Comments on "Wind Direction and Equilibrium Mixed-Layer Depth: General Theory"

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In a recent paper, Garwood et al. (1985) argued that the equilibrium mixed-layer depth in the presence of wind stirring and a stabilizing buoyancy flux need not be given by the length scale $L = 2u_*^3/B_0$, which is similar to the Monin–Obukhov length scale introduced by Kitaigorodskii (1960). (The notation is the same as in Garwood et al.) The arguments were that $|\delta V|^2/4\delta B$ exceeds L and that the mean-flow instability can cause mixing to a greater depth. However, it should be noted that $|\delta V|^2/4\delta B > L$ is a necessary condition for the existence of turbulence in the mixed layer and it does not give information on mixing beyond L. Entrainment at the density interface is controlled by the nature of turbulence and instabilities at the entrainment interface, and the existence of turbulent mixing in this region is determined by $\Delta b\delta_i/(\Delta V)^2 \leqslant 1/4$, where Δb and ΔV are the buoyancy and velocity jumps across the density interfacial layer of thickness δ_i (or pycnocline) formed at the entrainment interface. Both oceanic and laboratory studies indicate that $\delta_i \approx 0.25h$ (Price, 1979; Deardorff et al., 1980; Fernando and Long, 1985), and hence, for marginal stability $\mathrm{Ri} = \Delta bh/(\Delta V)^2 \approx 1$, which is the same as the criterion used by Pollard et al. (1973).

It is interesting to investigate the difference between the cases in which the stratified turbulent mixing occurs in the presence and in the absence of a surface buoyancy flux. The difference arises due to the formation of a dynamically important buoyancy gradient in the turbulent layer when there is an imposed buoyancy flux. This buoyancy gradient interacts with the velocity field to reduce the mixing rate at the pycnocline significantly. We may write $B_0 \sim bw$, where b and w are the rms buoyancy and velocity fluctuations in the turbulent layer, and hence, the buoyancy gradient becomes $db/dz \sim b/l \sim B_0/wl$, where l is the integral length scale of mixed-layer turbulence. In turn, the limiting value of l is determined by a balance between the vertical kinetic energy and the potential energy of the mixed-layer eddies, viz.,

$$\frac{db}{dz}l^2 \sim w^2 \sim \frac{B_0}{wl}l^2 \qquad (1)$$

or

$$l \sim \frac{w^3}{B_0}. \qquad (2)$$

Since the dissipation within the mixed layer can be parameterized as $\epsilon \sim w^3/l$, we may also write $l \sim (\epsilon/N^3)^{1/2}$, where $N^2 = db/dz$ is the stability frequency corresponding to the buoyancy gradient in the turbulent layer. Note that l is proportional to the Ozmidov (1965) length scale and that the proper velocity scale for parameterizing the limiting mixed-layer depth is w rather than u_*. This fact is supported by the laboratory experiments of Kantha and Long (1980) and Hopfinger and Linden (1982), who found that the mixed layer, under the action of a stabilizing buoyancy flux and mechanical stirring, assumes a depth given by (2). Hence, one of the key questions remaining to be answered is whether $w \propto (G - D)^{1/3}$.

Another aspect of the problem of mixed-layer turbulence is the influence of rotation. According to Garwood et al. (1985), one significant consequence of the presence of rotation is the interaction between τ_x and Ω_y, which results in an enhancement of intercomponent energy transfer. The question is whether such an interaction is sufficient to increase the mixed-layer depth substantially. The relative importance of the intercomponent energy-transfer terms due to the pressure–strain rate correlation and the stress–rotation interaction in (2.1)–(2.3) of Garwood et al. (1985) can be evaluated as $\Pi \sim \langle\overline{u_i'u_i'}\rangle^{3/2}$ and $I = \Omega_y h\tau_x/\rho \sim \Omega_y h\langle\overline{u_i'u_i'}\rangle$, so that the ratio of their relative contributions is $\langle\overline{u_i'u_i'}\rangle^{1/2}/\Omega_y h = \mathrm{Ro} = \Pi/I$, where Ro is the Rossby number. However, recent laboratory experiments (Dickinson and Long, 1983; Hopfinger et al., 1982; Long and Boyer, 1986) suggest that the turbulent length scales, and so the mixed-layer depth, of a rotating fluid are limited by $l \sim h \sim \langle\overline{u_i'u_i'}\rangle^{1/2}/\Omega_y$ or, equivalently, $\mathrm{Ro} \sim 1$. This means that during the mixed-layer deepening $I \ll \Pi$, and when $I \sim \Pi$, the mixed-layer depth can be determined by the limiting length scale of rotating fluid turbulence, $\langle\overline{u_i'u_i'}\rangle^{1/2}/\Omega_y$, rather than by its stratified fluid counterpart, $w/(db/dz)^{1/2}$. The time scales for intercomponent energy transfer due to the pressure–

strain rate correlation and the stress–rotation interaction can be estimated as $t_\pi \sim l\langle\overline{u_i'u_i'}\rangle^{1/2}$ and $t_s \sim \Omega_y^{-1}$, or $t_\pi/t_s \sim \mathrm{Ro}^{-1}$, which indicates that during the mixed region growth $t_\pi \ll t_s$. Since the time scale for the mixed-layer deepening process can be written in terms of the entrainment velocity u_e as h/u_e and since $u_e/\langle\overline{u_i'u_i'}\rangle^{1/2} \sim \mathrm{Ri}^{-n}$ ($n \geq 1$) (Phillips, 1977), we obtain $h/u_e \sim h \times \mathrm{Ri}^n/\langle\overline{u_i'u_i'}\rangle^{1/2} \gg h/\langle\overline{u_i'u_i'}\rangle^{1/2}$. Accordingly, during the mixed-region growth, the stress–rotation interaction appears to be a weaker and slower mechanism for intercomponent energy transfer than the pressure–strain rate interaction, and the latter alone seems to be sufficient to convert turbulent energy from the u-component to the w-component at a rate faster than the rate of the consumption of w-component energy during turbulent mixing. On the basis of the above arguments, one may wonder whether the stress–rotation interaction can indeed play a significant role in the mixed-layer deepening process.

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