Generation of Annual-Period Rossby Waves in the South Atlantic Ocean by the Wind Stress Curl

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ABSTRACT

The properties of first-mode annual-period baroclinic Rossby waves generated by the observed wind stress curl in a numerical model of the South Atlantic and Southwest Indian oceans are presented. The forcing wind field for the area 15°-51°S, 45°W-41°E was obtained from a harmonic analysis at the annual period of the monthly mean wind stress curl values derived from Hellerman and Rosenstein's data, and was used to drive a linear, reduced-gravity model of the South Atlantic and southwest Indian oceans bounded by 15° and 51°S, 46°W and 50°E.

In the South Atlantic Ocean, the response consists of long Rossby waves, which generally propagate their phase southwestward across the ocean and which exhibit refraction of wave energy towards the equator. Short Rossby waves with eastward energy propagation are generated in the small area of the Indian Ocean included in the model domain. Medium to short waves generated to the southeast of Africa reflect their energy off this landmass into the Indian Ocean.

Slowness curve theory and wavenumber computations along wave rays in the South Atlantic are applied to match the model wave trains with probable sources. The most efficient wave generators are found to be the wind stress curl maxima 1) off the Namibian coast near 25°S, 10°E; 2) near the Agulhas Plateau at 38°S, 25°E; and 3) in the South Atlantic Ocean interior near 38°S, 10°W.

1. Introduction

The generation of baroclinic Rossby waves at the eastern boundary of a midlatitude ocean by a seasonally varying wind stress curl has been the subject of several recent studies (White and Saur, 1981; Krauss and Wuebbler, 1982; Cummins et al., 1986). In contrast to the Northern Hemisphere focus of these papers, this is the first study to consider baroclinic Rossby wave generation in the South Atlantic Ocean. The approach adopted here is to apply the numerical model of Cummins et al. (1986) to the South Atlantic—southwest Indian Ocean region.

Although the South East Atlantic and the Benguela Current region have been of interest to oceanographers in the past because of the pronounced seasonal upwelling that occurs there, no work to date has been done on the possibility of Rossby wave generation in this area. Motivation for considering the central South Atlantic as an area for Rossby wave propagation follows from the fact that, like the central North Pacific and North Atlantic Oceans, it is a relatively quiet ocean in terms of eddy activity and mesoscale variability (Cheney et al., 1983). In addition, there exists a pronounced signal in the seasonal wind-stress curl that is necessary for the generation of annual period Rossby waves. However, south of 40°S in the South Atlantic as well as farther north toward the southern coast of South Africa, the ocean is dynamically much more active. Numerous studies report eddies, meanders and frontal activity in these areas (e.g., see Duncan, 1965; van Foreest et al., 1984; Lutjeharms and Valentine, 1984; Shannon, 1985). Apart from these regions however, it can be expected that the central South Atlantic will be characterized by similar quasi-geostrophic dynamic balances as occur in the central North Pacific (Thomson, 1986). Since annual Rossby waves have been observed in this ocean (Kang and Magaard, 1980; Price and Magaard, 1983), it is possible that these waves may also exist in the South Atlantic Ocean.

The model of Cummins et al. (1986) makes use of

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the two-dimensional (dispersive) Rossby wave equation written in terms of spherical polar coordinates. This allows both refractive and dispersive effects to be treated. It also includes a realistic representation of the boundary geometry which in this study is the 1000 m isobath off the coast of Southern Africa.

Use of the observed rather than an idealized wind forcing represents a significant step forward from earlier analytic models. For example, Krauss and Wuebber (1982) approximated the longshore winds off Western Europe and North Africa by the function $\cos(2\pi y/\lambda)$ where $y$ is the latitude and $\lambda$, the wavelength, is of the order of 6000 km. Another feature of our model is the solid boundary on all sides of the ocean with appropriate damping layers to prevent reflection of outward propagating wave energy back into the domain.

Since the model of Cummins et al. (1986) is linear, it obviously has limited applicability to the southwest Indian Ocean which is dominated by the Agulhas Current, an intense western boundary current. Thus, we wish to emphasize here that the Rossby wave pattern obtained in this ocean is mostly of academic interest, and has been included for the sake of completeness rather than as an acceptable model of reality.

Section 2 gives a brief description of the governing equation for the model together with the underlying assumptions and methods of solution. A discussion of the wind stress curl forcing for the model is presented in section 3. The model results are described in section 4. A detailed wave analysis of the results is given in section 5. The conclusions arising from the study are presented in section 6.

2. Model equations

The Rossby wave vorticity equation used in the model is the same as that employed by Cummins et al. (1986); it takes the form

$$\left( \frac{\partial}{\partial t} + u \right) \left( \nabla^2 - R^{-2} \right) h - \frac{\beta}{a \cos \theta} \frac{\partial h}{\partial \lambda} = -\frac{1}{\rho_0 f R^2} \nabla^2 \tau, \quad (2.1)$$

where $\lambda$ and $\theta$ are the zonal and meridional spherical coordinates, $h$ is the interfacial displacement (positive downwards), $\tau$ is the wind stress vector, $f = 2\Omega \sin \theta$ is the Coriolis parameter, $\beta = 2\Omega \cos \theta / a$ is the planetary vorticity gradient, $a$ is the earth’s radius, $\rho_0$ is a Rayleigh friction coefficient and $\rho_0$ is the density of the upper layer of the ocean. By definition, $R(\theta) = (g' H_0)^{1/2}/f$ is the internal Rossby radius where $g'$ is the reduced gravity and $H_0$ is the unperturbed depth of the upper layer. The operator $\nabla^2$ is the horizontal Laplacian in spherical coordinates, viz.,

$$\nabla^2 = \frac{1}{a^2 \cos \theta} \left[ \frac{1}{\cos \theta} \frac{\partial^2}{\partial \lambda^2} + \frac{\partial}{\partial \theta} \left( \cos \theta \frac{\partial}{\partial \theta} \right) \right].$$

Since (2.1) is the dispersive form of the Rossby wave equation and $f$, $\beta$ and $R$ are allowed to vary with latitude, refractive effects can occur in the model. Also, because (2.1) is linear, the response of the model to forcing at any given frequency can be described solely in terms of that frequency. Hence the wind stress curl forcing used in the model takes the form of the harmonic

$$\nabla^2 \tau = \mathcal{A}(\lambda, \theta) \cos(\omega t + \phi(\lambda, \theta)), \quad (2.2)$$

where $\mathcal{A}(\lambda, \theta)$ and $\phi(\lambda, \theta)$ are the amplitude and phase respectively; these have been computed by the harmonic analysis described in section 3.

Based on the experience gained from running the model for the North Pacific region, boundary conditions have been chosen which prevent the wave energy from being reflected back into the model domain, thus leading to unphysical results. As described in Cummins et al. (1986), this involves treating the three open ocean boundaries as well as the land as solid walls with no normal flow, i.e., $h = 0$ there. By imposing damping layers at all open ocean boundaries, except that along 51°S, reflection of wave energy is avoided. The reason for not putting any damping on the southern boundary is that it lies poleward of the critical latitude for annual period Rossby waves (47.2°S for our choice of parameters), and so any wave energy approaching this latitude will be refracted back into the domain. On the other ocean boundaries, the damping attenuates incoming Rossby wave energy via the Rayleigh friction coefficient which increases from zero in the open ocean to a maximum at the solid wall. Details of how the appropriate values of the coefficient are arrived at can be found in appendix A of Cummins et al. (1986).

Figure 1 is the finite-difference representation of the South Atlantic and South Indian oceans used as the model domain. The African coastline (i.e., the 1000 m isobath) is resolved by the model grid to within one degree of latitude and longitude. Superimposed on the diagram as hatched regions are the damping layers described above. For convenience, the small area of the South American continent which impinges on the western damping layer is ignored in the model.

The values of the parameters used in the model are as follows:

- $g' = 1.5 \times 10^{-2}$ m s$^{-2}$
- $H_0 = 500$ m
- $\rho_0 = 1025$ kg m$^{-3}$
- $\omega = (2\pi/1$ year)$\text{rad s}^{-1}$
- $\Omega = 7.29 \times 10^{-5}$ rad s$^{-1}$
- $a = 6371$ km.

Note that the above values for $g'$ and $H_0$ are identical to those used by Cummins et al. (1986) and Willmott (1985) for the North Pacific. The first-mode Rossby radius of deformation resulting from these values of $g'$ and $H_0$ is the best fit to the zonal averages of the observed radii reported by Emery et al. (1984). We assume
that the given values of $g'$ and $H_0$, known to be appropriate for the North Pacific, are also reasonable for the South Atlantic and South Indian oceans. Evidence supporting this assumption is given in Levitus (1982) who shows that contoured values of the mixed layer depth and Brunt-Väisälä frequency for the North Pacific and South Atlantic are similar. Also, maps of Fuglister (1960) show that the 10°C isotherm and 35‰ salinity surface are at 500 m for the central South Atlantic between 16° and 32°S, and that below this depth, the isotherms and isopleths of salinity show a marked increase in spacing.

From the definitions of the Rossby radius and the critical latitude we find that the values for the parameters give a Rossby radius of 29.2 km at 40°S for example, and a critical latitude of 47.2°S.

It should be mentioned that it is not too serious a problem if our choice for $g'$ and $H_0$ is not appropriate since we can easily anticipate the nature of the wave response for different values of these parameters. Since it is the combination of $g' H_0$ only which appears in (2.1) through the Rossby radius $R$, a larger value of $g' H_0$ would, for example, decrease the amplitude of the waves at a given latitude [see right-hand side of (2.1)], and push the critical latitude poleward [see (3.12) in Mysak, 1983].

The model domain shown in Fig. 1 is represented by a finite difference grid of 481 longitude by 181 latitude points corresponding to a resolution of 0.2 degrees. As discussed in section 4, this model resolution satisfies the criteria derived by Wajisowicz (1986) for the accurate representation of freely propagating Rossby waves by a finite difference grid. In order to solve Eq. (2.1) in as numerically efficient a way as possible, it is rewritten as

\[
(\partial_t + u_0)\mathbf{\zeta} + \left(\frac{2\Omega}{\alpha^2}\right) \frac{\partial h}{\partial \lambda} = \frac{\text{curl} \tau}{\rho_0 R^2},
\]

where

\[
\mathbf{\zeta} = (\nabla^2 - R^{-2}) h,
\]

and the definition of $\beta$ have been introduced.

Equation (2.2) is an advection equation for the vorticity field $\mathbf{\zeta}$ which can be solved using a leapfrog differencing scheme for the time variable and a centered differencing scheme for the spatial variable. The Helmholtz-type Eq. (2.3) can be solved by a successive overrelaxation method in the Gauss-Seidel iteration to yield the height anomaly field $h$ from the updated version of the vorticity $\mathbf{\zeta}$ obtained from (2.2). To start the model off from rest a forward-difference time step was used. Each subsequent time step was 10 days whereas the spatial increment was 0.2 degrees for both the zonal and the meridional directions. Because the available observations of the wind forcing in the region of interest have a resolution of 2 degrees, a linear interpolation was used to represent the annual harmonic of the wind stress curl on the model grid. Within the damping layers, the wind stress curl was set to zero.

3. Wind stress curl forcing

Realistic wind-stress curl forcing in the model was obtained from Helfman's and Rosenstein's 1983 monthly wind-stress data. The wind stress has been computed with 2 degree resolution from sea surface observations spanning a 106 year period from 1870–1976 using a bulk aerodynamic model for the drag coefficient, which is a second-order polynomial function of wind speed and stability (Bunker, 1976). The vertical component of the wind stress curl required for
the forcing of the model was computed using the finite difference approximation given in Nelson (1977).

Since it is the low-frequency variation in the wind stress curl that is particularly important for the generation of baroclinic Rossby waves in the ocean, a harmonic analysis of the data at the annual period was performed to yield the amplitude and phase of the forcing.

The harmonic analysis involved a standard least-squares fit (Foreman, 1977) of the monthly wind stress curl estimates to the sinusoidal form of the forcing used in the model. Figures 2 and 3 show the amplitude and phase, respectively. Particularly large wind stress curl amplitudes are evident near the Namibian coast at about the 25°S latitude, in midocean near 37°S, 8°W and in a belt along the bottom of the model domain between 47°S and 51°S. Secondary maxima in the wind forcing are located to the south and east of the African coast at 38°S, 25°E and near 25°S, 35°E. Although the wind forcing maximum near the Namibian coast is expected to be the most important for Rossby wave generation, it is quite possible that localized wind maxima in the midocean could produce Rossby waves (e.g., Haidvogel and Rhines, 1983). On the other hand, the belt of maximum wind forcing in the far south of the domain is not likely to be an important source of Rossby waves because this region is near the 47.2°S critical latitude for annual waves in our model.

The phase of the annual harmonic shows steep north–south gradients in the vicinity of the South Atlantic anticyclone centred near 23°S, 0°E and also along the 35°S to 37°S and 41°S to 43°S latitude bands. This latter zone indicates the boundary between the subtropical and subpolar wind systems.

The localized maxima in the winds near the African coast produce forced responses in the ocean in these areas. Thus in order that the no normal flow condition be satisfied at the boundary, free Rossby waves must be propagated out of the forcing region to the west. This is one way in which Rossby waves are generated by the wind. Waves can also be generated if the energy of the wind field is spread over a wide range of wavelengths which include the resonance wavelengths of the Rossby waves themselves (Krauss and Wuebber, 1982). However, since the wind field in the South Atlantic Ocean is concentrated rather than evenly distributed, this mechanism is not expected to be important.

Although detailed comparison is not possible, earlier studies by van Loon and Rogers (1984b) and Kamstra (1985) point to considerable variation in the wind field in the South Atlantic on a seasonal scale. The annual harmonic of the wave in sea level pressure and zonal geostrophic wind analyzed by van Loon and Rogers (1984b) is strongest at 30°S near and over the three southern continents. This ties in with the observation by Hellerman and Rosenstein (1983) that the maximum wind stress curl shifts south from its July zonal position of 27° to 34°S in January. At latitudes higher than this the annual atmospheric wave becomes second in importance to the semiannual wave; the latter has peaks in amplitude between 45° and 50°S and near 60°S (van Loon and Rogers, 1984a). However, the semiannual harmonic in the wind field in these regions will not be important for baroclinic Rossby wave generation because the critical latitude for waves at this frequency is of the order of 30°S. On the other hand, for our choice of parameters the critical latitude of

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**Fig. 2.** Amplitude of the annual harmonic of the wind stress curl field over the area 15°–51°S, 45°W–41°E. Units are 10^-9 dyn cm^-3.
47.2°S for the annual oceanic Rossby waves is well to the south of the peak in the yearly harmonic of the winds.

A useful synopsis of the wind stress field in the Southern Benguela current region, which is one of the most likely source areas for the Rossby waves, has been provided by Kamstra (1985). During the austral summer, strong heating over the southern Africa subcontinent leads to low pressures there which contrast with the South Atlantic anticyclone situated some 2000 km offshore. This results in pronounced zonal pressure gradients which in turn lead to strong southerly winds along the west coast of southern Africa. In the southern winter, after the circumpolar low-pressure belt and South Atlantic anticyclone have migrated towards the equator, one tends to find westerly to north westerly winds prevailing over the south western coastal area of South Africa (i.e., 32° to 35°S) whereas north of this the southerly winds, though generally weaker than those of summer, still prevail.

4. The model Rossby wave field

To obtain a steady-state Rossby wave response to the wind forcing, the model was integrated until all transients in the solution had died out and only uniform zero-mean sinusoidal oscillations remained at all points in the domain. The steady-state behavior required some 42 years of integration time and the resulting Rossby wave field is plotted in Fig. 4a. The solid lines in the plot are positive contours of the thermocline depth anomaly for Julian day 1 (January 1st) of the model year whereas the dashed lines are the negative displacements (i.e., towards the sea surface) of the thermocline. Figure 4b is a contoured plot of the Rossby wave situation 90 days later, i.e., 1 April in the model year. Plots of the wave field for 1 June and 1 September are identical to Figs. 4a and 4b, respectively, provided that the solid and dashed lines are interchanged.

From these plots it is clear that the Rossby wave field in much of the South Atlantic is dominated by long waves with a maximum amplitude of 24 m. Both the phase and the energy of the waves propagate with a westward component across the ocean. Likely sources for these long waves are the maxima in the wind forcing (i) near the Namibian coast at 25°S, 8°E and 27.5°S, 10°E, (ii) to the south of Cape Agulhas at 38°S, 21°E, and (iii) in midocean at 37.5°S, 9°W. Because of these many wind maxima, the model wave field shown here is somewhat more complex in its structure than that obtained by Cummins et al. (1986) for the Northeast Pacific. Nevertheless, the same basic wave pattern of alternating crests and troughs in thermocline displacement is evident in Figs. 4a–b. Indeed, the wave pattern is coherently maintained for thousands of kilometers in the model domain. A rough estimate for the wavelength of the waves in midocean is 400–500 km. More refined values are given in section 5.

Near the western coast of southern Africa the wave crests appear to be inclined roughly parallel to the land boundary, i.e., in a NNW to SSE direction, whereas further offshore the wave crests slope NW to SE or in a more zonal direction. As will be described later, this is a consequence of the refraction of the wave energy towards the equator as the waves propagate westwards across the ocean.

To the south and east of the land boundary the wave field appears to be even more complex. There are two areas of strong wind forcing that are likely to produce a significant wave response, namely, the maximum to
the south of Cape Agulhas already mentioned and, second, that to the southwest of Madagascar near 28°S, 38°E. As Figs. 4a–b show, the wave field in this region (the southwest Indian Ocean) is far more densely packed than that in the South Atlantic Ocean and consists of short Rossby waves with a generally eastward group velocity (the long Rossby waves in the South Atlantic have a generally westward group velocity).

In addition, the nearby presence of the African landmass as a western boundary means that there is likely to be substantial reflection of these waves back into the southwest Indian Ocean. The sharp cutoff in the thermocline response that is evident to the east of the 40° meridian marks the eastern boundary of the wind forcing domain and the western boundary of the eastern sponge layer (see Fig. 1). The apparent rapid attenuation of the wave field east of 40°E is due to the relative shortness of the waves (see Fig. 8) whose energy is directed into the sponge layer since their group velocity is generally eastward (see Fig. 7c). If there was substantial reflection at the western edge of the sponge layer, the wave field to the east of Africa would also contain long waves (with a generally westward group velocity). However, such long waves are not evident in Figs. 4 and 8.

The action of the equatorial and western damping layers is also evident in the plots of the model response. North of 20°S, where the equatorial damping region
begins, the waves emanating from the Namibian coast are dissipated with increasing efficiency by the sponge layer so that by 15°S there is little or no displacement of the thermocline. Similarly, the western damping region, which extends from 26°W to the western boundary of the model domain at 46°W, also attenuates the westward-propagating waves effectively. There is no wave energy evident beyond 34°W—the disturbances in the extreme south west of the domain are due to local Ekman pumping in response to the strong winds present there. The reason for imposing such a wide (20°) sponge layer here is in order to dissipate the waves completely before any energy can reach the South American continent near 40°W. In this way the question of Rossby wave reflection off this landmass is avoided. However, a drawback of having such a wide sponge layer is that it may prevent the detection of refraction of wave energy propagating in a southwesterly direction towards the 47.2°S critical latitude back into the domain.

In summary, the model response consists of several wave trains whose origins appear to be linked with the observed maxima in the wind field. These connections will be established more rigorously in section 5 and are consistent with the fact that the governing equations of the model are linear. A good qualitative check of this can be seen in Fig. 5 which shows the steady-state response of the model to a small localized wind source. The winds were completely shut off over the whole domain except over the region 22° to 28°S, 8°E to 0° where they were set to a constant amplitude of 10.0 × 10⁻³ dyn cm⁻³ (roughly corresponding to the wind peak offshore of the Namibian coast). After the transients have dispersed the resulting Rossby wave pattern is fairly simple. Several interesting points emerge from this example. First, the waves increase in wavelength as they propagate away from the generation area. Second, there is a bending of the wave crests from a north–south towards a northwest–southwest direction. This is evidence of the refraction of the wave energy, which initially has an approximately westward direction from the wind source, towards the equator (Schopf et al., 1981). Also, the wavelength is shorter near the equator which implies that the waves are slower here.

Before going onto section 5 we must comment on the numerical accuracy of the short Rossby waves present in the model southwest Indian Ocean. Since the resolution of the model grid (0.2 degrees or 22 km approximately) approaches the wavelength of these waves, the numerical solution is unlikely to be able to represent the dispersive nature of the waves with any reasonable accuracy. This very point has recently been discussed by Wajsowicz (1986) who finds that the accuracy of using a finite difference grid for numerically modeling freely propagating Rossby waves depends on two nondimensional resolution parameters. Specifically, the wave resolution of a numerical model can be defined as the ratio of twice the grid spacing to the wavelength of the waves whereas the model grid resolution equals the grid spacing divided by twice the Rossby deformation radius. Both these parameters should be much less than unity for a realistic representation of the Rossby waves. Taking values appropriate to 15° and 51°S (the bounding latitudes of the domain) yields grid resolutions of 21/145 and 14/48; both of which are much less than unity. Now the short waves evident in the model off the south eastern coast of Africa have a wavelength of about 50 km and so the short Rossby wave resolution in our model is roughly 44/50 km. Hence the accuracy of the finite difference representation of these waves is open to doubt. Also, the presence of the strong poleward flowing Agulhas

![Fig. 5. Steady state response of the model to an artificial wind forcing of 10 × 10⁻³ dyn cm⁻³ extending over the region 22°–28°S, 0°–8°E.](image-url)
Current is likely to make nonlinear effects (which have been ignored in our model) important, a point which will be discussed further in section 5. Because of this, it is not meaningful to attempt detailed wavenumber computations for this part of the model ocean—only results based on latitude–time and longitude–time plots, to be described below, are possible. Our main focus therefore, will be on the long waves propagating westwards across the South Atlantic Ocean. We take the 25°E longitude (near the Agulhas Bank) as the boundary between the two regions. Since the waves west of this longitude have wavelengths of hundreds of kilometers they should be effectively resolved by the model grid.

5. Analysis of the Rossby wave field

a. Direction of phase and energy propagation of the model waves

In order to determine the likely sources of the waves together with their refractive and reflective properties, plots of the thermocline anomaly distribution in both latitude–time and longitude–time space (Hovmöller diagrams) were made and the wavenumbers in the relevant parts of the field were computed. We consider the latitude–time plots first as these will indicate whether the generally westward-propagating Rossby waves have any northward or southward wavenumber component. The longitude–time plots are expected to confirm this westward phase propagation as well as indicate the zonal wavelength in different parts of the domain. With the direction of phase propagation for a wave group determined by these plots, it is then possible to deduce the direction of energy propagation, i.e., the group velocity direction. This is particularly important for the purpose of matching wave trains with sources since with such a complex wind field one must take care that the direction of the outwardly propagating energy from the wind source corresponds to the orientation of adjacent wave crests as closely as possible. With this in mind, it is most appropriate to use slowness curve theory to analyze the wave field as it not only enables the direction of wave energy and phase propagation to be determined but also allows wave properties like refraction and reflection to be studied in a heuristic manner [see Mysak, 1983, for an example of its application to the propagation of Rossby waves in the North Pacific]. To apply the slowness curve theory one must assume that once the wave disturbances have left the coastal ocean or any other generation region, they do not encounter any other vorticity sources and can be treated as freely propagating, nearly plane waves.

We now consider the relevant slowness curves for several different areas in the model domain; namely near the Namibian coast at 23°S, 2°E (region 1), in mid-South Atlantic Ocean around 33°S, 20°W and at 40°S, 20°W (region 2), to the southeast of Cape Agulhas near 38°S, 30°E (region 3), and finally east of the South African coast at about 32°S, 35°E (region 4). In each case the slowness curve consists of a circle of the appropriate radius centered on a beta plane at the given latitude.

b. Region 1: Near the Namibian coast

Figure 6a is a latitude–time plot taken along the 2°E meridian. Phase is seen to propagate from the west coast of Southern Africa with a southward component (because of the negative slope of the plot. For example, the dashed line shows that a wave front at 25°S on day 0 has propagated to 26°S by day 300). Because the waves are transverse the phase velocities and hence the wavenumbers are normal to the coast. The corresponding slowness diagram (Fig. 7a) shows the group velocity drawn towards the center of the circle from where the phase wavenumber vector \( \mathbf{k} = (k, l) \) meets the circumference. Since the phase velocity \( c_p = \omega/k/|k|^2 \), it is collinear with \( \mathbf{k} \) but of a different magnitude. From Fig. 7a we thus see that the group velocity and therefore the wave energy are directed slightly to the north of west. Consequently, the source for these waves must be the wind maxima located off the Namibian coast near 25°S, 8°E. Let us now determine how this equatorward transport of wave energy affects the slowness characteristics. Since \( \omega \) and \( k \) are constant along wave rays (curves traced out by the group velocity) on a beta plane (Schopf et al., 1981) whereas \( f \) and \( \lambda \) are slowly varying functions of the meridional displacement \( y \), it follows (Mysak, 1983) that as \( f^2 \) decreases towards the equator \( \lambda^2 \) must increase. This results in the group velocity being refracted continually equatorwards.

In addition, the radius \( r \) of the slowness circle increases as \( f^2 \) decreases towards the equator so that as the group velocity vector turns clockwise in this direction, the phase velocity turns more and more to the southwest (see Fig. 7a) and its magnitude which is inversely proportional to the wavenumber (see definition above), decreases. In other words, as the waves travel towards the equator they slow down.

Further south along the 2°E meridian, Fig. 6a shows the phase also to have a poleward component as far as 40°S. Applying the same slowness curve argument indicates that the wave energy is again transported in northwesterly direction. Possible sources for this energy are the wind maxima near the Agulhas Bank at 38°S, 25°E and the “roaring forties” near 46°S, 10°E. However, the latter source should probably be discounted since it is near the critical latitude in our model (47.2°S). Indeed, as the thermocline anomaly plots (Figs. 4a and 4b) confirm, these poleward propagating waves extend from at least 15°E in a coherent train to 0° before merging with other wave sets. Between 41°S and the critical latitude there is no wave activity, as confirmed by the thermocline anomaly distribution in
Fig. 6a. Latitude–time plot of the thermocline height field for 2°E. The negative slope of the straight dashed line indicates a southward component of phase propagation. Contour interval is 4 m.

Fig. 6b. Latitude–time plot of the thermocline height field for 30°E. Contour interval is 4 m.

Fig. 4a. The closed contours around 50°S in Fig. 6a are evidence of Ekman pumping.

c. Region 2: South Atlantic Ocean between 12° and 30°W

A similar analysis, as above, of the wave field in this area indicates that the source for the waves propagating with southwestward phase velocity between 15° and 36°S along 12°W is probably the midocean wind maximum near 38°S, 8°W, while the zonally oriented waves between 38° and 42°S arise from the Agulhas Bank source.

Further west along 20°W, the northwesterly propagating waves between 30° and 34°S with corresponding southwestward group velocity derived from the slowness curve (Fig. 7b) could arise from the large wind stress curl near 25°S, and 10°–15°W. Since energy transport refracts equatorward, the slowness circle local to the wave disturbance shrinks in size. However, refraction of this energy equatorwards and conservation of the zonal wavenumber mean that the group velocity and phase become increasingly zonal in direction until a point is reached where the circle is at its minimum radius and both vectors are entirely westward in direction. This marks the furthest south that the wave disturbance reaches. Thereafter, continued refraction equatorwards causes the group velocity to rotate clockwise to a northwest direction, the phase becomes southwest and the circle expands again. It is difficult to see whether this process is actually occurring in the thermocline plots because the friction zone which extends to 26°W would actively damp out these waves.

Further south between 36° and 40°S there are poleward-propagating waves. Since these waves must have a northwest direction of group velocity, it follows that they are likely to be generated by the strong zonal winds just south of 40°S and between 0° and 15°W.

West of this at 30°W, southwesterly propagating waves with an associated equatorward direction of group velocity exist between 15° and 31°S. Hence, a likely source for these waves is the strong wind field in midocean near 38°S, 21°W.

d. Region 3: Southeast of Cape Agulhas

Before considering the application of the slowness curve theory to the short waves evident in Figs. 4a and 4b off the southeastern coast of southern Africa it should be emphasized that we have assumed a zero mean flow and have ignored other nonlinear effects in our model. Thus, for example, we have omitted any representation of the intense poleward-flowing Agulhas Current that exists in the southwest Indian Ocean. In the case of a zonal baroclinic mean flow, the slowness curve changes into an approximately elliptical shape (Kang and Magaard, 1980), with the departure from the zero mean flow case (the circle) being largest at very short wavelengths. Thus, a slowness curve analysis similar to the above is not strictly applicable in this region, and the results we obtain concerning the reflection of wave energy from the southeastern coast of Africa have been included mainly for the sake of completeness. With this caveat in mind, we now proceed with an analysis of the model wave field in the South West Indian Ocean.

Figure 6b suggests that the waves between 37°S (roughly where the land ends) and 39°S along 30°E
have a poleward component and are short, as indicated by the narrow spacing between the contours in the figure. The short wavelength is also confirmed by the narrow spaced contours of the longitude–time distribution at this location (Fig. 8a). The slowness circle at 38°S (Fig. 7c) shows that the group velocity is northeastward so it is probable that these waves arise from the strong winds near the Agulhas Bank. At higher latitudes in Fig. 6b the waves become increasingly zonal until by 40°S the phase has a slight northward component corresponding to a group velocity with a slight southward component. This is consistent with the Agulhas Bank wave source.

e. Region 4: East of South Africa

Further east, along 35°E, short northwestward-propagating Rossby waves with southeastward group velocity are found between 29° and 33°S. The energy for these waves could arise from the wind maxima near the Natal/Mozambique coast between 24° and 31°S, 30° to 35°E.

Because of the nearby location of Africa as a western boundary, the possibility of reflection arises. To decide whether this is a realistic possibility we consider the generally WSW–ENE slope of this coast together with the direction of the group velocity of each wave set. Clearly, only the waves near 37°S, 30°E with their northeastward direction of energy propagation could be reflected. Energy from these waves could meet the South African coast near the 33°S latitude and so it is feasible that reflection could occur as illustrated in the second circle of Fig. 7c. This diagram has been derived using standard properties of Rossby waves (see LeBlond and Mysak, 1978). Figure 8b, a longitude–time plot along 31°S, indicates that the Rossby waves propagating their phase towards the southeastern coast of Southern Africa (along the 30°E meridian, say) have short to medium wavelengths. A medium length wave meets the circle at point I resulting in a group velocity with a large northward component. After undergoing reflection the wave is somewhat shorter in wavelength (it meets the circle at point R) and has a group velocity that is eastnortheastward in direction. In addition, the reflected group velocity with equatorward direction will be refracted still further toward the equator so that the local slowness circle expands (see the third circle in Fig. 7c). Hence, like the wave sets both to the north and to the south described earlier, there is a general energy flux northeastward into the Indian Ocean with a possible concentration near Madagascar.

f. Wavenumber vectors of the Rossby wave field

Wavenumber vectors were computed at several points on the ray paths in the model domain along which the wave energy is transported (i.e., along the group velocity vectors). Following Longuet-Higgins (1964) it is appropriate to use ray theory to determine these paths. The rays are obtained simply by drawing curves through coherent patterns of adjacent maxima crests and troughs of the thermocline displacement.
shown in Figs. 4a and 4b. By expressing the thermocline displacement \( h(\lambda, \theta) \) in terms of its annual harmonic,
\[
h(\lambda, \theta) = H(\lambda, \theta) \cos(\omega t + \Phi(\lambda, \theta)), \tag{5.1}
\]
where \( H(\lambda, \theta) \) and \( \Phi(\lambda, \theta) \) are the amplitude and phase, respectively; the wavenumber vectors can be computed from the definition (Cummins et al., 1986)
\[
k = \nabla \Phi. \tag{5.2}
\]

Figure 9 shows the vectors for 5 different rays in the South Atlantic Ocean; namely those originating near 23°S, 11°E; 24°S, 11°E; 32°S, 13°E; 30°S, 21°W and 40°S, 18°E. For future reference we label the rays A to E, respectively. Note that the scale of 1.5 units in Fig. 9 corresponds to a wavenumber magnitude of 1.5 \( \times 10^{-5} \) m\(^{-1}\) (i.e., a wavelength of 420 km).

In general the results are consistent with the slowness circle arguments given above. For example, on rays A and B near 20°S the phase can be seen to become generally more poleward in direction as the wave rays and energy refract equatorward. Similarly, the general southwest propagation of phase all along the 40°S latitude and the change in phase from northwest to southwest between 30°S and 34°S, 20°W to 30°W (corresponding to the midocean wave source) is evident. Since the magnitude of the wavenumber vectors determine the wavelength as \( 2\pi|k|^{-1} \) the following estimates for wave sets in different parts of the model domain can be made.

For rays A and B, wavelengths range from 900 km at 23°S, 5°E to 400–500 km around 18°S, 12°W with associated phase speeds of 2.9 and 1.3 to 1.6 cm s\(^{-1}\). Along ray C the wavelengths and phase speeds are about 560 km and 1.8 cm s\(^{-1}\) near the coast, 1050 km and 3.3 cm s\(^{-1}\) at 30.1°S, 5°W. Between 25°S and 35°S, 15°W and 30°W there is a pronounced wave train arising from the midocean wave source near 38°S, 10°W with wavelengths and phase speeds of 240–480 km and 0.8 to 1.5 cm s\(^{-1}\), respectively. Note that for the sake of clarity, Fig. 9 shows only two wavenumber vectors from this train. These are located at 34°S, 26°W and 35°S, 30°W. Further north, along ray D longer and faster waves (1000 km and 3.2 cm s\(^{-1}\)), which are likely to have arisen from the midocean source near 25°S, 15°W (i.e., to the west of the Namibian coastal source), are evident. Finally, along ray E (39° to 40°S band), the waves increase in length and speed from about 400 km and 1.3 cm s\(^{-1}\) near 20°E to 900 km and 2.9 cm s\(^{-1}\) near 15°W.

Although the zonal component of the wavenumber vector should be conserved as the waves refract (Schopf et al., 1981), Fig. 9 shows that this is not always the
case. Possible reasons explaining this inconsistency include inaccuracies in determining the exact ray path traced out by the wave disturbances and the existence of inhomogeneities at certain points in the wave field due to local Ekman pumping and the generally complex structure of the wave field. Also, variations in the slope of the coastline in several places and the likelihood of ray paths from different sources intersecting each other present difficulties.

6. Conclusions

The model has been able to demonstrate that, similar to the North Pacific, westward propagating Rossby waves are efficiently generated in the South Atlantic Ocean by a seasonally varying wind-stress curl. This wind-stress curl field is of complex structure with several localized maxima which are potential wave sources. From the phase information available in the latitude–time and longitude–time plots at various sections in the model field and by application of the slowness curve theory for free Rossby waves it has been possible to identify the probable wind source for each train of waves.

A summary of these wave sources is as follows:

1) Namibian coastal source near 25° S, 10° E and its westward extension: The strong winds here reach a maximum of at least $30 \times 10^{-9}$ dyn cm$^{-3}$ and generate waves with energy propagating northwest towards the equator between 23° and 15° S but poleward between 25° and 29° S.

2) Agulhas Bank source near 38° S, 20° E: Maximum winds here are $(15-19) \times 10^{-9}$ dyn cm$^{-3}$ and waves are propagated with energy both in the westward and northwestern direction in the South Atlantic between 30° and 38° S, 25° to 2° E. There is also wave energy propagation eastward into the South Indian Ocean between 31° and 46° S.

3) Midocean source near 38° S, 10° W: Peak winds reach $(25-29) \times 10^{-9}$ dyn cm$^{-3}$ with associated equatorward propagation of wave energy between 15° and 36° S but zonal propagation along the 37°–39° S band.

4) Natal/Mozambique coastal source near 23° to 31° S, 35° E: Over this area winds vary over $(15-24) \times 10^{-9}$ dyn cm$^{-3}$. Short Rossby waves with southeastward propagating energy are generated.

Comparing the model wave field presented here to that obtained by using the same basic model in the North Pacific (Cummins et al., 1986) shows that the wave field in the South Atlantic is somewhat more complex in its structure. This is consistent with the greater number of potential wave sources (i.e., more local maxima in the wind field) present in this region as well as the greater variation in the continental boundaries. Examples of the latter include (i) the termination of Africa as a continent in midocean at 35° S, which allows waves generated by the winds near the Agulhas Bank to propagate into the South Atlantic Ocean and which also causes reflection and short-wave propagation off its eastern coast, and (ii) the narrower expanse of ocean between Africa and South America compared to the breadth of the North Pacific Ocean.

Since the dynamics of the model are linear, such processes as interaction of the Rossby waves with eddies shed by the Agulhas Current retroflexion zone and with a mean current have been ignored. These processes could, of course, strongly modify the results presented here and make it difficult to observe the predicted wave field in certain regions.

Killworth (1979) has shown that refraction of wave energy from mean currents can be important if these currents have a substantial zonal component. Thus, the Benguela Current which flows northward along the west coast of southern Africa is unlikely to be of importance in this regard but the westward-flowing South
Equatorial Current which extends as far as 15°S or so may well be. Also, in the Indian Ocean the Agulhas Current would strongly affect the waves near the eastern coast of southern Africa.

Other features which could be included in a more general Rossby wave model are the Circumpolar Current and open ocean topography such as the Mid-Atlantic Ridge and the Walvis Ridge. However, the Circumpolar Current is unlikely to have any significant effect on annual period Rossby waves since it lies well south of 45°S and hence beyond the critical latitude for these waves. The possible effect of topography on annual period Rossby waves in the North Atlantic has recently been described in a study by Anderson and Corry (1985). They show that for relatively short time scales (weeks to months), the response of a two-layer ocean to the winds is essentially barotropic and strongly modified by topography. At the seasonal time scale, the transport in certain restricted regions (e.g., the Florida Straits) is affected by the passage of baroclinic signals over topography. However, since it takes several decades for annual period Rossby waves to travel across the ocean, subthermocline topographic features will not significantly affect such waves in the open ocean.

Finally, the possibility of vertical mixing of wave energy and the inclusion of a continuously stratified ocean would also need to be considered in a more sophisticated model. As far as the latter is concerned, one would expect the waves generated by the wind stress curl in a continuously stratified ocean to be of smaller amplitude than those obtained here in our reduced gravity model where the wind acts as a body force in the upper layer.

Despite the many limitations of the theory, it is conceivable that the model wave field in the central South Atlantic Ocean (20°–40°S, 10°–25°W) may be observed because of its relative simplicity. A first step in this direction would be to look for wavelet patterns in an array of synoptic CTD measurements made in this area.

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