friction is surprising. The original potential energy associated with the level difference between the two basins is converted to kinetic energy of circulation in the two basins, cyclonic in the one that was originally low and anticyclonic in the one that was originally high.

This slow conversion of potential energy to kinetic occurs even if $4b(a_1 + a_2) > 1$, for which ω has real and imaginary parts. For $4b(a_1 + a_2) \gg 1$ the oscillation is that of a Helmholtz mode; the frequency $b^{-1/2} \times (a_1 + a_2)^{1/2}f$ is equivalent to that given by Huthnance (1980), who did not point out the slow decay that occurs in a rotating system.

The foregoing analysis depends upon the time scale of the motion being much less than the travel time of a Kelvin wave around each basin. If this assumption is not made, other, higher frequency, nondecaying modes are possible. These have been discussed in detail by Huthnance (1980). However we emphasize the importance, at low frequencies, of allowing for the decaying normal modes discussed here. These are likely to be important in any forced solution.

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