

Comments on "Evanescent Pressure Gradient Response in the Upper Ocean to Subinertial Wind Stress Forcing of Finite Wavelength"

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In a recent paper White and McNally (1987, hereafter WM) developed and discussed a particular solution to low frequency, wind-forced motion in a stratified ocean of constant Coriolis parameter f_o . There is a mixed layer of depth H , where the horizontal velocities U_o , V_o and pressure, P_o , are independent of depth; the turbulent stress and vertical velocity are a linear function of depth. Below is a stratified deep layer of constant Brunt-Väisälä frequency N , where the turbulent stress vanishes. The wind forcing is a sinusoidal stress $\vec{\tau} = [0, \tau^y]$, where τ^y is the north-south component of frequency ω and wavenumber k which travels eastward at constant phase speed, $C_x = \omega/k$. White and McNally present a solution in which they assumed ab initio that pressure gradients in the mixed layer vanish and then force the lower stratified layer with a time-dependent Ekman suction (which is independent of mixed layer pressure gradients). Then they compute a pressure gradient in the lower layer, and assume that this lower layer pressure gradient also acts in the mixed layer, producing a geostrophically balanced flow. Such a method of solution is incorrect, as it is self-contradictory (first there is no pressure gradient and then one is arbitrarily inserted later) and it is incorrect to suppress any pressure gradient in fluid motion ab initio, as pressure is simply a reaction to conservation of vorticity in a fluid that is incompressible.

In this note we present the correct solution to the problem WM set out to solve. Qualitatively, none of the results presented by WM need be changed. Quantitatively, significant differences occur, especially as ω approaches the inertial frequency f_o .

With pressure terms included, the expression for mixed layer velocities are [Eq. (3.1) in WM], in the usual fluid mechanical notation,

$$\rho_o \left(\frac{\partial^2}{\partial t^2} + f_o^2 \right) U_o = \frac{1}{H} \frac{\partial \tau^x}{\partial t} + f_o \frac{\tau^y}{H} - \frac{\partial}{\partial t} \frac{\partial P_o}{\partial x} - f_o \frac{\partial P_o}{\partial y}, \quad (1)$$

$$\rho_o \left(\frac{\partial^2}{\partial t^2} + f_o^2 \right) V_o = \frac{1}{N} \frac{\partial \tau^y}{\partial t} - f_o \frac{\tau^x}{H} - \frac{\partial}{\partial t} \frac{\partial P_o}{\partial y} + f_o \frac{\partial P_o}{\partial x}, \quad (2)$$

$$\rho_o \left(\frac{\partial^2}{\partial t^2} + f_o^2 \right) \frac{W_o}{H} = \frac{\partial}{\partial t} \frac{\nabla \cdot \vec{\tau}}{H} + f_o \frac{\nabla_z \times \vec{\tau}}{H} - \frac{\partial}{\partial t} \nabla_H^2 P_o. \quad (3)$$

In the above, W_o is the vertical velocity at $z = -H$, and in its derivation we have used the condition that $W = 0$, at $z = 0$, the surface.

The vorticity equation in the stratified layer, as expressed in terms of pressure is [Eq. (2.2) in WM],

$$\frac{\partial}{\partial t} \left\{ \left(\frac{\partial^2}{\partial t^2} + f_o^2 \right) \frac{\partial^2 P}{\partial z^2} + N^2 \nabla_H^2 P \right\} = 0, \quad (4)$$

and W is computed from

$$\rho_o W = \frac{-1}{N^2} \frac{\partial^2}{\partial t \partial z} P. \quad (5)$$

The horizontal velocities in the stratified layer have expressions identical to those in the mixed layer, except $\vec{\tau} = 0$. The critical difference between (3) and (3.1) in WM is the expression in the last term on the right-hand side of Eq. (3).

The wind stress is

$$\vec{\tau} = [0, \tau^y \operatorname{Re}(e^{ikx-i\omega t})] \quad (6)$$

and following WM, we are interested in the solution of the form

$$P = \operatorname{Re}(\hat{P}(z)e^{ikx-i\omega t}) \quad (7)$$

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In the stratified layer, the complex valued expression for pressure amplitude that satisfies (4) is:

$$\hat{P} = \hat{P}_o e^{n(z+H)}, \tag{8}$$

with

$$n^2 = k^2 N^2 / (f_o^2 - \omega^2) \text{ for } \omega < f_o. \tag{9}$$

The above expression matches the mixed layer pressure at $z = -H$ and pressure perturbation vanishes as $z \rightarrow -\infty$. Substituting (8) into (5) and evaluating it at $z = -H$, where $\hat{W}(z = -H) = \hat{W}_o$, yields

$$\frac{i\omega n}{N^2} \hat{P}_o = \rho_o \hat{W}_o. \tag{10}$$

The second relationship between \hat{P}_o and \hat{W}_o is obtained from (3), (6) and (7); it is

$$\rho_o (f_o^2 - \omega^2) \frac{\hat{W}_o}{H} = \frac{ikf_o \tau^y}{H} - i\omega k^2 \hat{P}_o. \tag{11}$$

Eliminating \hat{P}_o between (10) and (11) yields the expressions for the mixed layer velocity response

$$\hat{W}_o = \frac{ikf_o \tau^y}{R_o^2}, \tag{12}$$

$$\hat{U}_o = \frac{f_o \tau^y}{HR_o^2}, \tag{13}$$

$$\hat{V}_o = -i \frac{\tau^y (\omega^2 - \Omega_o^2)}{\omega HR_o^2}. \tag{14}$$

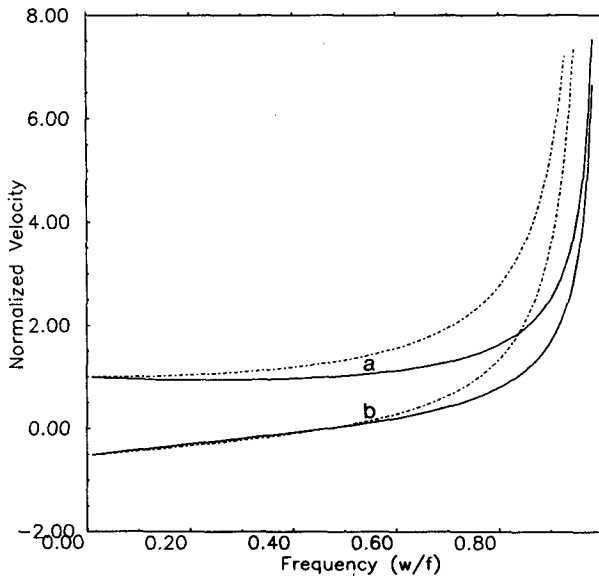


FIG. 1. Normalized velocity response $\rho_o f_o H \hat{U}_o / \tau^y$ (a) and $-i \rho_o f_o H \hat{V}_o / \tau^y$ (b). The dashed curve is the solution presented by WM and the solid curve is the correct solution.

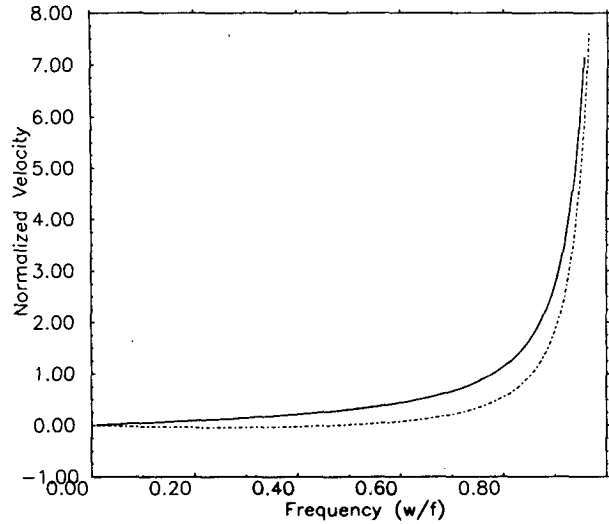


FIG. 2. The difference between the correct solutions for curves (a) in Fig. 1 (solid) and curves (b) in Fig. 1 (dashed).

with

$$\Omega_o^2 = kHN (f_o^2 - \omega^2)^{1/2}, \tag{15}$$

$$R_o^2 = \rho_o (f_o^2 - \omega^2 + \Omega_o^2). \tag{16}$$

These solutions differ from those presented by WM in the expression for R_o^2 . If the last term on the right-hand side of (3) is a priori set equal to zero incorrectly, then $R_o^2 = \rho_o (f_o^2 - \omega^2)$. WM's point was that terms proportional to Ω_o^2 are important in the expressions and the effect of the pressure gradient in the mixed layer as $\omega \rightarrow 0.1 f_o$ results in a downwind component of velocity in Eq. (14) by the term proportional to Ω_o^2 . Figure 1 shows the correct response function for $\rho_o f_o H \hat{U}_o / \tau^y$ and $-i \rho_o f_o H \hat{V}_o / \tau^y$ as a function of ω/f , with $HN/C_x = \pi/6$ (the same value used by WM), together with WM solution. Figure 2 shows the difference between the two sets of curves. Qualitatively, the solutions do not differ because the large inertial frequency in the parameter range used by WM, regardless of the pressure correction. Furthermore, given a W at $z = -H$, the physics in the region below the mixed layer is treated correctly, which also represents a portion of a quantitatively correct solution and contributes to the qualitative agreement. But we think it valuable to publish the hydrodynamically correct response function, and to give the correct physical interpretation.

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