Vertical Mixing in Basin Waters of Fjords

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ABSTRACT

The rate of work against the buoyancy forces due to vertical mixing (W) has been determined from repeated measurements of vertical density profiles in a large number of fjordic sill basins (basins dammed by sills). It is found that there is a weak "background" rate of work W_0, probably driven by the local wind. Superposed upon this is work driven by the tide. Thus $W = W_0 + R_f E$, where E is the mean energy flux from the surface tide to turbulence in the sill basin and R_f is an efficiency factor. We distinguish between "wave basins" and "jet basins." In the former category progressive internal tides are generated in the mouths, while in the latter there are tidal jets at the mouths. For wave basins, about 5.6% of the energy flux E from the surface tide is used for work against the buoyancy forces in the basin water (i.e., $R_f \approx 0.056$). The corresponding figure for jet basins appears to be less than 1%.

We have also studied the dependence of the vertical diffusivity κ upon the vertical stratification N. For well-behaved vertical distributions of N, it is found that $\kappa \sim N^{-1.5}$. A formula for κ, which appears to be applicable to many wave sill basins in fjords, is derived. From this, κ may be predicted if the vertical stratification $N(z)$, the characteristics of the topography and the sea level statistics are known.

1. Introduction

In this paper we investigate the vertical mixing in a large number of sill basins in fjords located in a relatively small area of the Norwegian west coast (Fig. 1). A sill basin is a basin dammed by a sill and the water in a sill basin is termed the basin water. Basin water is exchanged both by advective and diffusive transport processes. Advective exchange of basin water is usually an intermittent process occurring only when sufficiently dense water appears above the sill level outside the basin. Diffusive exchange, on the other hand, is thought to be a continuous process, driven by local vertical mixing of the basin water. The diffusive exchange generally implies a slow rate of decrease of the basin water density due to mixing with less dense overlying water. When studying vertical mixing in sill basins, one takes advantage of the intermittent character of the advective water exchange and selects for study periods with only diffusive water exchange. Such periods are commonly denoted as periods of stagnation.

A field program was conducted during the period July–December in 1986. The program included 30 different fjords with 47 different sill basins. During each of the five expeditions most of 81 fixed sampling positions were visited and profiles of salinity, temperature, and a number of biogeochemical variables were obtained. Characteristics of the fjords differ widely with respect to their surface area (1–48 km^2), volume (0.01–1.7 km^3), maximum depth (30–170 m), number of sill basins (1–4), sill depths (4–50 m), and normalized (with respect to the horizontal surface area of the fjords) local supply of freshwater (0.1–6 m^3 s^{-1} per km^2). The field measurements and the general oceanographic conditions of the fjords are presented in Aure and Stigebrandt (1988). The main purpose of the project is to investigate the effects of marine fish farming upon the natural environment, in particular the oxygen consumption in sill basins. In the present paper we only display and discuss results concerning the vertical mixing in the sill basins.

When planning this project we decided to approach our problem in an unconventional way. Instead of investing all available ship time and equipment into a detailed investigation of one single fjord, which is the usual approach, we decided to conduct measurements in as many fjords as possible at the expense of the amount of measurement effort placed in each single fjord. In this way we could obtain data from a large number of sill basins having different characteristics with respect to topography and supplies of freshwater and nutrients. This gave us the opportunity to explore

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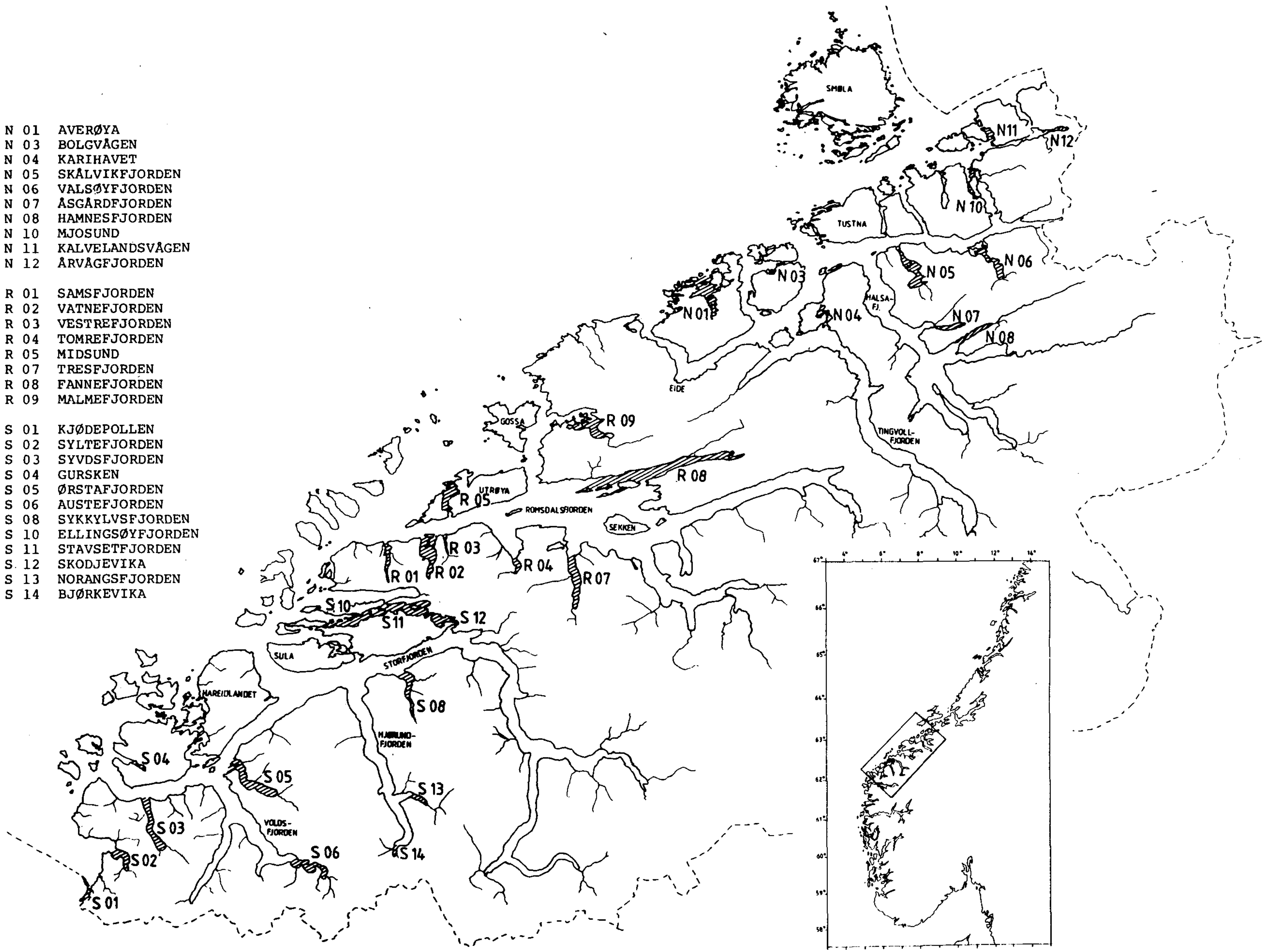

FIG. 1. Map showing the locations of the fjords.

functional relationships between different properties of the basin water and, for instance, topographical parameters. In addition to the results concerning the relationship between tidal dissipation and vertical mixing (presented in the present paper) we have also been able to show (Aure and Stigebrandt 1989) that the specific rate of oxygen consumption in the basin water is a strong function of two topographical parameters: mean depth of the sill basin (H_b) and sill depth (H_t), respectively. We have also shown that for identical sill depths the rate of denitrification is greater in shallow sill basins than in deeper sill basins (Stigebrandt and Aure 1988). These results could not have been obtained with a conventional "one-fjord" approach.

Sill basins offer a unique opportunity to study vertical diffusion since only diffusive processes change the basin water density in periods of stagnation. If precision vertical profiles of density (salinity and temperature) have been obtained at two or more occasions during a stagnation period, one may use the horizontally integrated diffusion equation to determine the turbulent vertical diffusion coefficient (the vertical diffusivity κ). The vertical diffusivity has been determined for a number of sill basins, see Gade and Edwards (1980) and Gargett (1984) for reviews of earlier work.

Much effort has been placed into analyses aiming at an increased understanding of the vertical mixing processes. Gade (1970) used data from the Oslo fjord to study the dependence of vertical diffusivity upon buoyancy frequency N. Inspired by the theoretical work of Welander (1968), Gade expressed the diffusivity κ in the following way

$$\kappa = \alpha(N^2)^\beta. \tag{1}$$

From the Oslo fjord data Gade estimated $\beta \approx -0.8$. Numerous similar determinations have later been performed in different sill basins. Many of these give β-values close to -0.5; e.g., Aure (1972) and for a summary of recent investigations see Gargett (1984). It turns out that the empirically determined β usually is in the range of $-0.8 < \beta < -0.4$, while α varies much from basin to basin.

The first successful attempt to relate the rate of vertical diffusion in fjords to a specific energy source for the turbulence was apparently made by Stigebrandt (1976, 1979), who demonstrated that the vertical mixing in the basin water of Vestfjord (a part of the Oslo fjord) is driven by progressive internal tides generated at the Drøbak sill. Evidence was gathered which indicated that the waves break against the sloping bottom of the sill basin. Thus, the vertical mixing actually occurs at the bottom boundary (boundary mixing). Further support for the hypothesis that vertical mixing in the basin water of fjords is mainly driven by sill-generated internal tides can be found in Svensson (1980), Smethie (1980), Lewis and Perkin (1982), and others.

Based on a theoretical discussion and analyses of our extensive field data from the 30 fjords, we will propose a parameterization of κ in terms of the flux of energy to internal tides, the vertical stratification, and the bathymetries of the sill basin and the mouth, respectively. Such a parameterization is of great advantage to modeling (cf. Aure and Stigebrandt 1988).

In section 2 the method to compute the vertical diffusivity from field data is briefly presented. In section 3 we discuss the parameterization of the vertical diffusivity from a theoretical point of view. In section 4 we discuss possible energy sources for the turbulence in the basin water. The results of our analyses of the field data are presented in section 5. Some concluding remarks are made in section 6.

2. The method to compute the vertical diffusivity

In periods of stagnation the density of the basin water may change only as a result of diffusive vertical exchange. A prerequisite for this is that there is no exchange of heat and salt with the bottom sediments and no radiation of heat from above into the basin water. As mentioned in the Introduction, much of the vertical mixing may actually be performed in a bottom boundary layer. If we consider horizontal averages, however, this complication may be avoided. Another advantage of working with horizontal averages is that the volumetric effect of nonvertical basin walls may be easily handled. In periods of stagnation the horizontally averaged conservation equation for the density $\rho = \rho(z, t)$ in the basin water is

$$\delta\rho/\delta t = (1/A)\delta/\delta z(A\kappa\delta\rho/\delta z), \tag{2}$$

where t is time, z is the vertical coordinate, $\kappa = \kappa(z)$ is the horizontally averaged vertical diffusivity and $A = A(z)$ is the horizontal surface area of the basin.

By vertical integration of Eq. (2) from the greatest depth $z = b$, where there is no diffusive flow of mass, to some level $z = u$, one obtains an expression for the vertical diffusivity at the upper integration level:

$$\kappa_{z=u} = 1/(A\delta\rho/\delta z)_{z=u} \int_b^u \delta\rho/\delta t A dz. \tag{3}$$

This equation may be used for computations of κ at different levels u. $\delta\rho/\delta t$ and $\delta\rho/\delta z$ are estimated from repeated measurements of salinity and temperature at different depths of the basin. This budget method to determine the diffusivity has been widely used; see Gade (1970) or Gargett (1984) for thorough discussions of the method.

In the subsequent analysis of turbulent mixing in sill basins, it is of primary interest to compare the mean rate of work performed by the turbulence against buoyancy forces with estimated rates of energy input from tides. We introduce the mean specific rate of work w, performed by the turbulence against buoyancy forces, defined by

$$w = \frac{1}{V} \int_b^u \rho\kappa N^2 A dz \tag{4}$$

where

$$V = \int_b^u A dz.$$

The total rate of work performed against the buoyancy forces between the levels b and u is $W = wV$. Thus, when κ has been determined from field data, W can be computed using Eq. (4).

The location of the fjords in this investigation can be found in Fig. 1. The fjords are given both "civil" names and code names (the latter are used in this paper). Hydrographical casts were made in the sill basins roughly once a month. Calibrated precision thermometers (accuracy ±0.01°C) were used to determine temperature. Water samples were taken for laboratory determination of salinity (accuracy ±0.003‰). In addition to standard errors due to the methods of measurement, errors due to ship drift may also be of importance (as samples may have been taken from shallower depths than intended). Due to possible internal wave activity in the basin water during sampling, there is also some uncertainty whether or not measurements from one single vertical profile may represent a true horizontal mean. A detailed description of the field program can be found in Aure and Stigebrandt (1988).

3. Theoretical considerations

As mentioned above, much work has in the past been devoted to find good parameterizations of the vertical diffusivity in the basin water of fjords and lakes. In particular the parameterization in Eq. (1) has been often used. From experimental data a number of investigators have determined α and β. Because of theoretical arguments leading to the prediction of certain values of the exponent β, this parameter has attracted most of the attention; see Gargett (1984). We will discuss the parameterization in Eq. (1) from the point of view of dimensional analysis. The aim of this is to reach a parameterization that is well suited for the analysis of experimental data. We will not try to model explicitly the mixing processes driven by the internal tides, but we need to find the vertical length and time scales of the energy containing turbulent eddies.

The (horizontal and time) mean speed of the energy containing turbulent eddies is denoted by v. The magnitude of v should be determined by the mechanisms distributing and releasing energy in the fjord, e.g., internal tides, tidal jets, and wind induced internal waves; Double-diffusive convection will not be considered. On larger time-scales both v and N are surely functions of the mixing. On the short time scale of the energy-containing turbulent eddies, however, we may consider v and N as independent variables.

It is known that in stratified fluids (assume N a constant for simplicity) there is a minimum internal vertical length scale, l, of a flow carrying a certain transport per unit width, q ($\mathrm{m^2\,s^{-1}}$) (cf. the classical theory for two-dimensional nonviscous selective withdrawal). The relationship between l, q, and N is $l^2 \sim q/N$. Since $q = lu$, where u is the mean speed of the current, one obtains $l \sim u/N$. We believe that uphill motion of the stratified fluid, caused by internal tides, may force the flow to take place in layers ultimately somewhat thinner than l. This should make the flow hydrodynamically unstable (supercritical) whereby turbulence is thought to be generated. Thus one may imagine that the vertical length scale above, set by the stratification and the mean speed, determines the thickness of intermittent, turbulent currents ultimately driven by the internal tide. The speed of the energy containing eddies v should be proportional to u. The vertical length scale of the energy containing eddies should thus be equal to $l \sim v/N$. Thus we do not believe that the turbulence caused by the internal tides is primarily generated by the bottom stress in an ordinary turbulent bottom boundary layer. Instabilities of wave generated currents, similar to those developing in the laboratory experiments performed by Stigebrandt (1976), should perhaps be more relevant for the generation of turbulence (note that the shadowgraphs in that paper were reproduced upside down). The time scale of the energy containing eddies is $\sim l/v \sim N^{-1}$. Such a time scale cannot be explained by ordinary (stress-generated) bottom-boundary turbulence, although, it should be in conformity with turbulence generated by a baroclinic instability mechanism controlled by the stratification and thereby by the magnitude of N.

From v and N we may estimate the scales of other quantities. The vertical diffusivity, κ, should have the following functional form:

$$\kappa \sim v^2/N, \tag{5}$$

which implies that

$$v \sim (\kappa N)^{1/2}. \tag{6}$$

The mean rate of work against the buoyancy forces, w, should have the form

$$w \sim v^2 N \sim \kappa N^2, \tag{7}$$

which conforms with what we already used in Eq. (4). Thus it appears that we have obtained a consistent parameterization of turbulence in stably stratified fluids. We will now proceed to discuss Eq. (1) within this framework.

One finds that α in Eq. (1) will have the dimension of velocity squared only if $\beta = -1/2$. Thus Eq. (1) is a dimensionally unacceptable parameterization if $\beta \neq -1/2$ since we insist, by comparison with Eq. (5), that α should have the dimension of velocity-squared. This problem may be solved by a suitable parameterization of α which should be of the type $\alpha = \gamma f$, where f is a dimensionless function and γ has the dimension of velocity-squared. If we require for some reason that α

should vary with N, this could be accounted for by writing $f = (N/N_m)^\delta$, where N_m could be the mean or maximum or some other suitable value of N. For the moment, however, it is pointless to discuss the most appropriate functional form of f. Instead we conclude that a dimensionally correct parameterization of κ could have the following appearance:

$$\kappa = \gamma f N^{-1}. \tag{8}$$

The function f should be normalized in such a way that if κ defined by Eq. (8) is used in Eq. (4) the value of W, determined from measurements in the basin, should be recovered. Thus

$$w = \gamma \rho_0 (1/V) \int_b^u fNAdz, \tag{9}$$

where we have replaced the (almost constant) density by a reference value ρ_0. We also require that

$$\gamma = w/(M\rho_0), \tag{10}$$

where M is the weighted average of the buoyancy frequency defined by

$$M = \frac{1}{V} \int_b^u NAdz. \tag{11}$$

From Eqs. (9), (10), and (11) we obtain the following integral condition for the function f:

$$\frac{1}{V} \int_b^u fNAdz = (1/V) \int_b^u NAdz = M. \tag{12}$$

For instance, f may be chosen to be

$$f = c(N/M)^\delta, \tag{13}$$

where c is a constant and δ is a real number. Utilizing Eq. (13), the integral condition expressed by Eq. (12) can be written as

$$c \int_b^u (N/M)^{1+\delta} Adz = V, \tag{14}$$

which determines the constant c if δ is known. This requires of course that N and A are known. From Eq. (14) it is obvious that f is not only a function of the actual kind of wave and turbulent processes but f may also be a function of the form of the basin as described by $A = A(z)$.

From the parameterization suggested here, it follows that we may utilize Eqs. (8), (10), (13), and (14) to actually compute the vertical diffusivity, κ, if we know the vertical stratification, N, the hypsographic function, A, the mean rate of work against the buoyancy forces in the basin, w, and the value of δ.

We will utilize our measurements to (i) investigate whether or not it is possible to establish a relationship between the observed mean rate of work against the buoyancy forces and the estimated energy input to the basin waters by internal tides and/or jets; and (ii) estimate empirical values of δ for the different sill basins, using the framework given in this section.

4. Possible energy sources for the turbulence in sill basins

It has been suggested that progressive internal tides generated at sills may be a major energy source for the turbulence in the basin water of many fjords (Stigebrandt 1976). In that paper evidence from the Oslo fjord was presented. It was also suggested that most of the energy fed into the internal tides dissipate below the sill level due to the breaking of the waves against sloping bottoms. Later we will compute the energy flux to internal waves and compare with the estimated rate of work against the buoyancy forces below the sill level. We will also to some extent consider the energy flux into fjords by tidal jets in narrow mouths. Tidal jets arise when the tidal current through the mouth is too fast for internal wave generation. This occurs when a certain densimetric Froude number is greater than unity (see Stigebrandt 1976, 1980). Since we have data from many sill basins, it should be possible to study further the possible relationship between the energy flux to internal tides and jets, and the rate of work actually performed by the turbulence against the buoyancy forces. It should be pointed out that many classes of flow phenomena occurring under different conditions in stratified fluids actually may be found in fjords. The fine review by Farmer and Freeland (1983) gives a broad exposure, but the scope of the present work limits our discussion to internal tides and tidal jets.

For a two-layer stratification the mean energy flux into sill-generated, long internal (interfacial) tides, having the phase speed c_i, is (Stigebrandt 1976)

$$E_2 = (1/2)\rho\omega^2 a_0^2 A_f^2/A_s\{H_b/(H_t + H_b)\}c_i, \tag{15}$$

where ω is the frequency, a_0 is the amplitude of the tide, H_t (H_b) is the depth of the upper (lower) layer and A_f (A_s) is the surface area of the fjord inside the mouth (the vertical cross-sectional area of the mouth). The horizontal surface area at sill level is denoted by A_b and the volume of the sill basin is V_b. Thus the mean depth of a sill basin is defined by $V_b = A_b H_b$. It is assumed that the interface is situated at sill depth and c_i is given by the standard formula

$$c_i = \{g(\rho_2 - \rho_1)/\rho_2[H_t H_b/(H_t + H_b)]\}^{1/2}, \tag{16}$$

where ρ_1 (ρ_2) is the mean density above (below) the interface.

For a linear increase of the density with depth (N = constant) a spectrum of waves are generated and one obtains (using Stigebrandt 1980) for the mean energy flux into the nth internal tidal mode

$$E_{cn} = (1/2)\rho\omega^2 a_0^2 A_f^2/ A_s(2/(n\pi))^2 \sin^2(n\pi d/H)c_{in}, \tag{17}$$

where the speed of the internal wave is $c_{in} = NH/(n\pi)$ and d and H are defined by $d = H_b$ and $H = H_b + H_t$.

A tide in the fjord of amplitude $a_0 \sin(\omega t)$ generates a volume flow in the mouth which is $A_f a_0 \omega \cos(\omega t)$. The mean tidal speed in the sill section is

$$u_s = A_f/A_s a_0 \omega \cos(\omega t) = u_{s0} \cos(\omega t). \quad (18)$$

The mean flow of kinetic energy E_j into the fjord by a mouth jet is approximately given by

$$E_j \approx 0.42(1/4)\rho \omega^3 a_0^3 A_f^3 A_s^{-2} \quad (19)$$

since the jet is flowing into the fjord only half of the time and the time average of $|\cos^3(\omega t)| \approx 0.42$.

It might be interesting to compare the efficiencies of jets and internal waves in withdrawing energy from the tide. From Eqs. (15) and (19) one obtains

$$E_j/E_2 \approx 0.42(1/2)u_{s0}/c_i(H_t + H_b)/H_b. \quad (20)$$

If, for instance, $H_t < H_b$ and $u_{s0} < 0.2c_i$, then $E_j/E_2 < 0.1$. Thus internal tides are very efficient compared to jets in removing energy from the tide, in particular if the velocity amplitude in the mouth (u_{s0}) is much less than the speed of the appropriate internal wave mode. Besides, the mixing effect of a buoyant surface jet should probably be rather local in the vertical direction in contrast to the deep-reaching mixing effect of (breaking) internal waves. Thus we expect that only a small fraction of the jet energy is dissipated in the basin water (cf. Stigebrandt 1980).

The efficiency R_f (the flux Richardson number) of the turbulence with respect to working against buoyancy forces is estimated by

$$R_f = W/E_2, \quad (21)$$

where $W = wV_b$ and V_b is the volume of the sill basin. Equation (21) requires that the rate of work against the buoyancy forces is due to the energy input defined by E_2. We have here followed the suggestion in Stigebrandt (1976) where it was argued that most of the internal wave energy should dissipate in the basin water. It was found there that $R_f \approx 0.05$ for the inner Oslo fjord (Vestfjord) if E_2 represents the energy flux to internal tides from the whole spectrum of frequencies. For the Oslo fjord it was estimated that the M_2 tidal component contributed about 60%. For the present area the whole semidiurnal frequency band should contribute about 85% of the energy flux into internal tides.

The local wind may probably contribute to the turbulence in the basin water, possibly via transversal internal waves carrying energy vertically. We will not, however, discuss the detailed mechanisms for this. This assumed vertical energy flow may create a "background" rate of work against the buoyancy forces, W_0, upon which the effect of the tide is superimposed. The total rate of work against the buoyancy forces in the basin water may then be written

$$W = W_0 + R_f \phi E_2, \quad (22)$$

where ϕ is the inverted fraction of the total energy supply to internal tides provided by the semidiurnal frequency band($E_{2(j)}$) (for the present area we thus obtain $\phi = 1/0.85 = 1.18$). We will later estimate W_0 and R_f from our field data. This may be done by the means of statistical analysis provided a sufficient number of estimates of W and $E_{2(j)}$ are available.

5. Results

For the method outlined in section 2 to be applicable, no advective water exchange of basin water should have occurred in periods between the measurements of two utilized profiles. Density and oxygen data showed that for some sill basins there was no period of stagnation. For a number of particularly shallow sill basins there is density data from only one level below the sill. These sill basins are not included in the present investigation. For the computation of κ, from Eq. (3), it is required to know the hypsographic function, the horizontal surface area of the sill basin at different depths. The horizontal surface area was determined for every 10 meters down to the greatest depth for all sill basins utilizing high resolution sea charts. In Table 1 we give some information about the topography of the different sill basins.

From our observational data we computed $\Delta\rho/\Delta t$ and $\Delta\rho/\Delta z$ for periods of apparent stagnation. Using Eq. (3) we thereafter computed κ for as many depths below the sill depth as our measurements permitted. Utilizing Eq. (4) we could then compute the total rate of work performed below the sill level (W). In Table 2 we have listed W, normalized by A_b (the horizontal surface area of the basin at sill depth), together with some topographical properties of the fjords. We have not introduced new notation for quantities normalized by the area since this should be clear from the dimension of the quantity in question.

The energy flow to internal tides depends on the speed of internal waves. Due to the sill, dense water is trapped in a sill basin. The stratification may therefore often be described as approximately two-layered with a pycnocline at about sill depth. In Table 2 we give the mean phase speed of interfacial internal waves with interfaces at sill depths, from Eq. (16), for the periods of stagnation in the different fjords. For these computations we have used the vertical mean densities above and below the assumed interface at the sill level. Using $a_0 = 0.9$ (m) and $\omega = 1.410^{-4}$ (s^{-1}), which are typical values for the semidiurnal tide in the present area, we have from Eq. (15) computed the mean energy fluxes to internal tides (E_2) in the different fjords. The $E_{2(j)}$-values shown in Table 2 are normalized by A_b. Also the amplitude of the tidal current in the mouth u_{s0} is given. It should be noted that the shapes of the mouths of the sill basins differ widely from nicely U-

TABLE 1. Some topographical information about the fjords; A_b is the area of the sill basin at the sill level. "Tokt" is the Norwegian word for expedition.

Fjord	Station	A_b (km^2)	H_m (m)	H_t (m)	H_b (m)	V_b/V_T (%)	Number of levels	Between tokts	Below depth (m)
S02	2	2.052	104	40	30	34	5	2–3, 4–5	55
S03	2	6.504	100	29	38	39	5	2–3, 4–5	35
S05	2	12.934	170	24	75	74	8	2–3, 4–5	35
S06	2	2.277	100	29	34	48	6	2–3	35
S10	2	2.784	80	40	13	14	2	2–4	45
S11	1	5.636	130	25	48	62	5	2–4	35
S12	1	5.456	70	5	29	85	5	2–4	22.5
S14	2	0.891	41	11	19	61	3	2–4	17.5
R01	2	0.948	73	40	15	14	2	2–5	45
R02	2	3.604	113	50	23	22	2	3–4	65
R02	3	1.040	82	42	21	24	2	3–5	46
R05	2	1.682	62	37	12	14	2	2–4	38.5
R07	2	7.857	76	37	19	27	2	2–4	45
R08	3	27.496	73	30	20	33	3	2–4	35
R09	3	6.501	71	16	22	51	4	1–4	22.5
N01	2	1.405	39	12	12	42	2	2–5	17.5
N01	4	1.625	52	10	23	68	4	2–3	22.5
N04	3	1.883	74	8	34	80	6	2–3	17.5
N05	2	1.380	58	25	19	36	2	2–3	27.5
N05	3	1.707	37	18	10	28	2	2–3	22.5
N05	4	3.377	41	18	12	34	2	2–3	22.5
N06	2	1.147	29	9	9	43	2	2–3	12.5
N06	3	1.840	36	5	14	73	3	2–3	12.5
N06	5	4.698	84	17	38	67	5	1–3	22.5
N07	2 + 3	3.736	53	9	26	73	4	3–5	17.5
N08	2, 3	5.381	137	7	66	90	6	1–2, 3–5	17.5
N10	2	1.864	61	16	24	57	4	2–5	18
N11	2	2.220	45	11	16	54	5	2–5	13
N12	2	1.106	77	23	31	52	4	2–3	27.5

shaped vertical cross sections, through V-shapes, to quite irregular shapes caused by the presence of islands for instance. One would expect that the simple wave generation model for two-layer flow utilized here should describe the wave generation less well for mouths with vertical cross sections deviating much from U-shapes. One also expects that the wave generation model used should be less reliable if the stratification deviates much from the assumed two-layer stratification with the interface at the sill level.

For sill basins where u_{s0} is greater than c_i internal tides should not be generated. The density of the fjord water above the sill usually varies with the density of the coastal water at the same level. Thus, in periods of upwelling at the coast, the density contrast between the basin water and the water above the sill decreases and thereby the speed of the internal wave. In fjords where c_i in Table 2 is only slightly greater than u_{s0}, we assume that jets are generated at least sometimes. In addition, friction against the mouth boundaries as well as the acceleration effects (vena contracta) tend to increase the actual velocity of the mouth flow and make this faster than u_{s0} defined by Eq. (18). We tentatively classify a sill basin as a tidal jet basin if $u_{s0} > 2c_i/3$. The $E_{2(j)}$-value in jet basins is computed from Eq. (19) and underlined in Table 2. Note that some jet basins obtain contributions from two jets (S10 Station 2, N05 Station 2 and N06 Station 2).

From Table 2 we see that the rate of work per horizontal surface area of the sill basins, $W(=wH_b)$, varies much from basin to basin with minimum 0.016 and maximum 1.67 (mW m^{-2}). It can be seen that in jet basins there is a tendency for greater rates of work. For the 9 jet basins we obtain a mean value of W equal to 0.51 with a std dev of 0.44. For the 20 wave basins the corresponding values are 0.138 and 0.093, respectively. Thus, as one would expect, jet basins are more energetic than wave basins, although the ratio $W/E_{2(j)}$ is generally greater for wave basins. For wave basins the mean value of this ratio is 0.080 with a std dev of 0.030. For jet basins the corresponding values are 0.029 and 0.020, respectively. The great scatter of data for the jet basins can probably be explained by the fact that some of these at times are wave basins. The "pure" jet basins, tentatively those with $u_{s0} > 2c_i$, appear to have $W/E_{2(j)} < 0.01$.

There is an appreciable scatter in the ratio W/E_2 for the wave basins for several reasons. At the end of section 2 we mentioned errors connected to the measurements. Another concern is whether or not the basin water really has been truly stagnant as required for the determination of the vertical diffusivity. An important

TABLE 2. Computational results. The symbols are explained in the text. Underlined values of $E_{2(j)}$ denote tidal jet basins.

Fjord	Station	A_s (m^2)	A_f (km^2)	u_{s0} (cm s^{-1})	c_i (cm s^{-1})	$E_{2(j)}$ (mW m^{-2})	W (mW m^{-2})	$W/E_{2(j)}$
S02	2	18 150	3.93	2.7	54	0.76	0.054	0.071
S03	2	20 600	8.88	5.4	55	1.45	0.137	0.094
S05	2	17 950	15.53	10.9	65	4.06	0.224	0.055
S06	2	7 730	3.58	5.8	70	2.20	0.123	0.055
S10	2	20 520	34.59	21.2	33	29.73	0.363	0.012
S11	1	5 950	21.64	46.0	54	10.93	0.529	0.048
S12	1	350	6.18	222.0	30	76.30	0.685	0.009
S14	2	2 800	1.07	4.8	51	1.18	0.058	0.049
R01	2	7 100	3.77	6.7	37	1.69	0.192	0.114
R02	2	26 100	9.90	4.8	57	1.50	0.016	0.011$
R02	3	16 010	2.45	1.9	51	0.48	0.072	0.150
R05	2	10 500	8.23	9.9	37	2.76	0.262	0.095
R07	2	28 500	12.81	5.7	44	0.88	0.038	0.043
R08	3	43 100	47.79	14.0	48	2.93	0.285	0.097
R09	3	6 250	10.11	20.3	59	6.86	0.091	0.013&
N01	2	3 250	2.52	9.8	28	1.55	0.142	0.093
N01	4	1 110	3.94	45.0	32	6.58	0.375	0.057
N04	3	1 000	2.23	28.0	56	9.51	0.101	0.011*
N05	2	2 400	11.14	87.0	42	43.80	1.670	0.038
N05	3	4 050	8.77	27.0	36	5.19	0.176	0.034
N05	4	11 500	5.28	5.8	45	1.03	0.104	0.101
N06	2	1 690	9.52	71.0	32	141.70	0.331	0.002
N06	3	1 020	7.78	96.0	31	53.00	0.353	0.007
N06	5	5 420	5.59	13.0	69	4.70	0.057	0.012*
N07	2 + 3	1 460	4.42	38.0	54	2.33	0.128	0.054
N08	3	1 400	4.65	42.0	66	13.73	0.288	0.021*
N10	2	4 125	2.49	13.0	50	1.93	0.107	0.056
N11	2	2 400	3.57	19.0	35	4.02	0.113	0.028#
N12	2	3 660	1.66	5.7	52	1.63	0.076	0.047

$ Irregular sill—only 10% of A_s below 25 m.
& Irregular sill—only 10% of A_s below 10 m.
* Strong pycnocline at about 2–3 m depth.
# Irregular mouth—~60% of A_s above 2.5 m.

factor contributing to the scatter of the W/E_2 ratios is connected to the geometry of the mouths. The performance of the wave generation process should depend upon the actual geometry of the mouth. For instance, a very irregular cross-sectional area of the mouth may be much less efficient in generating internal waves than the U-shaped mouth implicitly required by our simple wave generation model. Finally, the applied wave generation model (two-layer model) requires a pycnocline at sill depth. This requirement may be poorly fulfilled in some fjords. This should be particularly true in fjords with relatively shallow mouths and with very shallow pycnoclines, above the sill depth, due to local supplies of freshwater. In Table 2 we have marked those fjords where the computed energy flux (E_2) to internal tides should be unreliable since the requirements of the two-layer internal wave generation model are poorly fulfilled. Thus, 6 of the 20 wave basins do not fulfill the requirements of well-behaved sill topography and well-behaved vertical stratification.

In Fig. 2 we have plotted W versus E_2 for the 14 wave basins where the computed E_2 values should be reliable. A linear regression line has been fitted. A comparison with Eq. (22) shows that $W_0 = 0.018$ mW m^{-2} and $R_f\phi = 0.066$. Thus, the background rate of work against the buoyancy forces is about 0.02 mW m^{-2}. We have not determined error bars for W_0 and R_f. With regard to the limited number of data points in Fig. 2, however, it is obvious that we should not put

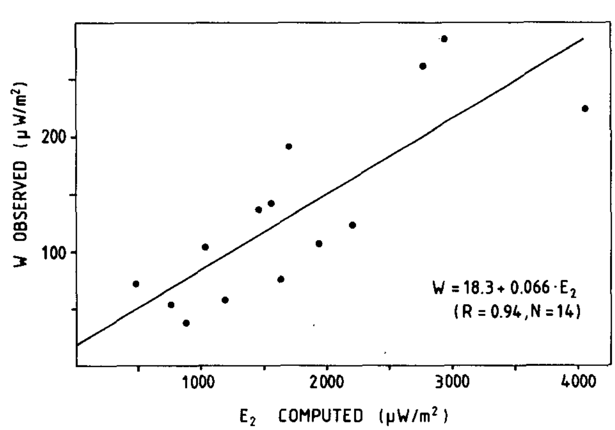

FIG. 2. The turbulent rate of work against the buoyancy forces in the basin waters, W, vs the estimated energy flux to interfacial internal tides with the interface at sill level, E_2. Also given is the linear regression line. R is the correlation coefficient and N is the number of data points.

too much emphasis upon the exact value of W_0. If $\phi = 1.18$ we obtain $R_f = 0.056$. This value of R_f conforms well with earlier estimates (e.g., Stigebrandt 1976). The high correlation between W and E_2 suggest that the relative error of our R_f estimate should not be greater than about ±20%.

We have also investigated in detail the N-dependence of κ in the different sill basins. Using Eqs. (8), (10), and (13) we expressed κ in the following way:

$$\kappa = (w/M^2\rho_0)c(N/M)^{\delta-1}, \quad (23)$$

where w is the mean specific rate of work defined by Eq. (4), ρ_0 is the reference density and M is the weighted average of the buoyancy frequency defined by Eq. (11). Using the least squares method we determined the constants c and δ for each basin in such a way that κ determined from Eq. (23) (κ_{23}) gave the best representation of the experimentally determined $\kappa(\kappa_{obs})$. The results are shown in Table 3. We also present the std dev (std) for the ratio κ_{23}/κ_{obs}. This should be a measure of how well Eq. (23) fits the observations. Note that if there are only two experimental points in a basin the fitted function passes through the two points and std dev = 0.

From Table 3 it can be seen that there is a large scatter in the estimated δ-values. Also the c-values vary. For well-behaved smooth vertical profiles of N, one expects c-values close to 1. It is evident from Table 3 that there is a correlation between large absolute values of δ and large deviations of c from 1. There is also an evident correlation between large std and large deviations of c from 1. Thus, extreme values of δ are often due to irregular N-profiles. A closer study of the vertical profiles of N confirmed this conclusion. We computed the average of δ for all wave basins with at least four measurements below the sill and with $0.9 < c < 1.1$. The result is $\delta = -0.47$ with a std dev of 0.35 (11 cases). For the jet basins only one fulfills these two requirements, so we may not state anything about the mean value of δ in jet basins with well-behaved vertical stratification. However, if we relax the requirement on the number of experimental data points we see that the same δ-value appears to fit the jet basins also.

Based upon our theoretical discussion in section 3 and the determination of the empirical constant δ made above we conclude that the vertical diffusivity in sill basins with well-behaved vertical stratification may be described by the following expression:

$$\kappa = (w/M^2\rho_0)c(N/M)^{-1.5}. \quad (24)$$

Here $w = W/H_b$ and W may be computed from Eq. (22) using appropriate values of W_0 and ϕ and using $R_f = 0.056$ (0.01 for jet basins). Using the known vertical stratification, M is determined from Eq. (11) and c is determined from Eq. (14). In the light of the considerable experimental scatter Eq. (24) should of course

TABLE 3. Values of c and δ determined for the sill basins. For some basins there are two independent sets of c and δ (doublets are given in the last three columns); Std dev is the standard deviation of the ratio (κ_{23}/κ_{obs}).

Fjord	Station	Measurement levels	c	δ	Std dev	c	δ	Std dev
S02	2	5	1.00	−0.98	0.19	0.69	0.63	1.92
S03	2	5	1.04	−0.44	0.11	1.06	−0.66	0.06
S05	2	8	1.07	−0.38	0.54			
S06	2	6	1.06	−0.62	0.23			
S10	2	2	1.00	−0.41	0			
S11	1	5	0.93	−0.36	0.62			
S12	1	5	0.43	−0.63	3.01			
S14	2	3	0.78	1.14	0.33			
R01	2	2	0.86	−2.02	0			
R02	2	2	1.00	−0.18	0			
R02	3	2	0.87	0.71	0			
R05	2	2	0.98	103	0			
R07	2	2	0.90	1.05	0			
R08	3	3	0.58	−1.25	1.25			
R09	3	4	0.82	−0.01	0.51			
N01	2	3	1.01	−0.81	0.09			
N01	4	4	0.78	−2.91	0.34			
N04	3	6	1.04	−0.11	0.08			
N05	2	2	1.01	−0.42	0			
N05	3	3	0.82	−4.83	0.36			
N05	4	2	0.75	−3.16	0			
N06	2	2	1.01	−0.54	0			
N06	3	3	0.97	0.67	0.11			
N06	5	5	1.10	−0.45	0.24			
N07	2 + 3	4	0.53	0.48	1.86	0.74	1.37	0.34
N08	2, 3	6	0.94	−1.15	0.49	0.99	−0.10	0.38
N10	2	4	0.95	0.01	0.21			
N11	2	5	0.59	1.81	1.49			
N12	2	4	1.04	−0.30	0.16			

be used cautiously and only for basins where $0.9 < c < 1.1$. One should also be aware of the problem to decide upon which value of R_f to use for basins that may switch between being wave and jet basins.

6. Concluding Remarks

Two independent variables, the horizontal and time mean speed, v, of the energy containing turbulent eddies and the buoyancy frequency, N, should characterize the field of turbulence in sill fjords. Using dimensional arguments we have clarified the commonly used parameterization of the vertical diffusivity in basin waters of fjords, in Eq. (1). The vertical diffusivity should be proportional to N^{-1} and a quantity of dimension velocity squared. If the latter varies with N for some reason, this N-dependence should be accounted for using a nondimensional function.

Thanks to the extensive data bank created in the present project, we have been able to demonstrate an evident relationship between the mean rate of work against the buoyancy forces in the basin waters and the flux of energy to tidal jets and internal tides. The efficiency of the (lost) tidal energy, in wave basins radiated by internal tides into the sill basins, with respect to working against the buoyancy forces in the basin water was found to be equal to about 0.06 (the flux Richardson number, R_f). The corresponding efficiency of tidal jets seems to be only about 0.01. We also estimated the "background" level of the rate of working against the buoyancy forces in the investigated sill basins. This was found to be about 0.02 mW m^{-2}. It should be remembered that this figure is quite uncertain. However, we suggest that the background mixing ultimately is driven by the local wind. The detailed mechanisms for the transfer of wind energy down into the basin waters have not been discussed. The intensity of background mixing may certainly vary from region to region. It would be interesting to compare our W_0 with the value of W in the hypolimnion in vertically stratified inland lakes.

For the wave basins with well-behaved vertical distributions of N (i.e., $0.9 < c < 1.1$) we found that $\kappa \sim N^{-1.47\pm0.35}$ where we have used the experimental std dev to express the uncertainty of the exponent. Thus, our result does not support the N^{-1} or the N^{-2} power laws (cf. Gargett 1984) but we cannot immediately rule out that one of them may be true. Since the actual shapes of $N(z)$ and $A(z)$ both appear to influence the power of N in the expression for κ we conclude that there probably is no universal power law applicable to all kinds of systems.

Our Eq. (24) constitutes an empirical formula for κ probably applicable to most wave sill basins of fjords. Given the vertical stratification in a well-behaved basin the sea level statistics (from which the barotropic flow through the mouth as a function of frequency may be computed) and the bathymetries of the basin and the mouth respectively, it appears to be possible to predict the vertical diffusivity. The existence of such a formula will be valuable for modeling purposes. It should be stressed that the use of the formula should be restricted at present to wave basins where c is close to 1.

The statistical work presented in this paper should be particularly useful in pointing out factors of major importance for the vertical mixing in the basin waters of fjords. The work may be refined as more data of good quality become available. In order to increase our understanding of what really is going on with regard to vertical mixing, it is essential that one also studies in detail the mechanics of the different processes of importance for the mixing. In particular one would like to investigate the mechanics of the "background" work W_0. A description of the mechanics of the "breaking" of internal tides is still lacking. It would also be desirable to develop and use a more sophisticated method to compute the energy flux to internal tides at sills. This method should preferably permit the use of arbitrary vertical density profiles and complex topographies of the mouths.

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