NOTES AND CORRESPONDENCE

Ventilating Beta Plane Lenses

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ABSTRACT

The theory of warm water lenses on beta planes is extended to include heat exchange between the lenses and their environment. The motivation for this study comes from recent observations of Gulf Stream warm core rings, which clearly show that warm rings are strongly modified by diabatic processes. Scaling arguments suggest that the effects of cooling on rings are comparable in magnitude to the effects of beta during much of the winter.

The principal effect of beta on adiabatic lenses is to cause them to drift west. The addition of weak cooling causes the magnitude of the westward drift to decrease at a rate proportional to net heat loss. It is argued that typical cooling rates of warm core Gulf Stream rings can reduce their beta-driven motion by 20% during the course of a winter. While nontrivial, this effect is probably unmeasurable.

The corrections to the dominantly radially symmetric field, which are induced by beta and modified by cooling, are also computed. These fields are time-dependent but evolve in relatively simple ways.

1. Introduction

Warm core rings are formed from finite amplitude poleward meanders of oceanic currents. A defining characteristic of these rings is their anomalously warm core of water, which, as recent observations have shown, can be profoundly affected during winter by interactions with the atmosphere. The clearest observational example of this was obtained during the recent Warm Core Rings experiment (Joyce 1985), during which one ring in particular (i.e., warm ring 82-B) was surveyed several times during its lifetime. A comparison of the temperature structure of 82-B in March (soon after formation) and in April reveals several considerable differences (see Fig. 1). Note, for example, that from March to April the core of 82-B cooled by 2°C and the central ring mixed layer deepened by roughly 100 m. Associated with these structural changes is a significant loss of heat. One would also expect that such a major modification of the ring would have some effect on ring dynamics, although what that effect would be is not currently clear. Indeed, the primary objective of this paper is to explore the dynamical consequences of such heat loss.

Evans et al. (1985) suggest that the March to April cooling of warm ring 82-B occurred in at least two stages. The first stage was associated with a slow, nearly constant decrease of SST in 82-B and lasted from Julian day 59 (i.e., immediately after the formation of 82-B) until Julian day 95. Schmitt and Olson (1985) estimate that this period was characterized by heat fluxes of O(400 W m^{-2}) from the ocean to the atmosphere, the fluxes being driven by the typical cool wintertime atmosphere overlying the Slope Water. The second stage of cooling occurred between Julian days 96 and 97, and was much more intense than the first. Schmitt and Olson (1985) estimate heat fluxes from 82-B to the atmosphere during this period as great as 1500 W m^{-2}. These fluxes were associated with an outbreak of cold continental air and were apparently driven by the appearance of anomalously cool air temperatures over 82-B. (Joyce and Stalcup 1985 report the occurrence of a similar atmospheric event over warm ring 82-I.)

Previous modeling studies of the effects of heat exchange on rings include those performed by Schmitt and Olson (1985) and Dewar (1986a). Both of these studies concentrated primarily on mixed layers in rings and strongly supported the idea that the density restructuring seen in rings during the wintertime (see Fig. 1) could be explained in terms of one-dimensional vertical processes. More recently, Chapman and Nof (1988) and Dewar (1987, 1988) studied the effects of heat loss on dynamically active rings. Chapman and Nof suggested that cooling causes rings to sink, which in turn induces exterior fluid to overwash the ring. Dewar (1987, 1988) examined f-plane ring models subjected to cooling and argued by means of scaling that such models were applicable to periods of "strong" [O(1500 W m^{-2})] cooling.

In this paper, the other end of the cooling parameter range (i.e., the "weak" cooling range) is examined for its effects on rings. When the equations of motion are scaled for rings, the lowest order structure is dominated by f-plane dynamics and the effects of beta appear at

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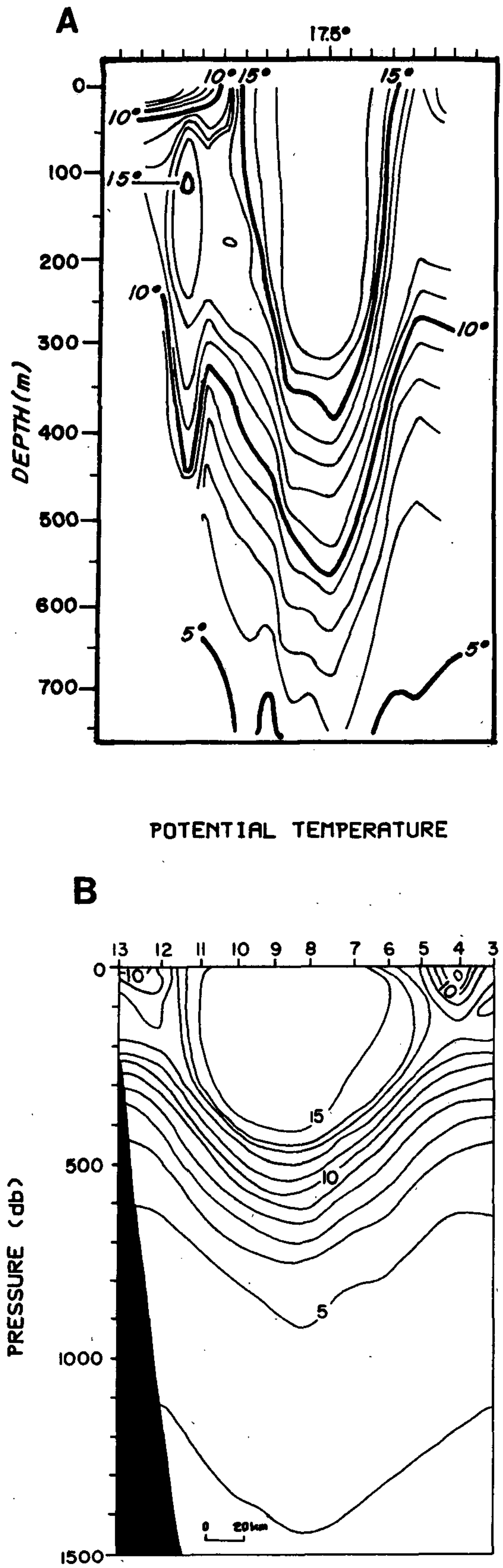

FIG. 1. A comparison of B2-B in (a) early March soon after formation, and (b) 40 days later in April. Note that the bowl of well-mixed water has cooled by 2°C and that the mixed layer has deepened by roughly 100 m, from 325 m to 425 m.

the next order. The term "weak" cooling is here meant to reflect that diabatic effects enter at the same order as beta. Scale analysis suggests that heat fluxes of 400 W m^{-2}, like those that affected 82-B during most of its winter lifetime, are "weak."

The primary dynamic effect of beta on rings is to drive them to the west. When rings are perturbed with weak cooling, it is found that their westward propagation decreases at a rate proportional to net ring heat loss. Ring deceleration occurs because cooling alters the ring density, and hence pressure, fields. The subsequent evolution of the velocity field is such that the net ring circulation decreases. This decelerates the ring, since ring propagation and circulation are related. Further, the formula for ring propagation emerges as a solvability condition on the next order equations. Therefore, a specific example of the beta-induced corrections to the lowest order fields is also computed in this paper.

Finally, it should be noted that while Gulf Stream ring observations provide the primary motivation for this study, there are several other phenomena to which the theory might apply. First, diabatic effects are apparently significant for warm rings throughout the world ocean. Hata (1974), for example, reported 500 m thick mixed layers in a Kuroshio warm core ring, and Cresswell (1981) has documented 350 m thick mixed layers in East Australia current rings. Second, the present analysis more or less applies to lenses imbedded in the ocean thermocline or trapped on the ocean bottom that are exchanging heat with their environment. Both types of lenses have been observed in the past (Ebbesmeyer et al. 1986).

2. Model development

The ring is modeled using a standard reduced-gravity, inviscid, Boussinesq, beta-plane model, the geometry of which is schematically depicted in Fig. 2. The basic governing equations of this system are

$$u_t + uu_x + vu_y - (f_0 + \beta_0 y)v = -g'h_x \tag{1a}$$

$$v_t + uv_x + vv_y + (f_0 + \beta_0 y)u = -g'h_y \tag{1b}$$

$$h_t + (uh)_x + (vh)_y = \begin{cases} -S, & \text{CI} \\ 0, & \text{VC} \end{cases}, \tag{1c}$$

which describe the dynamics of a layer of warm water on top of a deep and resting lower layer. In (1) u and v denote east and north velocities, respectively, x and y denote east and north coordinates, f_0 is the local Coriolis parameter, β_0 is the north–south variation rate

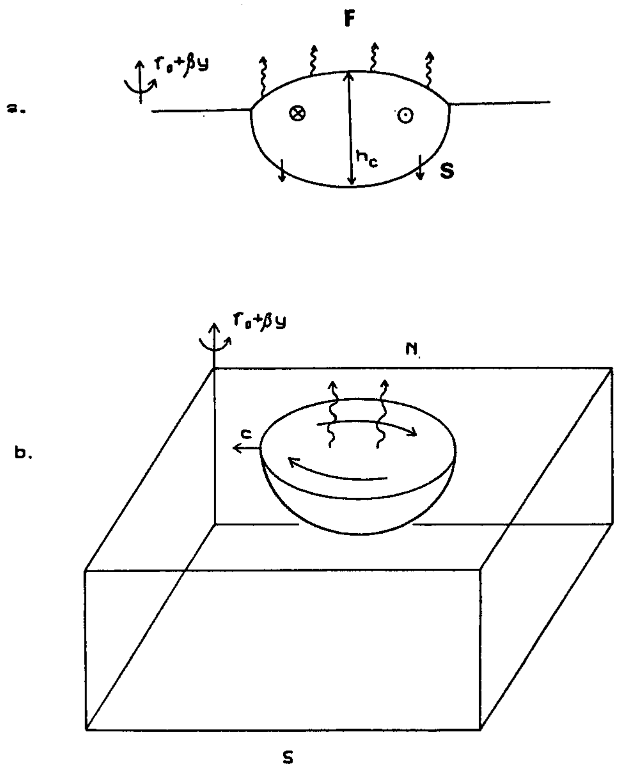

FIG. 2. Model schematic. The lens is modeled using reduced gravity, beta-plane equations. A cross section through the lens is shown in (a). Heat extraction, denoted by F, results in a cross-interface mass flux, denoted by S; h_c denotes the maximum eddy thickness. (b) contains a perspective plot of the lens; c denotes the lens propagation rate.

of f, h is upper-layer thickness, and g' denotes the so-called reduced gravity.

If diabatic effects are to be studied using the model in Fig. 2, it is necessary to include thermodynamics in a manner consistent with the layered framework. At least two cooling parameterizations that meet this criteria have recently been suggested, and both will be studied in this paper.

A number of researchers working on ventilated thermocline theory (Dewar 1986b; Luyten, Stommel and Wunsch 1985; Pedlosky 1986) have included diabatic effects in their layer models by means of a cross-interfacial mass flux. Dewar (1987, 1988) also recently used this parameterization in his study of cooling f-plane rings. In this parameterization, the fluid is allowed to exist only in the two initial density states, and heat extractions from the fluid are balanced entirely by converting an appropriate amount of warm water to cold water. This clearly results in a loss of mass from the upper layer, the effect of which appears in (1c) in the term S. If the flux of heat from the ocean to the atmosphere is denoted by F, S is proportional to $-F/g'$. Positive values of F, and thus negative values of S, correspond to heat loss from the ocean to the atmosphere. Note that such S values induce a "thinning" of the upper layer (i.e., $h_t < 0$) consistent with the required conversion of warm water to cold water. When this parameterization is used, it will be referred to as the cross-interfacial (or CI) parameterization.

A second analytically convenient cooling parameterization has recently been suggested by Chapman and Nof (1988). In their study, upper-layer density was allowed to increase in response to cooling. They restricted the surface heat fluxes acting on their ring, however, to be linearly proportional to h, the upper layer thickness. In this way, the upper-layer density, while time dependent, was uniform. This cooling parameterization results in no cross-interfacial mass flux; thus, the right hand side in (1c) vanishes. The reduced gravity parameter, g', however, becomes time dependent. Clearly, the Chapman and Nof parameterization is consistent with upper-layer potential vorticity conservation. When their parameterization is used, it will be referred to as the VC (or vorticity conserving) parameterization.

Scaling. The Rossby deformation radius of the system is given by $R_d = (g'h_c)^{1/2}/f_0$, where h_c is the scale thickness of the ring (see Fig. 2). Scaling both x and y by R_d and h by h_c in (1a) and (1b) suggests that u be scaled by f_0R_d. Time t is scaled by $(\beta R_d)^{-1}$, in keeping with the idea that the fundamental time dependency (i.e., translation) is introduced to the system by beta. Adopting the CI parameterization, the resulting set of nondimensional equations is

$$\beta u_t + uu_x + vu_y - (1 + \beta y)v = -h_x \qquad (2a)$$

$$\beta v_t + uv_x + vv_y + (1 + \beta y)u = -h_y \qquad (2b)$$

$$\beta h_t + (uh)_x + (vh)y = -\beta S_0 S, \qquad (2c)$$

where $\beta S_0 = S_*/f_0 h_c$ and S_* is a scale estimate of heat flux. Two nondimensional parameters $\beta = \beta_0 R_d/f_0$ and βS_0 appear in (2) and measure respectively the variation of f over the ring and the strength of the cooling. If $h_c = 400$ m and $g' = 1$ cm s^{-2}, $R_d = 20$ km and $\beta = 4 \times 10^{-3} \ll 1$. For a surface heat flux of 400 W m^{-2} (Schmitt and Olson 1985):

$$\beta S_0 = S_*/f_0 h_c = \frac{Fg\alpha}{\rho_0 C_p g'}\frac{1}{f_0 h_c} = 5 \times 10^{-4} \ll 1,$$

where g is gravity, α the coefficient of thermal expansion of seawater, and C_p the heat capacity of seawater; βS_0 is roughly the same magnitude as β. Further, both βS_0 and β are small parameters, suggesting that the lens is governed at lowest order by adiabatic f-plane dynamics. Note that, aside from the factor $\beta S_0 S$, (2) are identical to the lens equations in Flierl (1984).

3. Structure and propagation

Substituting perturbation expansions in powers of β for u, v and h and assuming that $S_0 \approx O(1)$ yields at lowest order

$$u_0u_{0x} + v_0u_{0y} - v_0 = -h_{0x} \tag{3a}$$

$$u_0v_{0x} + v_0v_{0y} - u_0 = -h_{0y} \tag{3b}$$

$$(u_0h_0)_x + (v_0h_0)_y = 0, \tag{3c}$$

which are formally identical to steady state f-plane equations. Following Flierl (1984), we will examine eddies with lowest order radial symmetry and constant potential vorticity. The special case of zero potential vorticity is analytically tractible and will occasionally be used here for the sake of illustration. Numerical solutions of the equations are required for nonzero potential vorticity (see appendix A).

The tangential velocity in a zero potential vorticity eddy is

$$V_0 = -r/2, \tag{4a}$$

where V_0 is the swirl speed. The radial velocity component, U_0, vanishes and thickness h_0 is given by

$$h_0 = h_c(t) - r^2/8, \tag{4b}$$

in which $h_c(t)$ denotes the maximum eddy thickness and is, in this analysis, potentially a function of time. Note that the eddy outcrops (i.e., $h_0 = 0$) at $r_\partial = 2(2h_c)^{1/2}$.

The translation speed of the lens can be determined in a manner similar to that used by Flierl (1984). At $O(\beta)$, (2) become

$$u_{0t} + u_0u_{1x} + u_1u_{0x} + v_0u_{1y} + v_1u_{0y} - v_1 - yv_0 = -h_{1x} \tag{5a}$$

$$v_{0t} + u_0v_{1x} + u_1v_{0x} + v_0v_{1y} + v_1v_{0y} + u_1 + yu_0 = -h_{1y} \tag{5b}$$

$$h_{0t} + (u_0h_1)_x + (u_1h_0)_x + (v_0h_1)_y + (v_1h_0)_y = -S_0S. \tag{5c}$$

Manipulating (5) yields the integral balance

$$c\iint h_0dA + \bar{x}\frac{d}{dt}\iint h_0dA + \iint \psi dA = -\bar{x}S_0\iint SdA - S_0\iint v_0SdA, \tag{6}$$

where:

$$c = \frac{d}{dt}\left[\iint xh_0dA \Big/ \iint h_0dA\right] = \frac{d}{dt}\bar{x}$$

is the eddy zonal center of mass speed and ψ is a streamfunction defined by

$$\psi_x = v_0h_0$$

$$\psi_y = -u_0h_0.$$

Assuming that S is at most a function of r (which seems reasonable), the quantity

$$\iint v_0SdA = 0,$$

since v_0 is an odd function of x.

Integrating (5c) over the eddy yields

$$\frac{d}{dt}\iint h_0dA = -S_0\iint SdA, \tag{7}$$

and thus,

$$c = \frac{-\iint \psi dA}{\iint h_0dA} \tag{8}$$

upon substitution of (7) into (6). The above equation for c is identical to that found by Nof (1981) and Flierl (1984) in their studies of adiabatic lenses. The new aspect here is that the same formula holds in the presence of cooling. The interpretation of (7) is that the swirling flow in an eddy develops a net Coriolis force that is balanced by bulk eddy translation. For a warm lens, the net Coriolis force is directed to the south and is balanced by a westward drift.

Equation (7) states that mass is lost from the lens at a rate proportional to the net heat flux to the atmosphere. This is, of course, a result of the CI cooling parameterization, and we will return to this point shortly. If S were a constant (such would be the case if heat flux were proportional to air–sea temperature difference), then

$$\frac{d}{dt}\left[\iint h_0dA\right] = -S_0S\pi r_{\partial 0}^2, \tag{9}$$

where $r_{\partial 0}$ is the outcrop curve [i.e., $h_0(r_{\partial 0}) = 0$].

Further manipulation of (5) yields

$$\frac{d}{dt}\left[\iint yh_0dA \Big/ \iint h_0dA\right] = \frac{d}{dt}\bar{y} = 0,$$

which states that the meridional eddy propagation rate vanishes. The lack of meridional translation reflects the fact that the analysis began with the reduced gravity equations, thereby assuming negligible lower-layer motion. Wave radiation, which resulted in nonzero meridional translation in both McWilliams and Flierl (1979) and Flierl (1984), is absent from this problem.

In principle, all of the pieces necessary to compute the evolution of a ventilating ring are now available. Consider an initial value problem in which an adiabatic eddy, propagating steadily at c determined by (8), is subjected suddenly to weak cooling. Since the initial outcrop is known, (9) can be used to compute the warm water volume at some later time. Equations (3) can then be solved subject to this new volume as a boundary condition (and also subject to new potential vorticity structure; see appendix A) to give new values for V_0 and h_0. These may be substituted into (8) to determine a new value for c and used to compute a new $r_{\partial 0}$ for (9). This cycle can be repeated, thus filling in the evolution of velocity and thickness, until $\iint h_0\, dA = 0$.

The case of zero potential vorticity is a specific and analytically tractible example of this procedure. Evaluating (8) yields

$$c(t) = -\frac{2}{3} h_c. \tag{10}$$

Evaluating (9) yields

$$h_c(t) = h_{c0} - SS_0 t, \tag{11}$$

where h_{c0} is the maximum eddy thickness prior to cooling. Substituting in (10) yields

$$c(t) = -\frac{2}{3}(h_{c0} - SS_0 t). \tag{12}$$

Note that c is inherently time dependent due to the presence of cooling. Further, by taking a derivative of (12) with respect to time, one obtains a major point of this paper, i.e.,

$$\frac{d}{dt} c(t) = \frac{2}{3} SS_0 > 0. \tag{13}$$

Note that according to (12), $c(t) < 0$, so (13) demonstrates that the ring drift rate decreases in magnitude toward zero as cooling proceeds.

This result, i.e., the deceleration of a ventilating ring, might appear in this analysis to be somewhat dependent on the two assumptions of CI cooling and zero potential vorticity. It is a more general result than either of those assumptions, however, as may be shown by considering an eddy subject to potential vorticity conserving, or VC, cooling.

The lowest order momentum equations in the case of VC cooling (see appendix B for details) are

$$u_0 u_{0x} + v_0 u_{0y} - v_0 = -(1 - pt) h_{0x} \tag{14a}$$

$$u_0 v_{0x} + v_0 v_{0y} + u_0 = -(1 - pt) h_{0y}, \tag{14b}$$

where p is a parameter measuring the cooling rate and is O(1). Solving the radially symmetric form of (14) for the special case of a zero potential vorticity eddy yields the lens propagation equation

$$c = -\frac{2}{3}(1 - pt)^{1/2} h_{c0}. \tag{15}$$

Note that at $t = 0$ (the onset of cooling), c is negative (i.e., westward). Subsequent evolution consists of c decreasing in magnitude toward zero as the square root of time.

The propagation formula appropriate to VC cooling, which was used in the calculation leading to (15), is identical to (8) (see appendix B). For more general and nonzero potential vorticity values, the solutions to (14) must be obtained numerically. These solutions can then be used to numerically evaluate $c(t)$ via (8). The results of such an analysis are presented in Fig. 3 for constant potential vorticity values of 0.25, 0.5 and 0.75. (The allowable range for potential vorticity in this problem is from 0 to 1.) Note the steady decrease in the magnitude of c in all three cases as time progresses. This is in qualitative agreement with the analytical predictions obtained from the zero potential vorticity analysis subject to both CI and VC cooling. This result, i.e.,

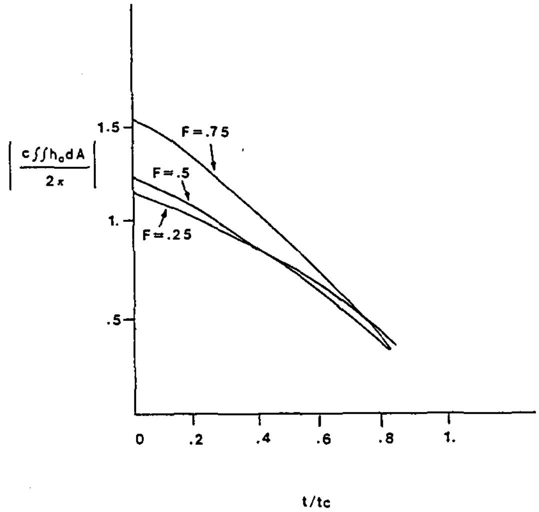

FIG. 3. Propation speed vs time for nonzero potential vorticity. The structure and propagation equations were solved numerically for potential vorticity values of $F = 0.25$, 0.5 and 0.75. VC cooling has been used, and t_c is the time at which the density anomaly would disappear. In all three cases, c decreases in magnitude toward zero as cooling proceeds.

$$\frac{d}{dt} |c(t)| < 0 \tag{16}$$

for ventilating rings, seems therefore to be robust and relatively independent of special assumptions.

4. First-order modifications

In this section, we compute the structure of the small modifications to the O(1) fields, which are governed by (5). In the absence of cooling, these are exactly the O(β) fields computed in Flierl (1984). Our purpose here is to demonstrate how those fields are modified by cooling.

As an explicit example, consider again the zero potential vorticity eddy subject to the CI cooling parameterization. The $c(t)$ is given by (12). Plugging separable solutions of the form

$$U_1 = u'(r)\cos\theta$$

$$V_1 = v'(r)\sin\theta$$

$$h_1 = h'(r)\sin\theta$$

into (5) and cancelling the $-S_0 S$ source term by means of (12) eventually yields a single inhomogenous equation for h:

$$2r\left(\frac{r^2}{8} - h_c\right)h'_{rr} - 2\left(h_c - \frac{3}{8}r^2\right)h'_r + \frac{2}{r}\left(h_c - \frac{3}{8}r^2\right)h' = \frac{10}{3}h_c r^2 - \frac{5}{8}r^4, \quad (17)$$

which possesses the general solution:

$$h' = \frac{A}{r} + Br - \frac{5}{24}r^3. \quad (18)$$

The constant A in (18) must vanish in order for the solution to be well behaved at $r = 0$. On the other hand, there are no obvious boundary conditions to apply to h_1, which will determine B. Flierl (1984) argued that B was associated with ambiguity in the meridional positioning of the ring and chose the B that minimized the distortion in the outcropping curve, r_∂. Killworth (1983) explored the arbitrary choice $B = 0$.

Here we will employ a variant on the choice of Flierl (1984) to determine B. The outcropping curve r_∂ is defined by $h(r_\partial) = 0$ and using the zero potential vorticity solutions is

$$r_\partial = 2\sqrt{2h_c} + \beta\left[4\left(B - \tfrac{5}{3}h_c\right)\sin\theta\right] + O(\beta)^2. \quad (19)$$

We choose $B = \frac{5}{3}h_{c0}$, which insures $r_\partial = r_{\partial 0} + O(\beta^2)$ for $t < 0$. Note, however, that for $t > 0$,

$$r_{\partial 1} = \frac{20}{3}SS_0 t \sin\theta.$$

As cooling proceeds, a positive correction to r_∂ develops to the north, and a negative correction develops to the south. This implies that the outcrop curve of the ring, which basically draws inward as cooling progresses, migrates inward more slowly on its northern side than on its southern side.

The $O(\beta)$ corrections for all the fields are thus

$$h_1 = \left[\frac{5}{3}h_{c0} - \frac{5}{24}r^2\right]r\sin\theta \quad (20a)$$

$$u_1 = \left(-2h_{c0} - \frac{4}{3}SS_0 t + r^2/4\right)\cos\theta \quad (20b)$$

$$v_1 = \left(2h_c - \frac{4}{3}SS_0 t - \frac{5}{12}r^2\right)\sin\theta. \quad (20c)$$

The perturbation fields can be conveniently displayed by computing the $O(\beta)$ correction to the streamfunction, which is obtained by solving

$$-\frac{1}{r}\frac{\partial}{\partial\theta}\psi_1 = U_1 h_0 \quad (21)$$

and is

$$\psi_1 = \left(2h_{c0} - \frac{4}{3}SS_0 t - \frac{r^2}{4}\right)r\left(h_{c0} - SS_0 t - \frac{r^2}{8}\right)\sin\theta. \quad (22)$$

The structure of ψ_1 at $t = 0$ and $t = (2SS_0)^{-1}$ is indicated in Fig. 4 and in both cases consists of a high center north of a low center ($\beta = 0.01$ in these figures). The perturbation velocities induce a clockwise circulation for $y > 0$ and a counterclockwise circulation for $y < 0$. The thickness corrections are indicated in Fig. 5, also at $t = 0$ and $(2SS_0)^{-1}$, and consist of a slight deepening of the thermocline to the north and a slight shallowing to the south.

5. Discussion

In this paper the effects of weak cooling on beta plane warm lenses were computed. The motivation for this study comes from the many observations throughout the world ocean of strong interactions between rings and the atmosphere. The present analysis is not, however, restricted to rings; similar analysis can be applied to cooling lenses imbedded in the thermocline and trapped on the bottom.

Rings on a beta plane propagate to the west due to beta. If these lenses are perturbed with weak cooling, their propagation rate decreases in magnitude toward zero. This tendency has been suggested by examining layered models with various parameterizations of cooling.

The mechanism behind lens deceleration involves the adjustment of the ring. Cooling induces low pressure perturbations at lens center. The ring responds by drawing radially inward and, in turn, loses some of its circulation. Propagation rate is proportional to net circulation; thus, the lens slows down.

Equations (13) and (15) both suggest that the net deceleration of a lens is proportional to its net heat loss. More quantitatively, within a two layer model the heat anomaly of a warm lens is

$$H = \rho_0 C_p V \Delta T,$$

where V is the lens volume, ΔT the temperature differential between the layers, C_p the specific heat of water, and ρ_0 a reference density. From (7) one sees that the net warm water volume change is proportional to the integrated heat loss and, further, using the zero potential vorticity solutions, that

$$\frac{d}{dt}V = 4\pi\frac{d}{dt}h_c{}^2.$$

Thus, the ratio of the net heat loss from the ring to its initial heat anomaly is

$$\frac{\delta H}{H} = \frac{\delta h_c}{h_c},$$

where δh_c denotes the net thickness change at ring center. But, upon using (10), the fractional heat loss can be related to fractional lens deceleration

$$\frac{\delta H}{H} \sim \frac{\delta c}{c}.$$

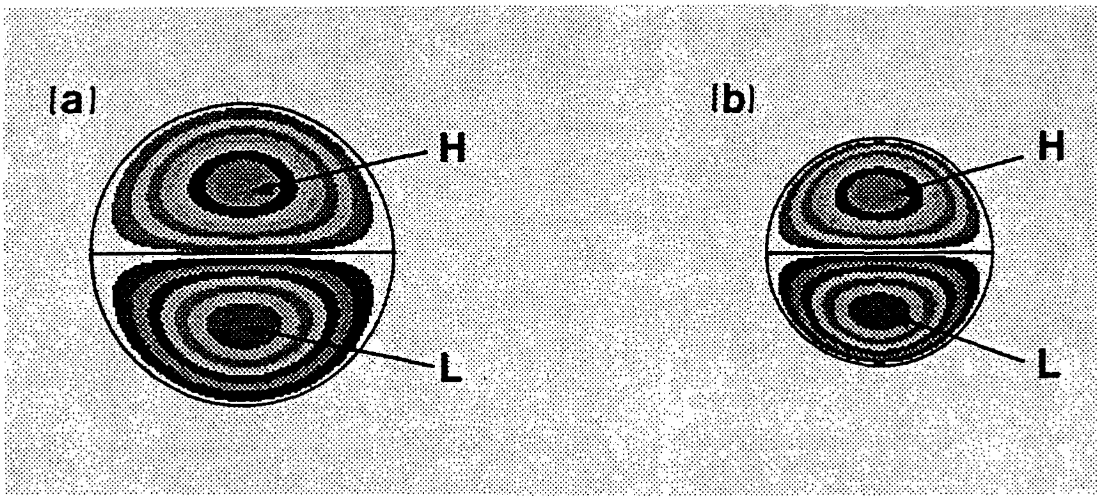

FIG. 4. Perturbation streamfunction at time (a) 0 and (b) $t_c/2$, where t_c is the time at which the lens density anomaly would be completely removed. Note the dipole pair structure, consisting of positive stream function values north of negative streamfunction values. In this plot, $\beta = 0.01$.

Warm ring 82-B was cooled for 40 days at a rate of 400 W m^{-2}. Assuming a deformation radius of 22 km ($g' = 1$ cm s^{-2}, $h_c \approx 500$ m)and using the zero potential vorticity structural solutions yields a net heat loss of

$$\delta H = F\delta tA = 400 \text{ W m}^{-2}\ 40 \times 10^5 \text{ s } 8 \times 10^9 \text{ m}^2 = 1.28 \times 10^{19} \text{ J},$$

while the initial warm water anomaly was roughly

$$H = \rho_0 C_p \Delta T V = 1 \text{ g cm}^{-3}\ 4.2 \text{ J (g °C)}^{-1}\ 5\text{°C}\ 3 \times 10^{12} \text{ cm}^3 = 6.5 \times 10^{19} \text{ J}.$$

A characteristic temperature anomaly for 82-B of 5°C was used in the above estimates.

Thus the scaling from the present analysis suggests that over the course of a winter, a typical Gulf Stream ring might experience a net 20% reduction in its rate of beta-driven propagation. Although this is a reasonably significant effect, it is probably not measurable. A 20% loss of speed for most rings translates to a change of ~1 cm/sec. The best current means of tracking rings is by satellite, and the uncertainties involved in determining propagation rate from such observations are generally of this magnitude (Hooker and Olson 1984).

The primary difficulty associated with the observa-

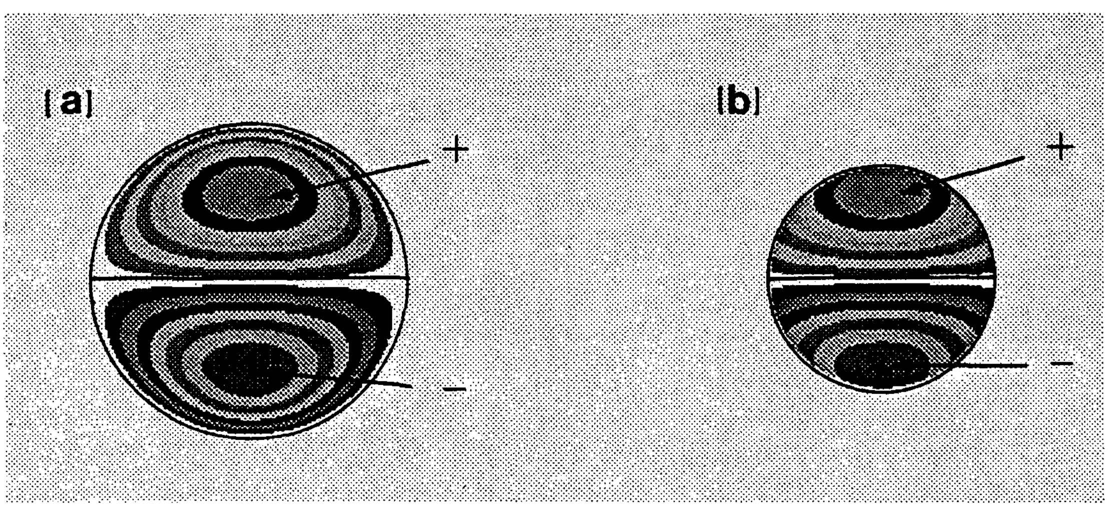

FIG. 5. Perturbation thickness at time (a) 0 and (b) $t_c/2$. The thermocline is deepened to the north and made shallower to the south by the perturbations. In these plots $\beta = 0.01$.

tion of the predicted deceleration in rings is the number of competing influences in the Gulf Stream/Slope Water region. On the other hand, it appears that cooling can have marked effects on lens dynamics. As an alternative to rings, it might prove enlightening to attempt to seed thermocline lenses with Lagrangian drifters. Such eddies are thought to form in the vicinity of the Mediterranean Sea and to arrive eventually at the western Atlantic boundary (McWilliams 1985). It might well be that thermocline lenses evolve away from strongly sheared currents and topography for a long enough time that the effect predicted here can be observed. Although there is currently some question about the propagation mechanisms in thermocline lenses (Nof 1983), beta propagation most likely accounts for some of their motion. The present mechanism should be persistent in its effects and coincide with the loss of lens density anomaly. Perhaps these characteristics will identify the mechanism in the data.

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APPENDIX A

Potential Vorticity Evolution in Ventilating Warm Lenses

The dimensional potential vorticity equation that can be obtained from (1) is

$$\frac{d}{dt} q = q_t + uq_x + vq_y = \begin{cases} Sq/h, & \text{CI} \\ 0, & \text{VC,} \end{cases} \quad \text{(A1)}$$

where $q = (v_x - u_y + f_0 + \beta y)/h$ and S is the dimensional cross-interface mass flux associated with the CI cooling. Note that potential vorticity is conserved if the VC parameterization is used, a result implicit in the analysis contained in section 4.

If the CI parameterization is examined, however, (A1) states that the potential vorticity of fluid parcels is altered by cooling, and that analysis must be somewhat modified. There are (at least) two instances in which the potential vorticity evolution can be analytically computed. The first occurs for the case of zero potential vorticity. Note that an exact solution of (A1), which meets the initial condition of zero potential vorticity, is $q = 0$ for all time. In this case, fluid parcels conserve q, even in the presence of S. The second analytically tractible case occurs for the special choice,

$$S = \alpha h,$$

i.e., for cooling proportional to local thickness. In this case,

$$q = q(t = 0)e^{\alpha t},$$

or equivalently, q grows exponentially following fluid parcels. If it is assumed that the initial potential vorticity is uniform in the upper-layer lens, then it remains uniform as it grows in magnitude. It is simple to incorporate such q behavior in the calculations for ring structure. Other heat flux parameterizations require the solution of (A1) in addition to (3) in order to calculate ring structure.

APPENDIX B

Lens Evolution in the Presence of VC Cooling

The density equation for a cooling fluid can be written:

$$\frac{d}{dt} \rho = -(\overline{w'\rho'})_z = \frac{F}{h}, \quad \text{(B1)}$$

assuming the region of cooling in the fluid is vertically well mixed. In (B1) F is proportional to the surface heat flux. If $F = \rho_0 \gamma h$, as postulated by Chapman and Nof (1987),

$$\frac{d}{dt} \rho = \gamma \rho_0$$

so

$$\rho = \rho_0(1 + \gamma t) \quad \text{(B2)}$$

following fluid parcels, where $\rho_0 = \rho\ (t = 0)$. Substituting (B2) into (1) yields

$$u_t + uu_x + vu_y - fv = \frac{-g[\rho_1 - \rho_0(1 + \gamma t)]}{\rho_0} h_x = -(g' - g\gamma t)h_x \quad \text{(B3)}$$

$$v_t + uv_x + vv_y + fu = \frac{-g[\rho_1 - \rho_0(1 + \gamma t)]}{\rho_0} h_y = -(g' - g\gamma t)hy. \quad \text{(B4)}$$

Scaling the equations as discussed in Section 2 yields the nondimensional set:

$$\beta u_t + uu_x + vu_y - (1 + \beta y)v = -(1 - pt)h_x \quad \text{(B5)}$$

$$\beta v_t + uv_x + vv_y + (1 + \beta y)u = -(1 - pt)h_y \quad \text{(B6)}$$

$$\beta h_t + (uh)_x + (vh)_y = 0, \quad \text{(B7)}$$

where $p = g\gamma/(g'\beta_0 R_d)$. Evans et al. (1985) document an SST decrease in 82-B of 0.035°C/day, which translates to an approximate γ value [via (B2)] of $\gamma = 7 \times 10^{-11}$ s^{-1}. For $g'/g = 10^{-3}$, $\beta_0 \sim 2 \times 10^{-13}$ cm^{-1} s^{-1} and $R_d \sim 10$ km, $p \approx .35 \sim O(1)$.

Expanding all variables in powers of β yields (14). From these equations, one obtains the potential vorticity equation

$$\frac{1}{r}\frac{\partial}{\partial r} rV_0 + 1 = h_0 q, \quad \text{(B8)}$$

where q is the (constant) potential vorticity, and the cyclostrophic balance equation:

$$\frac{V_0^2}{r} + V_0 = (1 - pt)h_{0r}. \quad \text{(B9)}$$

If $q = 0$ (zero potential vorticity),

$$V_0 = -r/2 \quad \text{(B10)}$$

$$h_0 = \frac{h_{c0}}{(1 - pt)^{1/2}} - \frac{r^2}{8(1 - pt)}, \quad \text{(B11)}$$

where h_{c0} is the ring thickness prior to cooling.

Manipulating the O(β) equations as described in the text yields

$$c = -\iint \psi dA \Big/ \iint h_0 dA \quad \text{(B12)}$$

and, using (B10) and (B11),

$$c = \frac{-2h_{c0}(1 - pt)^{1/2}}{3}. \quad \text{(B13)}$$

In the more general case of nonzero potential vorticity, the coupled equations (B8) and (B9) must be solved subject to a volume conservation constraint. It is then a simple matter to compute c by evaluating (B12) numerically.

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