A Model for the Seasonal Pycnocline in Rotating Systems with Application to the Baltic Proper

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ABSTRACT

A one-dimensional seasonal pycnocline model, primarily intended for use in long-term circulation models, is developed. The model is of the two-layer (integral) type with a turbulent, well mixed surface layer and a nonturbulent, stratified lower layer. The parameterization of the entrainment velocity at the pycnocline accounts for the effects of buoyancy fluxes through the sea surface upon the entrainment flow. Also the "retreat" of the pycnocline, caused by large positive buoyancy supplies through the sea surface, is allowed for in the model. Effects of the rotation of the system are included. Under stable or neutral conditions the rotation may limit the penetration of mechanically generated turbulence. Hence, for weak positive buoyancy fluxes through the sea surface the rotation will cause the "retreat" of a deep pycnocline.

The model is utilized to simulate the climatological mean annual cycles of temperature and salinity in the upper parts of the Baltic (above the perennial main halocline at about 60 m depth). The test of seasonal pycnocline models introduced by Gill and Turner is applied. Two empirical constants are determined, the well known m_0, which appears in formulas for the entrainment velocity, and $\mathscr{K}$, which determines the thickness of the Ekman layer. For the best-fit case, which is obtained for $m_0 = 0.6$ and $\mathscr{K} = 0.20$, the computed annual cycles of temperature and salinity appear quite realistic.

The estuarine circulation of the Baltic is accounted for. Brackish surface water is exported to the ocean and a dense bottom current carries the import of saltier "sea" water from the mouth. The seasonal pycnocline model correctly predicts the depth of the main halocline.

1. Introduction

The Baltic (see map, Fig. 1) is an estuary with a rather narrow and shallow connection to the sea. (The greatest sill depth is 18 m.) Owing to the relatively large freshwater supply, about 15 000 m^3 s^{-1} on the average, the salinity of the upper layer in the Baltic proper (about 60 m thick) is only about 7‰. Below the upper layer is an approximately 10 m thick halocline (the perennial main pycnocline) resting on the stratified lower layer of salinity 10–13‰. There are also horizontal salinity gradients in the Baltic with the highest salinities on all levels in the inflow region in the southwest. For a detailed description of the hydrographical state of the Baltic, see Kullenberg (1981).

The vertical salt stratification in the Baltic proper is the result of an estuarine circulation; see Stigebrandt (1983) for a description of the exchange of water and salt with the ocean. Relatively dense water (mean salinity about 17‰) flows into the Baltic proper from the Belt Sea/Kattegat forming a dense bottom current (gravity current). This entrains fresher, upper-layer Baltic-proper water on its way towards the deeper parts of the basin as described by Pedersen (1977) and Walin (1981). At some depth, in or below the main halocline, the water in the bottom current becomes neutrally buoyant. Vertical mixing processes lift the denser water through the halocline into the upper layer where it is mixed with the freshwater supplied through the sea surface. The surface layer loses water to the ocean via the Belt Sea and Kattegat.

For a large part of the year the seasonal thermocline/pycnocline shields the main halocline in the Baltic from direct contact with the well mixed surface layer. Hence, in order to model the seasonal exchange through the main halocline, one has to take into account the buoyancy flux through the sea surface. This is induced by heating/cooling processes and by removal/addition of freshwater at the sea surface.

The seasonal pycnocline model to be used here is partly quite standard with the most direct links to the shelf front model of Stigebrandt (1981a) and the model for the Arctic Ocean of Stigebrandt (1981b). In these two models generalized Kato-Phillips type of expressions for the entrainment velocity in the presence of a buoyancy flux, positive and negative, respectively, were developed and used. An extension of the theory, which in a simple way accounts for the effects of the rotation of the system upon the development of the pycnocline, is presented in Section 2c. In Section 3 the theory is tested against the climatological mean Baltic proper and the empirical constants m_0 and $\mathscr{K}$ are determined. Some properties of the Baltic proper are discussed in Section 4 and new estimates of the heat and freshwater

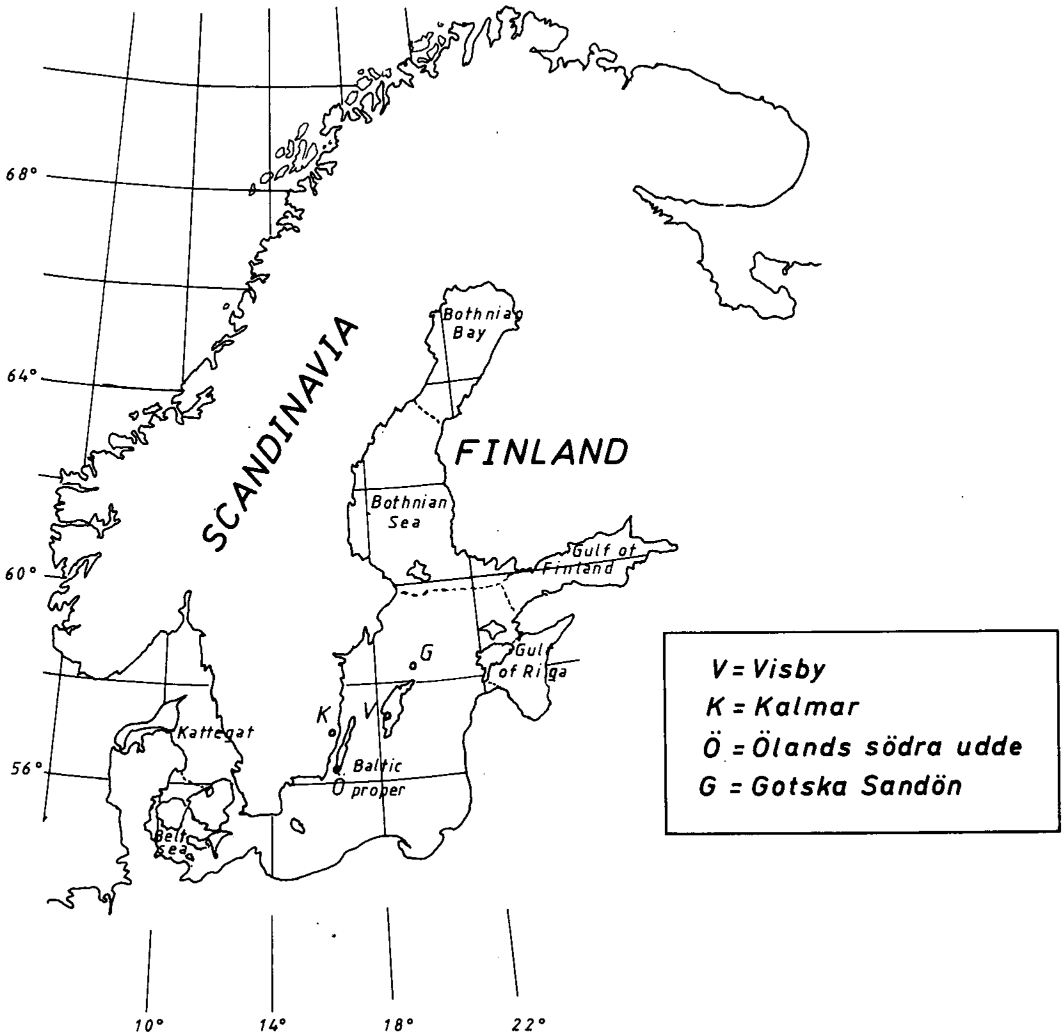

FIG. 1. Map of the Baltic Sea.

budgets are presented. A few conclusions are drawn in Section 5.

2. Theory

Let us consider a physical system composed of two dynamically different regimes physically on top of each other and separated by a very thin (seasonal) pycnocline. The upper regime is a turbulent, thoroughly mixed (homogeneous) surface layer of thickness h, salinity S and temperature T. The lower regime is a passive, motionless and turbulence-free reservoir of stratified water of salinity $S(z)$ and temperature $T(z)$. Here z is measured positive downwards from the sea surface. The system is one-dimensional and thus horizontally homogeneous. A model sketch is shown in Fig. 2.

Because of the assumed horizontal homogeneity the state parameters of the mixed layer (h, S and T) may change with time only because of exchange processes acting at (i) the sea surface and (ii) the pycnocline. The depth to the pycnocline, H, is one of the important vertical length scales in this problem. The wind is normally the main source of energy for the turbulence in the mixed layer. The velocity scale for the mechanically generated turbulence is denoted by u_*. Another source of energy for the turbulence operates during periods with removal of buoyancy from the sea, through the sea surface, by thermohaline processes. The thermohaline processes at the sea surface, giving rise to a buoyancy flux, B, may also suppress the turbulence. This occurs during periods with buoyancy being supplied to the sea. In periods with large and increasing buoyancy supplies it may occur that the turbulent layer becomes thinner (the pycnocline retreats). A second vertical length scale determining the thickness of the well-mixed layer then enters. This is the Monin-Obukhov length scale which is proportional to u_*^3/B.

In geophysical flows a third vertical length scale, due to the rotation of the system, may also be of importance for the thickness and time-development of the well

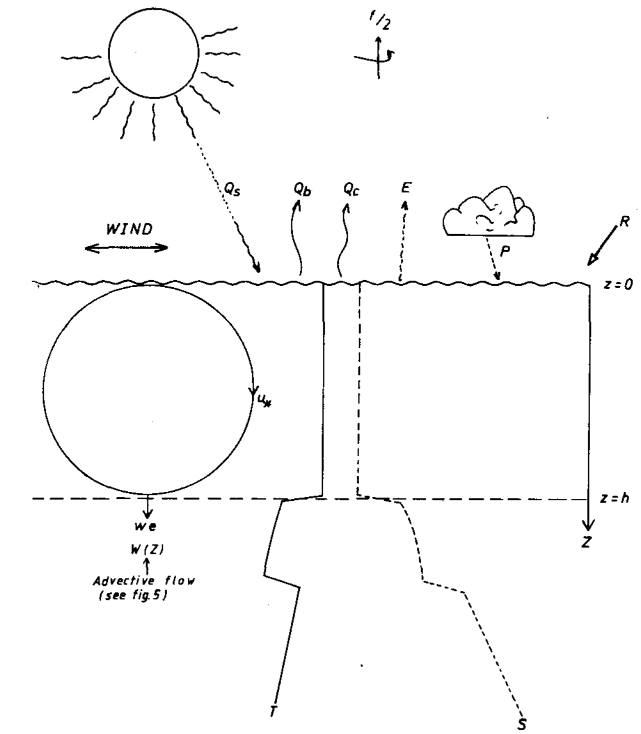

FIG. 2. Schematic of the seasonal pycnocline model with a homogeneous, thoroughly mixed upper layer and a stratified lower layer.

mixed layer. It is well known that, in the absence of (i) a vertical stratification in the sea and (ii) buoyancy fluxes through the sea surface, the rotation of the system effectively limits the penetration of the mechanically generated turbulence. The penetration depth in this case is given by the so-called Ekman length. This is proportional to u_*/f where f is the Coriolis parameter. Hence, there are at least three dynamical regimes, each characterized by its own macroscopic vertical length scale, that may determine the thickness of the well mixed layer.

First we develop the theory for a nonrotating system. At the end of this section the effects of rotation of the system are discussed and the theory is then extended also to rotating systems. Readers interested in different kinds of seasonal pycnocline models are referred to the works of Kraus and Turner (1967), Gill and Turner (1976), Pollard *et al.* (1973) and Price (1979) for integral models (in which category the present model belongs) and to, e.g., Mellor and Durbin (1975) and Omstedt *et al.* (1983) for "continuous" models which utilize a detailed turbulence model.

a. The regime of mixed layer deepening in a nonrotating system

Regardless of the kind of dynamics used in the model, conservation equations for the state parameters of the mixed layer (thickness, temperature and salt) have to be formulated. Conservation of volume in the mixed layer gives the following equation for changes of the thickness, h, of the layer

$$dh/dt = F + w_e - [F + w(h)] \quad \text{if} \quad w_e \geq 0 \quad (1)$$

where t is time and w_e the entrainment velocity that is taken positive when fluid from below the pycnocline is entrained into the mixed layer; F is the net freshwater supply which is taken positive if precipitation plus runoff from land is larger than evaporation. The terms within the square brackets do not apply to a closed system. In an estuary, however, the sea surface is held at a constant level and there is an import of dense water that is stored below the mixed layer. These advective effects are accounted for by the terms within the squared brackets. The vertical advective velocity $w(h)$, due to the estuarine circulation, is further discussed in Section 3. Equation (1) is irrelevant for situations with a retreating pycnocline. This latter case is treated in Sections 2b, c.

The temperature T of the upper layer may change owing to heat flow through the air–sea interface, Q_{in}, and to entrainment of water (heat) from the lower layer. Temperature changes in the upper layer are described by the equation

$$dT/dt = [Q_{\text{in}}/(\rho c_p) + w_e(T2 - T)]/h, \quad \text{if} \quad w_e \geq 0, \quad (2)$$

where ρ is the density and c_p the specific heat of water in the upper layer. $T2$ is the temperature of the water at the top of the lower layer (i.e., at the depth $z = h+$). The Q_{in} is taken positive for heat flow from the atmosphere to the sea. We regard ρ and c_p as constants in this equation.

The salinity S of the upper layer may change by the addition/removal of freshwater through the sea surface. Entrainment of water from the lower layer may also change the salinity. Salinity changes are then described by the equation

$$dS/dt = [-F \cdot S + w_e(S2 - S)]/h, \quad \text{if} \quad w_e \geq 0 \quad (3)$$

where $S2 = S(h+)$. Thus, Eqs. (1), (2) and (3) show that we can compute changes of the temperature, salinity and depth of the upper, well mixed layer if we manage to obtain a useful expression for the entrainment velocity.

The so-called Kato-Phillips formula for the entrainment velocity, which (except for a numerical constant) can be derived from dimensional arguments, appears to work quite well for conditions with a thin mixed layer and no buoyancy flux through the sea surface (cf. Stigebrandt, 1981c, 1983), is

$$w_e = (2 \cdot m_0 \cdot u_*^3) \Big/ \left(g \frac{\Delta\rho}{\rho} h\right), \quad (4)$$

where $\Delta\rho/\rho$ is the relative density difference between the upper and lower layers and m_0 is an empirical constant. The friction velocity u_* is calculated from the relation

$$u_*^2 = \frac{\rho_a}{\rho} \cdot C_d \cdot W^2, \tag{5}$$

where C_d is the drag coefficient for air flow over water, W the wind speed and ρ_a the density of air.

The buoyancy flux B through the sea surface may be calculated from the expression (cf. Turner, 1973)

$$B = g\left(\frac{\alpha}{\rho C_p} Q_{\text{in}} + \beta F \cdot S\right). \tag{6}$$

The coefficients α and β are defined by

$$\alpha = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}, \quad \beta = \frac{1}{\rho}\frac{\partial \rho}{\partial S},$$

where ρ, α and β are functions of temperature and salinity and may be obtained from the equation of state for sea water. The term $\alpha \cdot Q_{\text{in}}$ is positive for heating ($Q_{\text{in}} > 0$) at temperatures above the temperature of maximal density (T_m) and for cooling ($Q_{\text{in}} < 0$) at temperatures below T_m. For all salinities and a positive net freshwater supply ($F > 0$) $\beta \cdot F$ is positive. Hence, with the sign conventions adopted here, B is positive when the sea receives buoyancy through the sea surface.

The entrainment velocity into the mixed layer in the presence of a positive buoyancy flux through the air–sea interface is (deSzoeke, 1980; Stigebrandt, 1981a),

$$w_e = (2m_0 \cdot u_*^3)\Big/\left(g\frac{\Delta\rho}{\rho}h\right) - B\Big/\left(g\frac{\Delta\rho}{\rho}\right), \; B > 0. \tag{7}$$

In this regime, buoyancy is added at the sea surface (thus radiative penetration of heat into the water column is neglected) and the buoyancy flux in the well mixed layer dampens the intensity of the mechanically generated turbulence. For large buoyancy supplies and/or weak winds (small u_*), the entrainment velocity may formally become negative (change of direction). There is of course no physical possibility for this to occur and Eq. (7) is not valid for this regime of "pycnocline retreat." [Thus, Eq. (7) is valid only for $w_e \geqslant 0$.] The "retreat" regime, which occurs when the depth to the pycnocline (H) is no longer the relevant vertical length scale for the well mixed layer, is treated in Sections 2b, c.

There is also another regime for which B is negative. The negative buoyancy flux creates convection in the mixed layer. With regard to entrainment flows, the buoyancy flux now appears primarily as a turbulence source. However, the efficiency of the convectively generated turbulence in the mixed layer in creating entrainment flows through the pycnocline should probably not be larger than that caused by the mechanically generated turbulence (i.e., of the order of 5%, which means that the thermohaline convection is only slightly penetrative). Thus, for this regime ($B < 0$) the buoyancy term should be multiplied by a factor ϵ, much smaller than one.

Based on a literature survey, the present author found that ϵ should be about 0.05 and the following expression for the entrainment velocity in the presence of a negative buoyancy flux through the air–sea interface was proposed (Stigebrandt, 1981b):

$$w_e = (2m_0 \cdot u_*^3)\Big/\left(g\frac{\Delta\rho}{\rho}h\right) - \epsilon \cdot B\Big/\left(g\frac{\Delta\rho}{\rho}\right), \quad B < 0. \tag{8}$$

If we let ϵ be equal to 1 for $B \geqslant 0$, Eq. (8) can be used for all values of B.

b. The regime of pycnocline retreat due to large buoyancy supplies through the sea surface

Some of these equations are valid only if the mixed layer deepens ($w_e > 0$). Therefore, we need to state the set of equations that is valid for the retreat regime, in relation to a nonrotating system.

For the retreat regime [$w_e < 0$ in Eq. (7)], the depth to the pycnocline (H) is not the relevant length scale in the system. Here the depth of the well mixed layer is determined by the depth to which the mechanically generated turbulence is able to mix the buoyancy supplied at the sea surface (thus, $B > 0$ and $\epsilon = 1$ for this case). This depth is obtained by setting $w_e = 0$ in Eq. (7). One then obtains

$$h = (2m_0 \cdot u_*^3)/B, \tag{9}$$

which is equivalent to the so-called Monin-Obukhov length. This expression for the thickness of the mixed layer in the retreat regime was used already by Kraus and Turner (1967). The temperature and salinity changes of the upper layer are determined from Eqs. (2) and (3), respectively, with $w_e = 0$.

c. A simple inclusion of rotational effects

One may ask what role the rotation of the earth plays in the evolution of the seasonal pycnocline. It is well known that for a situation with no buoyancy flux through the sea surface and with vertically homogeneous water, the thickness of the wind-stirred layer is limited to a boundary layer (the Ekman layer) whose thickness is $\mathscr{H} u_*/f$ (where $\mathscr{H}$ is an empirical constant). The penetration depth of the turbulence (that determines the value of $\mathscr{H}$) is related to the intensity of the vertical mixing, which in classical Ekman theory is expressed by the magnitude of a vertical (eddy) exchange coefficient for momentum.

As a first approximation it appears quite reasonable to assume that for zero, or positive, buoyancy fluxes through the sea surface the shortest of the length scales $\mathscr{H} u_*/f$, $2m_0 u_*^3/B$ and H determines the thickness of the well mixed upper layer. The rotational length scale will be the shortest for the cases (i) $w_e < 0$ and $B > 0$ and $\mathscr{H}u_*/f < 2m_0u_*^3/B$, and (ii) $w_e > 0$ and $B \geqslant 0$ and $\mathscr{H}u_*/f < H$. For these cases we set

$$h = \mathscr{H}u_*/f. \tag{10}$$

For a negative buoyancy flux ($B < 0$), however, it seems reasonable, from arguments of static stability, to assume that the rotation of the system is an irrelevant factor in determining the thickness of the mixed layer. Hence, for this regime, the depth to the pycnocline (H) should be the thickness of the mixed layer. We assume that the rotation has no effect upon the entrainment velocity (i.e., when H is the shortest length scale), which means that Eq. (8) is also used for the rotating case. More complicated effects of the rotation upon the seasonal pycnocline were considered by Garwood (1977) and Gaspar (1984) who worked out parameterizations for the effect of rotation upon the entrainment velocity. We have covered the whole w_e, B-plane except for the physically impossible domain where $w_e < 0$ and $B < 0$; see Fig. 3 for a graphical summary of the different dynamical regimes.

When the mixed layer retreats, the lower layer takes over the water left behind. The properties of this resemble those of the mixed layer just before the retreat. Thus the retreat of the mixed layer will generally create a stratified lower layer. In the model, the vertical salt and temperature profiles in the lower layer are carried by a vertical grid. Thus the model is two-layered only in the sense that there is one turbulent layer while the rest is nonturbulent. With respect to the vertical distribution of density (salt, temperature), however, the model is multilayered. For application of the theory we also need the equation of state for sea water from which we determine ρ, α and β. We will use the equation of state published by UNESCO (1981).

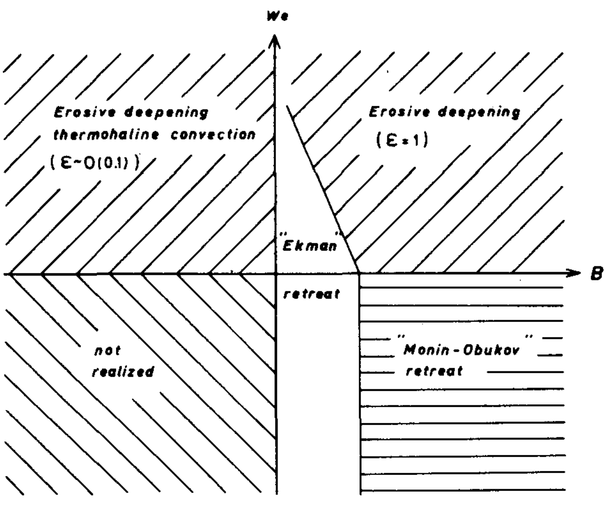

FIG. 3. The different dynamical regimes considered in this paper shown on the w_e, B-plane.

3. Application of the theory to the Baltic proper

In the application of a one-dimensional theory it is implicitly assumed that the system can be regarded as horizontally well mixed. If the model is applied to a partly shallow system, some of the effects induced by bathymetry may be included. Thus one may use horizontally integrated equations, utilizing the hypsographic function of the system, whereby Eqs. (2) and (3) become slightly modified. In the Baltic the vertically varying horizontal area is a factor of large importance in, e.g., determining the volume flow caused by entrainment through the main halocline. Ehlin and Mattisson (1976) give the horizontal surface area of the Baltic proper for some depths. They also give the volumes for these depths. This information is displayed in Fig. 4 where, for the area, we have made linear interpolations between the given values. Volumes were computed under the assumption that the horizontal surface area varies as in Fig. 4.

The assumption of horizontal uniformity is not always valid for short time scales in large systems since buoyancy supplied to shallow, nearshore areas may produce persistent, baroclinic–geostrophic currents with only a slow exchange with the deeper, central parts of the basin. Hence the use of horizontally integrated models might be particularly dubious for short time-scale predictions. In the present application, however, we are interested in an annual cycle in the Baltic and, as will be seen, a horizontally integrated model appears to be adequate. An alternative treatment could imply the addition of one or several separate models for the coastal areas that interact with the model for the open sea. Such a partition might be considered in future when more is known about exchange processes between coastal areas and the open sea.

In the real Baltic, the vertical distributions of salt and temperature below the mixed layer will change because of:

(i) there is turbulence and vertical diffusion below the mixed layer. Kullenberg (1981) describes the results from efforts of his own and others to estimate the rate of vertical diffusion.

(ii) the dense bottom current, renewing the Baltic deep water, removes by entrainment water from the depth interval from about 15 m down to the perennial halocline (at about 60 m). The effect of this removal is a compression of the vertical profiles. At the same time the filling of the layer below the main pycnocline leads to a general lifting of the profiles above the halocline (cf. Fig. 5).

In the model, these effects of the dense bottom current (which is part of the estuarine circulation) are taken into account. We assume that the volume flow of dense water ($S = 17‰$) into the Baltic proper just inside the Darss Sill (at the border between the Belt Sea and the Baltic proper) is $V_0 = 10\,500$ m^3 s^{-1}. By entrainment

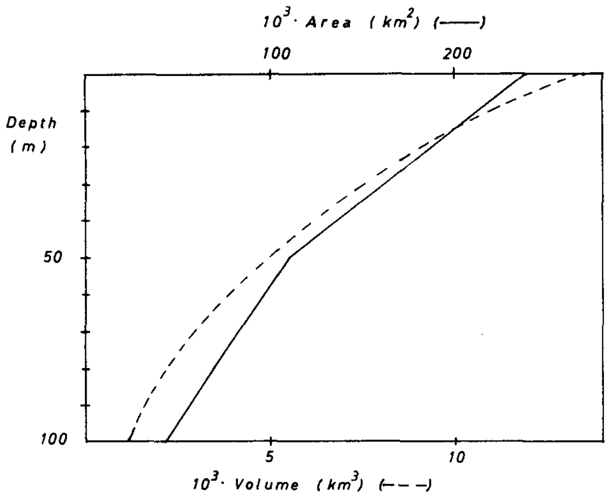

FIG. 4. The vertical distribution of the horizontal surface area in the Baltic proper (solid). Also shown is the total volume below a certain depth (dashed).

of Baltic Sea upper water (S = 7‰), the volume flow increases to V_{max} = 35 000 $m^3 s^{-1}$ at the depth of the halocline where the salinity is 10‰. These figures appear to be in harmony with estimates by Walin (1981). The inflow has a seasonal variation (Stigebrandt, 1984) that is ignored here. By continuity, the upward volume transport in the interior must be equal to the downward volume transport by the dense bottom current (cf. Fig. 5). Thus the upward directed vertical velocity in the interior, $w(z)$, may in the model be described as

$$w(z) = V(z)/A(z), \tag{11}$$

where

$$V(z) = \begin{cases} V_0 + (V_{max} - V_0)(z - 15)/45, & \text{for } 15 \leq z \leq 60 \text{ m} \\ V_{max}, & \text{for } z > 60 \text{ m.} \end{cases} \tag{12}$$

Here $A(z)$ is the horizontal surface area of the Baltic proper at the depth z. On the annual time scale the halocline has a fairly steady position. This means that the upward vertical advection of the halocline on this time scale must be balanced by vertical "diffusive" processes. In the present model the diffusive process is the entrainment process which, as will be seen, is active at the halocline during a few months in winter.

a. The buoyancy fluxes

For application of the seasonal pycnocline model, one has to know the buoyancy flux through the sea surface. For the Baltic the climatological means of some of the buoyancy flux components are relatively well known while others must be computed. There is no good dataset for one single year available for the vertical distribution of monthly, horizontal mean temperatures in the Baltic proper. Therefore we have constructed monthly mean vertical profiles from the temperature charts of Lenz (1971). Horizontal mean temperatures were estimated every 10th meter. No attempts were made to account for coastal anomalies, e.g., warm (cold) coastal jets in spring (winter). Thus these profiles should represent the long-term mean open-sea conditions in the Baltic proper. Since we are going to compare our model results with the climatological mean Baltic proper, we will use the climatological mean buoyancy fluxes.

The net freshwater supply F to the sea is

$$F = R + P - E, \tag{13}$$

where R is the runoff from land, P is the precipitation and E is the evaporation from the sea surface. For the Baltic R appears to be rather well known. Mikulsky (1982) presents monthly means for the Baltic inside the Belt Sea for the period 1951–75 (see Fig. 6). There is mixing between fresh and sea water in river mouths and adjacent seas. This premixing is not considered in our model. Since the buoyancy flux in the Baltic proper generally is strongly dominated by heating/cooling processes, the neglect of this premixing appears to be harmless. The annual mean precipitation on the open Baltic proper appears to be in the range 0.5–0.65 m (see Ehlin, 1981). The values used in our computations, adopted from Ehlin's paper, are shown in Fig. 6.

Evaporation from the sea surface E is an important factor not only in the freshwater budget but also in the heat budget. The net heating of the water body through the sea surface, Q_{in}, is

$$Q_{in} = Q_s - Q_b - L \cdot E - Q_c, \tag{14}$$

where Q_s is the absorbed solar radiation, Q_b the effective longwave back radiation, $L \cdot E$ the heat loss due to evaporation, for L is the latent heat of vaporization of water, and Q_c is the sensible heat exchange.

Dera (1983) gives monthly means of Q_s for the Baltic at 54, 57 and 60°N, respectively. We adopt his values for 57°N as representative for the Baltic proper. The linear interpolation between these values used in the computations is shown in Fig. 7. Dera's values are in agreement with those given earlier by Gargas (1975).

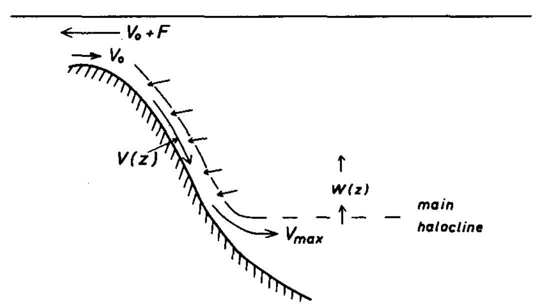

FIG. 5. Sketch showing the induction of a vertical advection field in the interior by the dense, entraining bottom current. F is the net freshwater supply to the system.

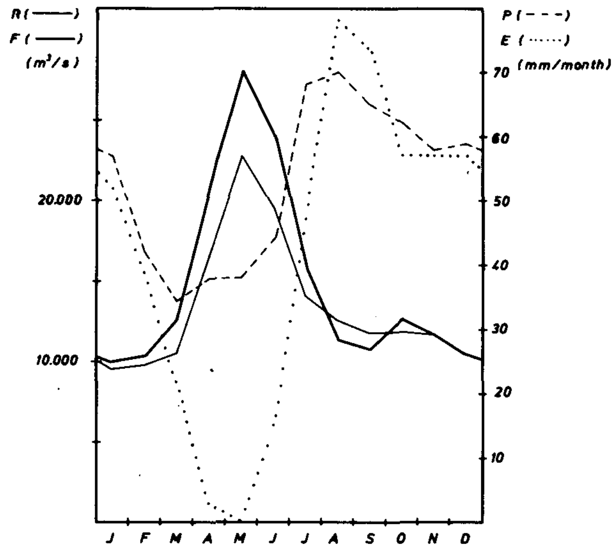

FIG. 6. The annual variation of freshwater runoff from land to the whole Baltic inside the Belt Sea (R, after Mikulsky, 1982), precipitation on the Baltic proper (P, after Ehlin, 1981) and evaporation (E, computed by the present model). Also shown is the net freshwater supply to the Baltic inside the Belt Sea (F), estimated in this paper.

Dera also shows how much of the net downward radiation is transmitted to different depths in the Baltic. From his figures we conclude that about 60% of the insolation is absorbed in the uppermost meter and about 90% is absorbed in the uppermost 5 m. The assumption that radiation is absorbed at the sea surface hence appears to be a good one.

We utilize standard parameterizations for the other terms in Eq. (14); e.g., see Krauss (1972) or Gill (1982) for explanations and references. Thus we use

$$Q_b = \epsilon\sigma T_s^4(0.39 - 0.05(e_a)^{1/2})(1 - 0.75nc^2) \quad (15)$$

$$Q_c = \rho_a c_{pa} C_h W(T_s - T_a) \quad (16)$$

$$E = \rho_a C_e W(q_s - q_a). \quad (17)$$

Here ϵ is the emissivity of the sea surface, σ is Stephan's constant, ρ_a the density and c_{pa} the heat capacity of air, T_s is the absolute sea surface temperature (K) (computed by our model), T_a the absolute air temperature, q_a the specific humidity of air, q_s the specific humidity at the air-sea interface (assumed to be the saturation value at T_s), e_a is the vapor pressure of air, n_c is the fraction of sky covered by clouds and W is the wind speed. The heat exchange coefficient used in this paper is

$$10^3 \times C_h = \begin{cases} 0.83 & \text{for stable conditions} \\ 1.1 & \text{for unstable conditions.} \end{cases}$$

These values were proposed by Smith (1980). We use the same exchange coefficients for heat and water vapor, thus we use $C_e = C_h$.

Monthly means of relative air humidity, $h_r = e_a/e_{as}$ where e_{as} is the saturation vapor pressure at T_a, are given by Taesler (1972) for a number of coastal sites. From the relative humidity one may calculate the vapor pressure of water and specific humidity. Figure 8 shows the linear interpolation between the monthly means of relative humidity used in our computations. In order to lessen bias because of offshore winds with significant mean value, we have used the greatest of the two values from Kalmar and Visby (see Fig. 1). Also shown in Fig. 8 is the linear interpolation between the monthly means of n_c at Visby. The n_c curve was computed from the statistics of the number of completely cloudy and completely cloudless days given by Taesler (1972). The partly cloudy days were assumed to have a cloud cover of 50%.

The air temperatures used in this paper, shown in Fig. 7, are obtained by linear interpolation between the monthly means from Visby published by Taesler (1972).

The mean frequency distributions of wind speeds for each month of the year, based upon 20 years of standard observations, from Ölands södra udde and Gotska Sandön (see Fig. 1) were obtained from the Swedish Meteorological and Hydrological Institute. The winds are generally stronger at Ölands södra udde than at Gotska Sandön. Horizontal and monthly mean frequency distributions were formed from the distributions from the two observational sites. These horizontal mean distributions are supposed to be representative for the winds over the Baltic proper. The highest monthly mean wind speeds (~9 m s^{-1}) occur in the period November–January and the lowest (~6.5 m s^{-1}) occur in June–August (see Fig. 8).

b. Model test

The model was implemented on a computer, and the constants used in the model and the applied initial

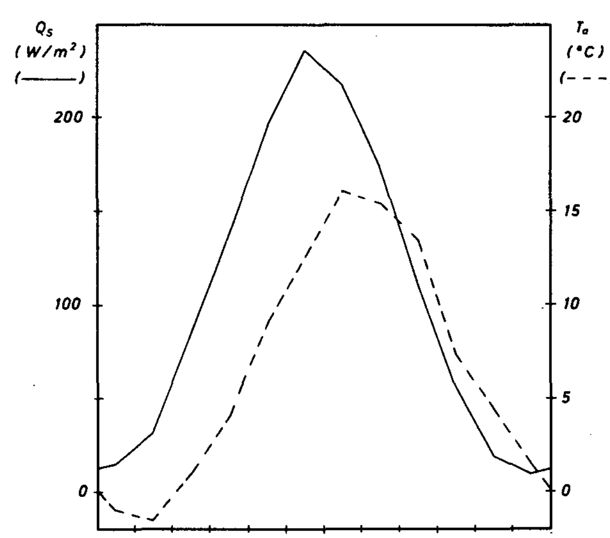

FIG. 7. The annual variation of the net insolation (Q_s, after Dera, 1983) and the anticipated air temperature over the Baltic proper.

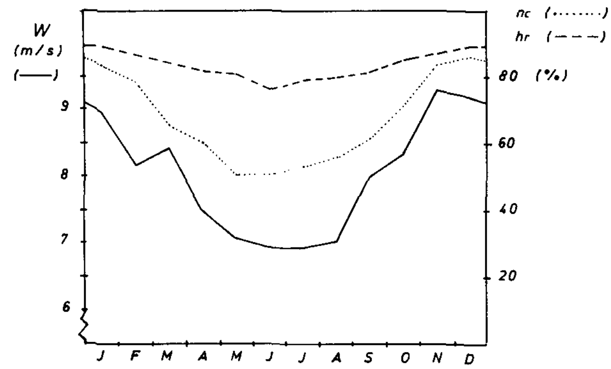

FIG. 8. The annual variation of the fraction of sky covered by cloud (n_c), the relative humidity (h_r) and the wind speed.

conditions are given in Table 1. The vertical separation of grid points in the model is 0.1 m and the time step is 0.5 days. Upward advection of the salinity and temperature fields, due to the dense bottom current, is calculated separately.

Because of the complicated response of the seasonal pycnocline model (and Nature) with respect to wind-effects, one expects that the model would probably give different results from one using a wind that varies only on an annual period as compared to the results from using a wind with the same monthly mean wind speeds but, in addition, containing the day-to-day speed fluctuations. If the day-to-day wind speed fluctuations are incorporated there will be a larger input of mechanical energy into the water. This will affect the magnitude of m_0, which will be determined next. Because of these and other considerations it was decided to use monthly frequency distributions of wind speeds described in the preceding section. For each day of integration the wind speed was picked from the actual monthly frequency distribution using random numbers. Thus two model runs will not give identical results. This enables us also to study natural fluctuations, caused by different wind conditions, around the mean state.

Gill and Turner (1976) have suggested a method for testing seasonal thermocline models. Curves relating sea surface temperature, heat content and potential energy are constructed from observed and computed vertical distributions of temperature. When applied to shallow systems like the Baltic, bathymetric effects should also be taken into account. Therefore contributions to heat content and potential energy from an interval around the depth z are weighted by the factor $A(z)/A_0$, where $A(z)$ is the horizontal surface area at depth z and A_0 is the area at the air–sea interface. In order to avoid influence from the vertical motion of the main halocline (which may have influenced the hydrographic measurements on which our test profiles are based), we have chosen 50 m as the lower limit for integration. For calculations of potential energy we use the absolute difference between actual temperature and temperature of maximum density in the mixed layer (T_m = 2.5°C). Thus there will be a minimum of potential energy when the temperature of the mixed layer passes the temperature of maximum density. There are no climatological mean monthly salinity profiles at our disposal. Therefore we have not included the salinity effect upon density in this test. However, the annual temperature amplitude above the main halocline is O(15°C) while the corresponding salinity amplitude is only a few tenths of one per mille. Thus the simplification in the calculation of potential energy should not be significant.

The Gill and Turner test was performed for different combinations of m_0 and $\mathscr{H}$. The whole parameter space $0 < m_0 \gg 1$, $0 < \mathscr{H} \gg 1$ was inspected. The test gave the best fit for the combination $m_0 = 0.60$ and $\mathscr{H} = 0.20$, as shown in Fig. 9. The computed curves represent the mean of a 15-year run starting from the initial conditions in Table 1. As can be seen from Fig. 9, the curve relating the "observed" surface temperature and heat content is well described by the model. Also, the curve relating the "observed" heat content and potential energy is well represented for a large part of the year. The main reasons for deviations between prototype and model "loops" are probably:

(i) The observations are from the open Baltic proper while the model calculates horizontally integrated values.

(ii) Uncertainties in the modeling of the heat exchange through the sea surface, partly because of uncertainties in atmospheric data which mostly are based upon observations from coastal or island sites.

(iii) We have interpolated linearly between the observed Baltic temperatures (estimated every 10th meter). This may give an unrealistic curvature of some parts of some of the profiles.

(iv) Diffusive processes below the mixed layer in the prototype may, in particular, change the potential energy. However, as discussed later, the diffusivity appears to be low.

TABLE 1. Initial conditions and empirical and physical constants used in the seasonal pycnocline model.

Initial conditions (1 March: t = 0):	
S = 7 (‰), T = 1.5 (°C)	for $0 \leq z \leq 45$ m
S = 7 (‰), T = 2.5 (°C)	for $45 < z \leq 65$ m
S = 10 (‰), T = 4.5 (°C)	for $z > 65$ m
Empirical constants:	
$m_0 = 0.60$	(see determination below)
$\mathscr{H} = 0.20$	(see determination below)
$\epsilon = 0.05$	for $B < 0$ (else $\epsilon = 1$)
$10^3 \times C_d = 1.1$	for $W \leq 6$ m s^{-1} (Smith, 1980)
$10^3 \times C_d = 0.61 + 0.063W$	for $W > 6$ m s^{-1} (Smith, 1980)
Constants not given elsewhere in this paper:	
$g = 9.82$ m s^{-2}	$c_p = 4180$ J kg^{-1} K^{-1}
$f = 1.22 \times 10^{-4}$ s^{-1} at 57°N	$c_{pa} = 1000$ J kg^{-1} K^{-1}
$A_0 = 2.1 \times 10^{11}$ m^2	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
$\rho_a = 1.3$ kg m^{-3}	$L = 2.5 \times 10^6$ J kg^{-1}

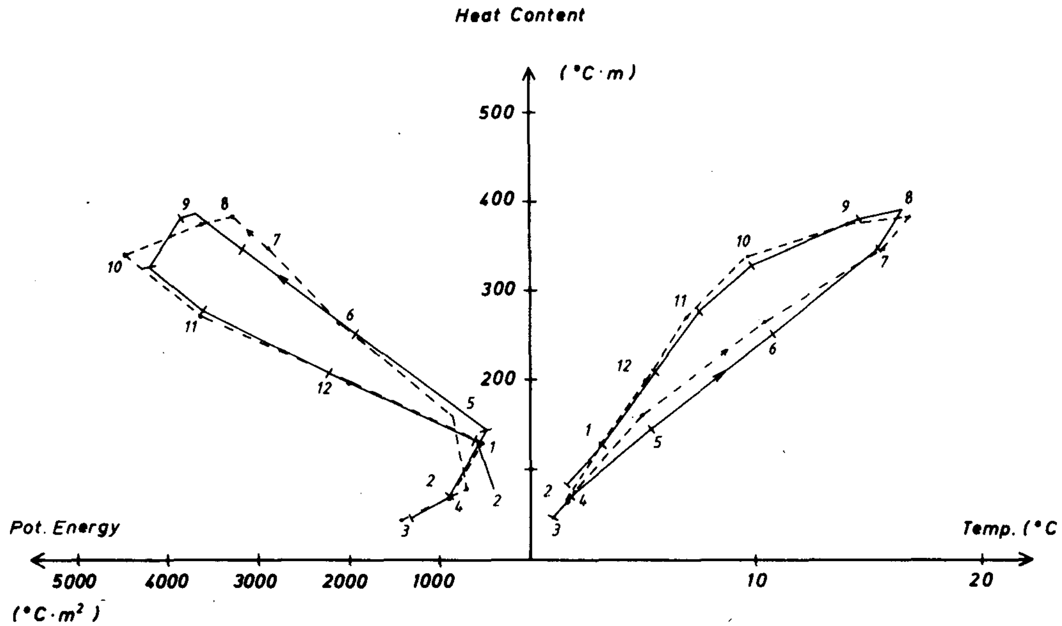

FIG. 9. The Baltic "loops" of potential energy vs heat content (left) and heat content vs surface temperature. The lower integration limit is taken to be 50 m. The "observed" loops are drawn by solid lines. The broken lines show the loops computed from the model for $m_0 = 0.6$ and $\mathscr{H} = 0.20$.

(v) Model defects.

The most realistic prediction of the seasonal pycnocline without invoking the rotational effect (thus for $\mathscr{H} \gg 1$) was obtained for $m_0 = 0.5$. However, it was not possible to reproduce the characteristic, low-temperature minimum found in summer at 40–60 m depth without imposing rotation. Nor was it possible to obtain the vertical stratification typical for midwinter conditions without inclusion of the rotational effect. Existence of the low summer temperature minimum and the midwinter stratification appear to be due to an effective retreat mechanism operating in periods when the surface buoyancy flux B switches from negative to positive values. Rotation of the system obviously provides such an effective retreat mechanism. The sensitivity of the model to changes of m_0 and $\mathscr{H}$ is shown in Fig. 10. The sensitivity of the model to changes of ϵ within the range suggested in literature (0–0.2) was found to be quite small.

Computed and observed temperature profiles for each month of the year are shown in Fig. 11. The computed profiles represent the average obtained from the 15-year run upon which Fig. 9 is also based. As can be seen, the main features of the observed profiles are well captured by the model. Owing to the rotational effect, the thermal stratification starts to develop in late April. The summer stratification is fully developed in July (see Fig. 11). In August, deepening of the mixed layer is quite evident and the deepening process accelerates from September when the sea starts losing heat to the atmosphere. The well mixed layer reaches the main halocline in November. Much of the entrainment of the lower layer into the mixed layer is performed in December. In January and February there is a weak thermal stratification above the main halocline that is due to the rotational effect. However, occasional storms may homogenize the water column down to the main halocline and cause some additional entrainment of saltier water through the main halocline before the spring stratification develops.

Allowing for the effect of vertical advection (caused by the dense bottom current), it can be observed that the temperature profiles below about 40 m depth virtually do not change at all during the period June–September. This indicates that the (diffusive) exchange through the main halocline in the Baltic is quite small in summer. This topic is further discussed in the next section.

The buoyancy supply through the sea surface is strongly dominated by the heating in spring and summer. Thus, during this period, the upper Baltic behaves like a large lake and salinity is dynamically rather unimportant. However, when the mixed layer reaches down to the main halocline, salinity is dynamically much more important than temperature. During this period the estuarine character of the Baltic is evident since saltier water from below is entrained into the mixed layer and replaces the salt lost by the export of surface water through the Kattegat. The computed salinity profiles (see Fig. 12) confirm this behavior. It can be seen that the lowest surface salinities are found in summer when added freshwater is mixed into a shallow layer. The computed annual salinity range at the sea surface, ~0.4‰, is in good agreement with observed results presented by Matthäus (1984). The perennial

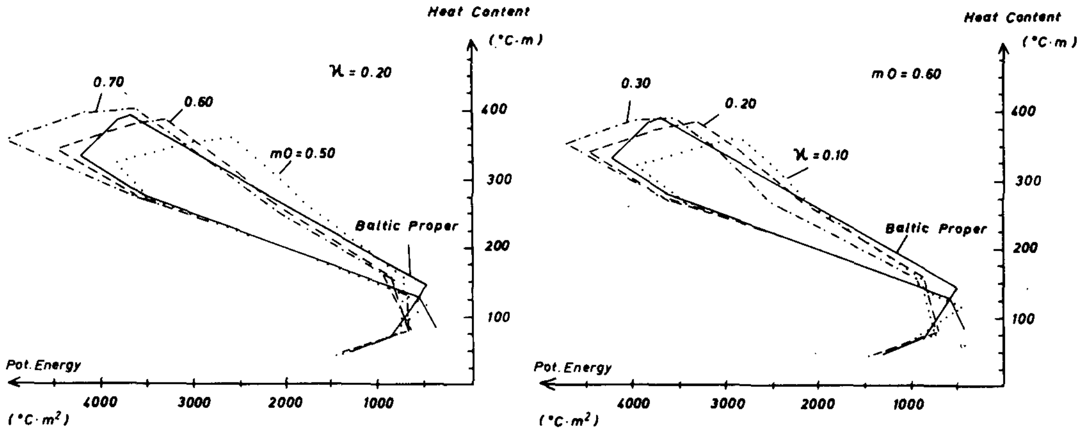

FIG. 10. The sensitivity of the system, revealed by the curves of pot. energy versus heat content, for some combinations of m_0 and $\mathscr{H}$.

halocline is reset by erosion from a shallow summer position to a deeper winter position.

The perennial halocline in the Baltic proper is situated well below the entrance sill. This means that the surface salinity of the Baltic proper is determined essentially by processes in the transition area between the Baltic and the ocean as described by the present author (Stigebrandt, 1983). The net freshwater supply (F) is the only forcing function within the Baltic that may have any influence upon salinity of the layer above the main halocline. Long-term changes of the wind forcing will only result in changes of the depth of the main halocline although there are salinity transients following wind changes. If the winter erosion process determines the depth of the main halocline in the Baltic the present model should predict this depth. In fact, the predicted mean depth of the halocline (~60 m, see Fig. 12) is perfectly within the observed range (see Kullenberg, 1981).

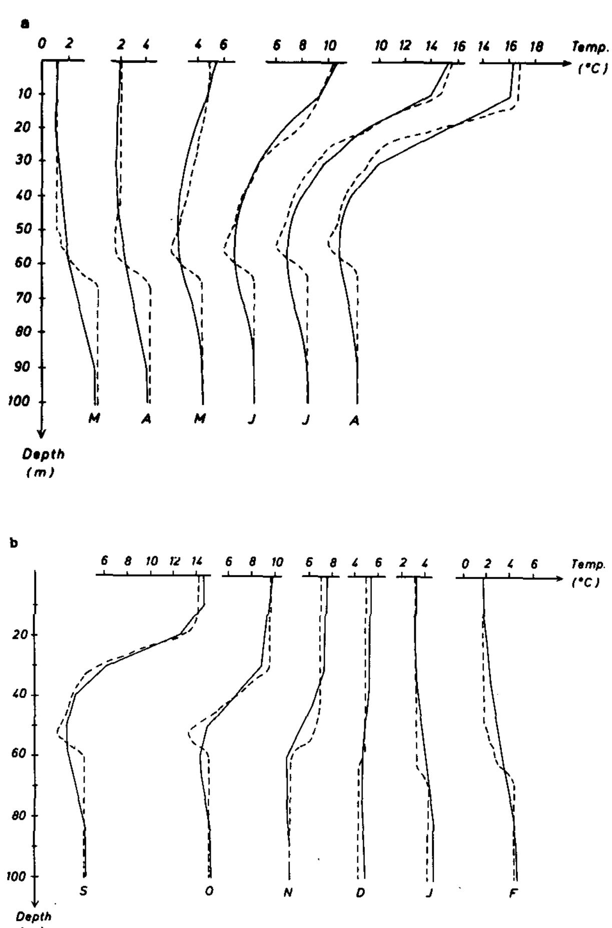

FIG. 11. Computed (dashed) and observed (solid) vertical temperature profiles for each month of the year. The computations were performed with m_0 = 0.6 and $\mathscr{H}$ = 0.20.

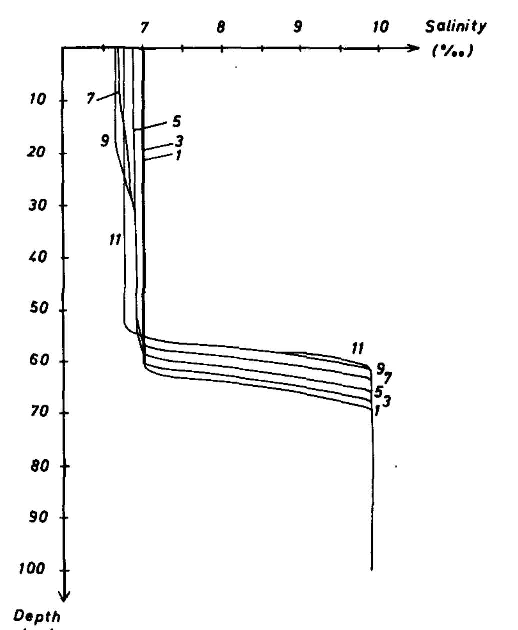

FIG. 12. Computed salinity profiles for every second month of the year.

The annual maximal surface salinity was found to vary typically by ±0.1‰. Monthly mean surface temperatures were found to vary typically by ±0.7°C from year to year while monthly evaporation rates may vary by ±5 mm (less in late winter and spring). These computed variations are all ultimately due to the different wind conditions from run to run. In the real Baltic, even larger variations may be expected since external parameters other than the wind may vary.

4. Discussion of the Baltic proper in the light of the model results

The annual mean diffusive transport of salt across the pycnocline, due to the entrainment process, is about 4×10^{-6} kg m^{-2} s^{-1}. However, the entrainment process is active at the main halocline only for about three months in winter and, hence, the mean diffusive flow in this period is about four times higher than this figure. Kullenberg (1977) estimated from vertical eddy coefficients (determined by him from tracer studies) that the vertical salt transport through the main halocline is about 0.55×10^{-6} kg m^{-2} s^{-1}. However, as also remarked by Kullenberg, the tracer studies were apparently not performed during periods when the mixed layer reached down to the main halocline. The vertical salt flow determined by Kullenberg is probably representative for the vertical exchange in the stratified layer, below the mixed layer. Hence on an annual basis the diffusive vertical flow below the mixed layer appears to contribute only about 10% of the total flow (estimated from the figures given before). This estimate appears to be in harmony with the observation we presented in the preceding section, namely, that the vertical mixing well below the mixed layer is quite small in June–September. Vertical diffusion below the mixed layer may be incorporated in the present model at the cost of greatly extended computational time.

There is now a rather widely spread opinion that much of the diffusive vertical transport below the mixed layer in the Baltic takes place in a coastal zone; see Kullenberg (1981) for a review. However, the observational evidence for this opinion seems to be rather weak. Furthermore, it is very hard to explain how coastal mixing processes may create a very sharp halocline and locate it at a specific depth. The model developed in this paper describes the general evolution of the seasonal pycnocline and the depth of the perennial halocline so well that it is hard to believe that the dominating vertical exchange below the mixed layer in the Baltic proper really is executed in the coastal zone.

If the precipitation over the Baltic proper favored by Ehlin (1981) and the evaporation calculated by our model, shown in Fig. 6, are reasonably correct, there is an annual excess of precipitation over evaporation amounting to about 0.1 m (corresponding to a mean freshwater supply of 700 m^3 s^{-1}). The excess of precipitation over evaporation may be expected to be still greater over the rest of the Baltic Sea (the eastern and northern parts). However, the surface area of this residual area is smaller than that of the Baltic proper. If we assume that the excess of precipitation over evaporation for the residual area contributes an equally large annual mean flow as the excess for the Baltic proper, we obtain about 15 000 m^3 s^{-1} for the mean freshwater supply to the Baltic inside the Belt Sea. If the annual variations of excess precipitation over evaporation are similar in the Baltic proper and the residual areas, we arrive at the annual variation of the total freshwater supply to the Baltic inside the Belt Sea (F) shown in Fig. 6. Strictly, the excess of precipitation over evaporation for the residual Baltic should be taken into account in our model computations. However, buoyancy fluxes in the Baltic proper are dominated by the heat exchange through the sea surface and an increase of the mean freshwater supply by 5% will not lead to any important change.

In Fig. 13 we present the monthly mean heat budget for the Baltic proper as it appears from our computations. Heat supply from rivers, precipitation and geothermal heating are contributions (probably quite small) not considered here. Nor do we present the advective exchange with the Belt Sea/Kattegat which, on an annual basis and integrated over the whole system, appears to be of the order of a few percent of the total budget. A comparison with earlier heat budget esti-

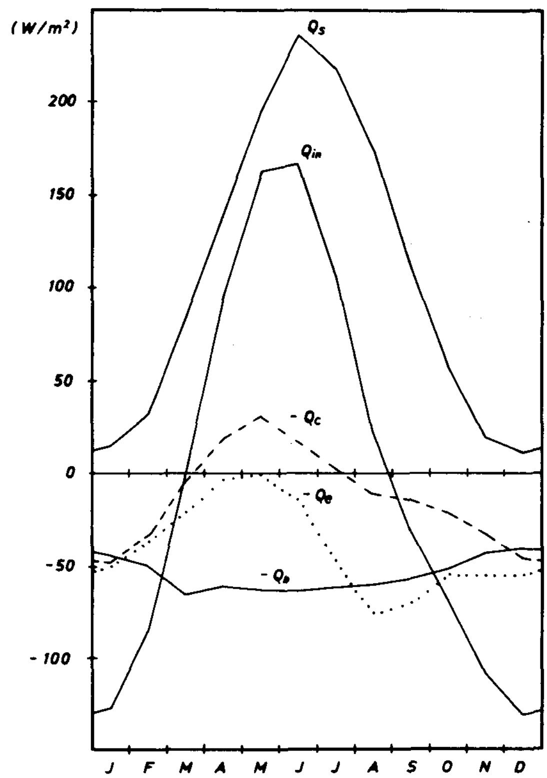

FIG. 13. Monthly means of the different terms in the heat budget for the Baltic proper [Eq. (14)].

mates (see Kullenberg, 1981) shows that the heat loss by evaporation probably was overestimated in some of these.

5. Conclusions

The seasonal pycnocline model presented here appears to describe the stratification in the upper Baltic quite well for the whole annual cycle. In addition, the model correctly predicts the depth of the perennial halocline. The model dynamics have very clear similarities to those of earlier integral models presented in the literature. The nonrotating version of the present model is dynamically roughly similar to the improved Krauss-Turner type of model presented by Gill and Turner (1976). In the terminology of the present paper, that model is characterized by two different length scales determining the thickness of the well mixed surface layer. In the present model, a third length scale, due to the rotation of the system, has also been introduced. From the investigation in this paper it appears that rotation is of crucial importance during early stages of the retreat of a deep pycnocline. In the model, rotation of the system determines the thickness of the upper layer during situations with a deep pycnocline and a small positive buoyancy flux through the sea surface (i.e., when the Monin-Obukhov length and the depth to the upper pycnocline both are greater than the Ekman length). In the Baltic this occurs in midwinter when there is cooling at temperatures below the temperature of maximum density (T_m) and in early spring when there is heating at temperatures above T_m. (Since there is also an addition of freshwater buoyancy to the system this description is only approximative.) Existence of the low summer temperature minimum at 40–60 m depth and the midwinter stratification in the Baltic proper are probably the best evidence for the crucial importance of the rotation.

The fluctuation of the wind speed from day to day is of importance for the actual development of the seasonal pycnocline. The values of the empirical constants m_0 and $\mathscr{H}$ were determined taking these wind speed variations into account. However, there are other variations in the forcing functions that are not considered in this paper. These are, for instance, the diurnal and day-to-day variations in the solar and terrestrial radiation and air temperature. If these variations were also considered it is quite likely that a determination of m_0 and $\mathscr{H}$ would result in slightly different values. Hence, when comparing values of empirical constants given by different authors it is important to remember that the actual value of a specific constant is dependent upon the underlying assumptions and averaging processes utilized in the actual analysis (model). The m_0 value estimated in this paper is lower than most earlier estimates. This may partly be due to the fact that the day-to-day wind variations are also included, which means that the "mean cube" of the wind speed is larger for the present case than for the case where only the monthly mean wind speeds are used. However, our m_0 value appears to be in harmony with the results of Davis *et al.* (1981); see also deSzoeke and Richman (1984) and Gaspar (1984). Furthermore, from an M_2 tidal current profile Mofjeld and Lavelle (1984) determined a coefficient analogous to our $\mathscr{H}$ to be about 0.2. These results indicate that the values of m_0 and $\mathscr{H}$ determined in the present paper may be used with confidence. Finally, it may be mentioned that the present model has been applied to the case described by Omstedt *et al.* (1983). The present model gave results almost identical to those produced by the "continuous" (closure) model utilized by these authors. A detailed comparison between these two models is under preparation.

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