Kinematics and Dynamics of a Mediterranean Salt Lens

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ABSTRACT

Two surveys of the absolute velocity field of an eddy of Mediterranean Water (meddy) in the Eastern North Atlantic were conducted one year apart in 1984 and 1985. Two velocity regimes were revealed. Within the radius of maximum velocity, the meddy rotated anticyclonically as a solid body with a depth-dependent rotation period near 6 days at its mid-depth (1000 m). One year later the radius of the core had decreased by one third. The rotation rate of the lens also decreased, except at its mid-depth where there was a small but perceptible increase.

There was a sharp (5 km or less) transition between the core and the outer region where the velocity decayed exponentially with radius. A strong potential vorticity front, due to the abrupt change in sign of horizontal shear, kept the core isolated from the outer region. Potential vorticity was nearly constant within the upper confines of the core over the study period, whereas, there was a notable increase in potential vorticity in the lower portion of the core due to erosion from underneath. Although there were significant azimuthal velocities beyond the transition, the potential vorticity was nearly that of the background field. The horizontal uniformity of the potential vorticity field suggests free exchange along isopycnal surfaces.

1. Introduction

In recent years, discrete lenses have been recognized as a mechanism which could contribute significantly to the transport of heat and salt in the Mediterranean salt tongue (Lilibrige and Rossby 1983; McWilliams 1985). Several eddies of Mediterranean Water, or “meddies”, have been observed in the Canary Basin (Armi and Stommel 1983; Armi and Zenk 1984). They are distinguished by their high salt and heat content, about 1 psu and 2.5°C greater than the surrounding waters. Typically, these lenses measure up to 50 km radially and 1000 m vertically, and are centered near 1000 m depth, the mid-depth of the salt tongue. Armi and Zenk (1984) have estimated that a meddy of such dimensions can carry the equivalent of 10 days of salt outflux from the Mediterranean. If several lenses are formed each year, they could account for a significant portion of the total amount of salt dispersed. Not only do meddies transport large quantities of heat and salt, but have been observed to do so over great distances. Armi and Zenk (1984) have identified meddies thousands of kilometers from their proposed formation sites.

Earlier, it was believed that the zonal distribution of conserved properties in the eastern North Atlantic could be explained in terms of an advective/diffusive balance (Defant 1955; Needler and Heath 1975; Richardson and Mooney 1975). However, more recent observations with current meters and subsurface floats do not support the mean west and southward velocities assumed for these models. For example, a seven-year current meter record at 1000 m showed negligible mean velocity at 33°N, 22°W; most of the energy was associated with the eddy fluctuations (Zenk and Müller 1989). Using SOFAR floats, Price et al. (1986) observed a strong, short-lived (6 months) westward jet near 32°N, only 150 km wide and surrounded by weak eastward return flow. These observations suggest that the processes distributing salt from the Mediterranean outflow may be more complicated than simple advection and diffusion from a point source at the eastern boundary. The numbers, size, and distribution of meddies observed in the eastern North Atlantic suggest that they may be an important transport mechanism and that models of the salt tongue may need to incorporate meddies to accurately describe the processes affecting the zonal property distributions. In order to determine their significance in the North Atlantic salt budget we first need to know more about the behavior of these lenses: their frequency of occurrence, migration patterns, and decay processes. This study concentrates on how a single meddy is held together as a coherent feature over a lifetime of several years.

In 1984 a cooperative investigation was launched to study the physical and dynamical properties of a meddy

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in the Canary Basin, where the probability of finding one was expected to be high (Armi and Zenk 1984). A meddy was found east of the Azores, at 32°N, 22°W, and tagged with SOFAR floats and deep drifters so it could be tracked and relocated. The temperature and pressure records of the floats, their speeds and their dispersion characteristics were used to determine how long each float stayed trapped within the meddy (Rossby 1988; Richardson et al. 1989). One float stayed in the meddy for two years as it moved 1100 km to the south. During this two year period the lens was visited four times: October 1984, June 1985, October 1985, and October 1986. Several researchers collaborated to conduct four hydrographic and two velocity surveys, as well as make detailed microstructure measurements. A general description of the experiment is given by Armi et al. (1988) and in greater detail by Armi et al. (1989).

The objective of this paper is to examine the velocity structure and local dynamics of the lens and how these change with time. The study is based on two PEGASUS absolute velocity surveys and the corresponding hydrography. These surveys were conducted in October 1984 and October 1985, and enable us to examine the evolution of this meddy over a one year period. The next section summarizes the data and the techniques used to position the observations within the framework of the lens. Section 3 describes the velocity structure of the lens for the two years. Section 4 looks at the dynamical properties of the lens, i.e. the momentum balance, the vorticity field, and the potential and kinetic energy integrals. Section 5 discusses our understanding of how the lens evolved and aged between the two surveys one year apart. The last section concludes the paper with a brief summary.

2. Data

a. CTD data

The CTD data were collected by L. Armi during the October 1984 and October 1985 cruises and were processed by D. Hebert (see Hebert 1988a, for full details of the processing). The CTD processing included computations of dynamic height, isopycnal displacement from a background mean depth, and Brunt–Väisälä frequency, all of which were used in this analysis. Two different sampling strategies were employed. First, discrete CTD casts to 2000 m were made at various locations within the lens, including each of the PEGASUS drop sites. Sampling was also done far from the influence of the lens to obtain information on the background density field. Second, to enhance the horizontal resolution, continuous sections were made by lowering and raising the CTD ("toyo-ing") between 500 and 1500 m, depth, while the ship was slowly making way. The location of each toyo profile was taken to be the ship’s position when the CTD was at 1000 m.

The hydrographic data were used to determine the location of the meddy’s center as a function of time and thus the radial position of any observation. This work was carried out by Hebert (1988a) in his analysis of the scalar properties of the lens and was directly applied to the PEGASUS data. He assumed a circular meddy, moving at a constant rate based on the mean advection measured by the SOFAR floats at the time of each survey. He then varied the mean velocity and the relative positions of six radial sections of salinity to find the minimum variance in the location of the strongest salinity gradient. Further details of the center determination may be found in Hebert (1988a). Hebert’s method for obtaining the meddy’s center is compared below with the velocity data and the assumptions used for its analysis, and the two were found to be in good agreement. Included in his analysis, Hebert (1988a) shows it is reasonable to assume the lens was axisymmetric and in a steady state over the time period of each year’s survey. The agreement of the two meth-

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odds reinforces each one and allows us to use all the CTD measurements in conjunction with the PEGASUS data.

b. Velocity data

Absolute velocity fields were obtained using the free-falling instrument, PEGASUS. It is a simple measurement in which velocity is derived from the acoustically determined position of the instrument as it sinks through the water. As the instrument falls, it emits a 10 kHz signal at 16 s intervals, or about every 10 m. Two transponders, which have been previously set on the bottom, answer this signal. The round-trip travel time is recorded in microprocessor memory inside PEGASUS. The travel times, together with the sound velocity profile can be used to determine the slant ranges between PEGASUS and the transponders (see Spain et al. 1981, for a detailed discussion of the range calculations). Combined with pressure (= depth), the ranges determine the instrument’s horizontal position relative to the transponders whose locations were previously surveyed. The first derivative with respect to time of a local linear fit of the instrument’s trajectory yields the absolute north- and eastward components of the currents as a function of depth. A detailed description of the instrument is given by Spain et al. (1981), which includes a thorough analysis of measurement errors. They show that the error in the velocity measurement is proportional to the descent/ascent rate of the instrument and is about 1 cm s\(^{-1}\).

In October 1984, 26 profiles were obtained within the meddy (Table 1). They sampled the full horizontal dimension of the lens, from the center out to a radius of 65 km, with the densest sampling in the high velocity region. The azimuthal component of velocity and radial position of the profiles are shown in Fig. 1a. In October 1985, 8 profiles were taken along a radial arm within the meddy (Fig. 1b and Table 1). Again the survey extended to the outer boundary of the lens, to a radius of about 40 km.

The velocity profiles are instantaneous snapshots of horizontal motion and as such, include contributions from all time scales, in particular, inertial oscillations. In order to focus on the meddy field, the high wavenumber structure (presumably including inertial oscillations) was removed by filtering in the vertical. A squared, second-order, low-pass Butterworth filter with a cutoff (3 db) wavelength of 333 m was used. This wavelength was chosen by visual inspection because it effectively eliminated the high wavenumber variations, which we attribute to inertial oscillations, internal waves, and turbulence, while preserving the main velocity signal of the lens. It is a conservative choice since the “half-wavelength” of the azimuthal velocity field in Fig. 1a is about 1400 – 500 ≈ 900 m, much greater than the associated half-wavelength of the filter. For consistency, the identical filter was used on the CTD data.

To study the Meddy’s internal dynamics, the measured east and north velocities were transformed into the cylindrical framework of the lens using the model described earlier (Hebert 1988a). Studies of other lenses and mesoscale eddies have shown that the cores of such features were in near solid-body rotation, and the same is true of this meddy as well. The radial positions of the PEGASUS velocity profiles, as determined from the salinity signal, are consistent with solid-body rotation; a linear regression between velocity and radius showed a very high correlation coefficient (0.98) within the core at 1000 m. Furthermore, the resulting radius of zero velocity was 2.6 ± 0.5 km. Similar results were obtained for the 1985 data, the radius of zero velocity was 2.2 ± 2.4 km. Not only are the data consistent with the assumption of constant horizontal velocity shear, but also with the requirement of zero velocity at the center. The 2 to 3 km offset from the origin of the radius of zero velocity may be considered an estimate of the error of the radial coordinates. The center used is at least consistent with the assumptions made for this analysis, and the uncertainty in the method is at least this accurate (±3 km). It should not be surprising that the velocity field, with its zero crossing at the center, should be more effective in locating the axis of the lens than the eddy scale density survey,

![Diagram](image_url)

**Fig. 1.** Azimuthal component of velocity of a representative sample of PEGASUS profiles in 1984 (a) and 1985 (b), shown in the reference frame of the lens. Each profile is offset according to its radius.
which is more sensitive to gradients around the periphery than to the level surfaces near the center. That they agree so well suggests that the lens is highly symmetric and circular.

3. Results

a. The meddy in 1984

The meddy was first found in October 1984. It was identified by its large salt and heat anomalies, which were comparable to those of other meddies in the Canary Basin (Armi and Zenk 1984, for example). At that time, it was approximately 130 km in diameter and 900 m in the vertical, as defined by its velocity signal. The hydrographic survey revealed a horizontally uniform core of Mediterranean Water (12°C, 36.2 psu) 48 km in diameter and 500 m thick. Shown in Fig. 2 are profiles of salinity, temperature, and density from 1984 and 1985. The smooth profiles indicate a weakly stratified core with very little horizontal variation in

![Fig. 2](image-url)

**Fig. 2.** Profiles of salinity (a), temperature (b), and density (c) through the core in 1984 (dotted line), and 1985 (solid line), and in the background field in 1984 (dashed line).

![Fig. 3](image-url)

**Fig. 3.** Mean profiles of stratification in the core and background for the meddy in 1984 (a) and 1985 (b). Note the vertical axis is $\sigma_1$. The core of the meddy is where the stratification is less than that of the background field.
temperature and salinity. Near the core’s edge however, a radial section at 1000 m, say, shows a sharp decrease in temperature and salinity, (see Fig. 9, Hebert et al. 1990). Although the region surrounding the core is cooler and fresher than the core itself, it is still distinguishable from the ambient field.

Figures 3a (1984) and 3b (1985) show how the weakly stratified interior was bounded by regions of high stratification above and below, where the isopycnals were “pinched” to accommodate the lens. It is noteworthy that, as a result of the pinching of the density field, the dynamic height signal in the center of the lens, about 5 dyn cm in 1984, was completely compensated for and produced no surface expression. This dynamic high in the center is consistent with the anticyclonic circulation measured with PEGASUS. Figure 4 shows a smoothed contour section of velocity. Peak speeds (radially smoothed with cubic splines) were just greater than 25 cm s\(^{-1}\) between 900 and 1000 m depth at a radius of 24 km. A radial section of velocity reveals two regimes: a solid-body core, surrounded by an outer region where velocity decays exponentially with radius, Fig. 5. The core extends from the lens center to the radius of maximum velocity, \(r_0\). Imposing zero velocity at the meddy’s center, a linear regression between velocity and radius (for radii less than 24 km) has a high correlation coefficient \((r = 0.95)\) and a standard deviation (from the data) of less than 3 cm s\(^{-1}\) at 1000 m. The associated rotation period was 6½ days.

In an attempt to empirically model the decrease in velocity with radius in the outer region, several different representations were explored, including exponential, Gaussian, and \(r^{-2}\) decay. By the method of least squares, the models were combined with Gaussian decay of velocity in the vertical, and fit to the data. The model that best characterized the outer region, from \(r_0\) to the lens’ edge, is an exponential decay of velocity (Schultz Tokos 1989). Over a large depth range (600 m) centered at mid-depth, the transition between the inner core and the outer region is so abrupt that its structure cannot be resolved with the PEGASUS data. Therefore, we will consider it as the juncture between the two velocity regions.

**b. The meddy in 1985**

By acoustically locating the SOFAR floats from aboard ship, the meddy was easily found again in the fall of 1985. During the one year interval, it had drifted 500 km to the south into a background field dominated by Antarctic Intermediate Water instead of the Mediterranean salt tongue. Its decay was obvious from the hydrographic survey; its full extent spanned only 80 km in diameter and 600 m in the vertical. The inner core, now only 350 m thick and 32 km in diameter, still contained large temperature and salinity anomalies, but was not as homogeneous as it was the previous year. Even at its center, there was evidence of mixing with the cooler, fresher waters in which it was embedded. The interior, or core region, was still more weakly stratified than the background field in 1985 (Fig. 3b). The upper and lower surfaces of the meddy were still marked by increased stratification, but the differences with the background field were less than they were in 1984. Beneath the core, the characteristic staircase re-
sulting from salt fingering was evident in the temperature and salinity profiles (Fig. 2). Hebert (1988a) has shown that this type of double diffusion is a significant decay mechanism of the meddy’s core.

The PEGASUS survey in 1985 also showed substantial decay of the meddy as a whole. The contour section in Fig. 6 shows peak (radially smoothed with cubic splines) velocities near 18 cm s⁻¹, at only 16 km radius and between 1000 and 1100 m depth. The velocity signal of the lens extended only to 40 km radius. Radial sections at 200 m depth intervals, Fig. 7, indicate a similar velocity structure as in 1984, but with a reduced vertical extent. At 800 m depth, there is still evidence of a solid-body core, although the transition to exponential decay may be smoother than indicated in the figure. At 1200 m, however, the structure is not as clear, and is further indication of the preferential erosion of the core from underneath. Although there were fewer profiles in 1985 and consequently greater uncertainty in the velocity structure, the data are still consistent with a solid-body core, surrounded by a region in which velocity decays exponentially with radius.

c. The solid-body core

The vertical distribution of angular velocity in the core of the lens was examined using measured velocities within 24 km of the center. Linear regressions of velocity with radius, at 100 m intervals between 500 and 1500 m show high correlation coefficients indicating that the core was rotating as a solid body at each depth interval. The slope of each regression line is proportional to the angular velocity and these are plotted against depth in Fig. 8. From this analysis it is evident that the angular velocity decreased with vertical distance away from the center depth. In other words, rather than the core as a whole rotating as a solid body, its stratification resulted in a continuum or stack of layers, each of which had its own rotation rate.

The analysis of the 1985 data shows that at the mid-depth the rotation rate of the lens did not decrease, and indeed there is even some indication of a slight increase. At other depths, however, the lens had spun down from its 1984 rate. The core was substantially eroded from beneath, resulting in a greater reduction in velocity below than above it.

In other studies of this meddy (Armi et al. 1989; Hebert 1990) the core has been defined in terms of its salinity signal. These authors place the boundary between the core and the surrounding region at the radius where the vertical variations in salinity exceed 0.01 psu. To illustrate the similarity between the horizontal extent of the salinity and velocity signals of the core and their change with depth and time, the variance associated with the radial distribution of salinity was examined. The salinity data for each year were sampled at 100 m intervals and grouped into 5 km bins every 2.5 km. This running mean and corresponding standard deviation of each bin are shown for representative depths in Fig. 9. Radius is nondimensionalized by the radius of maximum velocity, r₀, which coincides with the abrupt change in sign of relative vorticity, \( \xi_r \) (\( \xi_r = \frac{dv}{dr} + \frac{v}{r} \)), and varies with depth. The standard deviation is normalized by the mean salinity anomaly.
inside the core. The striking features of Fig. 9 are the very uniform salinity within the meddy core and the rapid rise in variance near \( r_0 \). We define the maximum in salinity variance as the salinity front, and find in both years this front was at a radius greater than \( r_0 \) at most depths. The same relative positions of these fronts have also been observed in a Sargasso Sea eddy (Elliott and Sanford, 1986a), in warm core rings (Joyce, 1984), and also in Hedstrom and Armi’s (1989) laboratory lenses.

In 1984, the radius of the salinity front varied little between 900 and 1200 m and was near 1.3\( r_0 \). Above the below this depth, the front was closer to the radius of maximum velocity. By 1985, the radius of the salinity front was maximum at 1000 m. The front’s radius decreased with vertical distance away from the mid-depth, both in actual distance and with respect to \( r_0 \). Further, the lens’ cylindrical core had become more toplike in 1985. Both fronts had migrated inward, but the migration rates were the same only over a limited depth range. Below 1100 m the salinity front had moved inward faster, and was inside the radius of maximum velocity.

The slight increase in angular velocity at mid-depth between 1984 and 1985 is intriguing, and to make sure the observation is real, it is estimated from the PEGASUS velocity data using two different methods with independent measurements, and the conclusion is the same. First, velocity was regressed against radius at 1000 m for each year’s data, assuming zero velocity at the center. In 1984, the slope, or rotation frequency was \(-1.14 \pm 0.02 \times 10^{-5} \text{ s}^{-1}\). In 1985, it was \(-1.22 \pm 0.05 \times 10^{-5} \text{ s}^{-1}\), which is significantly higher. At 1100 m, the 1985 value is also significantly higher. The only vertical information shared between the two is through the low-pass filtering process, but similar results were obtained using 100 m averages rather than filtered data.

The second way the rotation rate was calculated uses only PEGASUS profiles taken beyond \( r_0 \), and uses vertical information as well. A least squares regression of the data using an exponential decay of velocity with radius in the outer region and with depth over the entire lens was performed. Only data from radii greater than 24 (16) km were used from the 1984 (1985) observations. The rotation rate was calculated for fluid inside the solid-body core from the exponential fit at \( r_0 \) and zero velocity at the center. The resulting rotation rate was \(-1.10 \pm 0.02 \times 10^{-5} \text{ s}^{-1}\) in 1984 and \(-1.13 \pm 0.05 \times 10^{-5} \text{ s}^{-1}\) in 1985. These means are significantly different at the 87% confidence level (Student’s t-test).

4. Dynamical properties

a. Momentum balance

In many studies the velocity field is estimated from hydrographic surveys of the density field. Since we have measured both, we are able to examine the advantages and disadvantages of the dynamic method of computing currents from density. The relevant balances for this study are those of gradient wind and geostrophy. For two-dimensional, axisymmetric flow, the gradient wind equation can be written following the nomenclature of Johns et al. (1989):

\[
 f\nu + \frac{v^2}{r} = \frac{\partial \Delta \phi}{\partial \theta} .
\]

Here \( v \) is the azimuthal velocity, \( f \) the Coriolis force, \( r \) the radius, and \( \Delta \phi \) is the dynamic height anomaly referenced to 2000 dbar. In straight flow, or where the radius of curvature is very large, the centripetal term, \( v^2/r \), is negligible and the flow is geostrophic:

\[
 f\nu = \frac{\partial \Delta \phi}{\partial \theta} .
\]

The Rossby number (\( Ro = v/f\)) is a measure of the relative importance of the nonlinear terms (in this case \( v^2/r \)) in the momentum equation. From the Rossby number distribution in 1984, curvature effects were inferred to be significant from the center to at least 25 km, and therefore should be considered in the momentum balance. Beyond 25 km, the centripetal effects dropped rapidly to less than 10% of the total. This is not a new result, a cyclogeostrophic core surrounded by a region that is essentially geostrophic was also de-
scribed by Elliot and Sanford (1986b) and D'Asaro (1988). A similar distribution applies to the 1985 survey as well.

The momentum balance, specifically, a comparison between the gradient wind balance and the geostrophic balance, was examined in detail by combining the PEGASUS data with the density data, both from the more extensive survey in 1984. Two approaches were used. One approach magnifies errors, but may reveal some fine structure. The other smooths errors, but sharp transitions are filtered out.

The first is the conventional way of differentiating the pressure field to obtain velocity. This method, in principle, allows us to see as much as possible of the structure of the velocity field. The problem is that there is much variability in the density data due to measurement noise and internal waves. Rossby (1988) was able to measure the amplitude of the internal tide using isobaric drifters trapped inside this eddy. Typically, the peak-to-peak amplitude was on the order of 30 m, but at one time it was 80 m. Such vertical displacements could seriously alias a hydrographic survey done during that time, resulting in a large amount of scatter in the dynamic height field. Fortunately, the direct velocity measurements are not as affected by the tidal currents.

Dick and Siedler (1985) have shown that the rms variation of the combined $M_2$ and $S_2$ tidal currents as measured by a current meter mooring in the Iberian Basin are of the order of 3 cm s$^{-1}$, about half the variability of the observed velocity field. Consequently, tidal currents do not dominate the variability of the measured velocity, and no attempt was made to remove them.

Thirty-five unevenly spaced CTD profiles were used to compute a radial section of dynamic height. The station spacing and dynamic height variability made it necessary to smooth the data before it could be differentiated to obtain velocity. Cubic splines were used in the smoothing. The dynamic height field seemed to be represented adequately by at least three different choices of cubic splines. The corresponding velocity fields, integrated from the gradient wind balance, are shown in Fig. 10a. Without a priori knowledge, it would be difficult to know which smoothed dynamic height field to use, with the consequence that a comparison between the resulting geostrophic and gradient wind velocity fields with the PEGASUS data could be highly subjective.

The second method for assessing the momentum balance is to integrate the velocity field with respect to radius and thus filter out much of the local uncertainty.
The velocity field was integrated in two ways: first according to the geostrophic equation, and then with the effect of curvature added, using the gradient wind balance. The two resulting dynamic height sections were then compared to the measured one to determine which momentum balance was more characteristic. Although the two resulting dynamic height anomalies at the center differ by only 0.6 dyn cm, or 12% of the total signal, the difference is significant, and the gradient wind balance agrees better with the observations (Fig. 10b). It is therefore the more appropriate dynamic balance to use in the core of the lens.

As mentioned earlier, these dynamical balances have been shown to apply in other lenses before. This discussion is included not only to confirm the validity of the gradient wind balance in the core for this lens, but also to emphasize the amount of error and uncertainty that can arise when deriving velocity from density (compounded in higher derivatives)—hence the advantage of direct velocity measurements.

b. Vorticity

Potential vorticity, $PV$, is of particular interest because it is a property of fluid motion that is conserved in the absence of dissipation. It is defined as the component of absolute vorticity which is perpendicular to the density gradient (Ertel 1942) and can be expressed for circular flow as:

$$PV = \frac{N^2}{g} (\xi_r + f) + \frac{1}{\rho} \frac{\partial v}{\partial z} \frac{\partial p}{\partial r},$$

(3)

where $\xi_r$ is the relative vorticity ($\xi_r = dv/dr + v/r$), and $N$ the Brunt–Väisälä frequency. Although this is a scalar quantity, we refer to the first term on the right-hand side as the vertical component, since it arises as the product of the vertical components of the density gradient and absolute vorticity. Analogously, the second term is referred to as the horizontal component with respect to density. The horizontal component is usually of minor importance, and we have found it is at most 5% of the total. Therefore, when discussing potential vorticity, it is approximated by its vertical component. We first consider the individual components of potential vorticity and their change from one year to the next. Then, we examine how these components combine to govern the entire quantity.

The lens moved from 32°N ($f = 7.7 \times 10^{-5} \text{ s}^{-1}$) to 27°N ($f = 6.6 \times 10^{-5} \text{ s}^{-1}$) between October 1984 and fall of the following year. This means $f$, which is the dominant part of absolute vorticity ($f + \xi_r$), was reduced by 15%.

The stratification ($N^2/g$) in the lens was nearly constant and around $0.6 \times 10^{-6} \text{ m}^{-1}$ in 1984, and close to 15% greater in 1985 as shown in Fig. 3. Since the meddy was weakly stratified, there were regions of high stratification above and below the lens, where the background isopycnals were pinched together to accommodate the lens. This is seen in each year, but in 1985 the peak stratification below the core near $c_1 = 32.2$ was rather weak. Along with a general weakening of the lens, two other processes could have contributed to this. One is that the lens moved into ambient waters that were less stratified (by about 15% over the depth range of the meddy). Since the stratification within the lens increased with time, the difference between the depth of an isopycnal inside and outside the lens must decrease. The other process is a result of extensive salt fingering (Hebert 1988a) which is readily apparent in the temperature and salinity profiles (see Fig. 2). The salt fingering resulted in well-mixed salt layers tens of meters thick. Between the steps were very small, O(1 m), transition layers of high stratification. Since $N^2$ was calculated over 40 dbar, the layers of high stratification are filtered out.

At the mid-depth of the lens, relative vorticity of the core in each year was $-0.3f$ and $-0.4f$ respectively, where $f$ is the local Coriolis parameter. At $r_0$ the relative vorticity abruptly changed sign and reached a peak
(0.05f and 0.10f), then diminished to zero at large radii. Within the core around 1000 m, as also shown in Fig. 8, the relative vorticity appears to have decreased (rotation rate within the core increased slightly) between surveys.

Potential vorticity was calculated over the entire lens and is shown in Fig. 11. The uniformly low values in the core each year combined with low stratification is consistent with the concept of solid-body rotation. The strong front near 24 km in 1984 reflects the abrupt change in sign of shear vorticity (\(\partial \nu / \partial r\)) at that radius. Also shown (dotted line) is the radial limit of the core as determined by an iterative method of least squares using only the velocity measurements. The same structure, but on a reduced scale, is seen in the 1985 data. The low values of potential vorticity did not change in the center and indicate that water remained essentially isolated inside the core over that period of time.

Above \(\sigma_1 = 32.0\) (1000 m) within the core, changes in the three major components of potential vorticity balanced in such a way that the total quantity was conserved. Figure 12 depicts changes in PV within the core in detail. Shown are the differences of the components of potential vorticity between years, averaged over the core and expressed as a fractional change from their 1984 values.

In the upper portion of the core above \(\sigma_1 = 32.0\), the overall decrease in planetary vorticity balanced the general increase in stratification, resulting in no net change in PV. Note, however, that the background PV decreased significantly.

In the lower portion of the core, changes in stratification due to its erosion from underneath lead to significant changes in potential vorticity. In 1984 the lower boundary was near \(\sigma_1 = 32.3\), and by 1985 had moved upward to \(\sigma_1 = 32.2\). This is shown by a decrease in stratification at \(\sigma_1 = 32.3\) and an increase at \(\sigma_1 = 32.2\). There was a corresponding change in potential vorticity within the core on these surfaces. The background field, however, showed a decrease at all depths, but with little effect on the core of the lens.

c. Potential and kinetic energy integrals

The available potential energy (APE) of a fluid is the difference between that fluid's total potential energy and the amount it would possess if allowed to adjust adiabatically to a state where the density and geopotential surfaces coincide. Practically, it is not easily defined, and several methods to calculate it have been described in the literature (Reid et al. 1981; Bray and Fofonoff 1981; Hebert 1988b).

The amount of APE is determined relative to a specified reference state. But what is considered to be the reference state may depend on the application. Bray and Fofonoff (1981) show how to calculate it, assuming the observed field is a result of isentropic and adiabatic readjustment of an initial level reference field. This is computationally intensive and would not allow one to observe the difference in APE of the meddy from one year to the next while excluding the effects of the changing background field, which itself possesses APE. Hebert (1988b) shows that for an isolated feature the far field may be used as an approximation to the reference field. This implies that the feature exists in an infinitely wide ocean, and the redistribution of mass over such an area does not affect the far field. Using Hebert's formulation, the effects of the changing background field on the APE of the meddy may be removed. Following Hebert (1988b), APE is defined as

\[
APE = 0.5 \rho \int_V N^2 \pi^2 dV
\]

where \(\rho\) and \(N\) are the density and Brunt–Väisälä frequency of the reference state, and \(\pi\) is the displacement of isopycnals inside the lens from the reference state. The reference state for each year is the mean field as estimated from all CTD stations beyond 80 km from the meddy's center in 1984, and beyond 40 km in 1985.
The total kinetic energy (KE) contained by the meddy is defined as

\[ KE = 0.5 \rho \int_V v^2 \, dV. \]  

First the distribution of the energy density is considered. Figure 2.9 in Hebert (1988a) shows available potential energy was stored near the vertical axis and ±200 m from \( z_0 \), where the displacement of isopycnals is a maximum. The distribution of KE can be inferred from Figs. 4 and 6. The strongest contributions to KE were at mid-depth near the radius of maximum velocity. Thus, the two forms of energy had very different spatial distributions.

The energy densities, both available potential and kinetic, were vertically integrated before calculating the energy over the entire lens' volume. The isopycnal displacement, \( \pi \), used in the APE calculation is with reference to the corresponding mean background field. Of course, there exist small deviations of both signs of the depth of an isopycnal at any single background station from that isopycnal's mean depth. Since this quantity (\( \pi \)) is quadratic in the integral, the mean background vertically integrated APE (APE\(_V\)) is nonzero. We assume this nonzero background APE\(_V\) is a reasonable approximation of the error associated with the APE\(_V\) calculated inside the lens. To compensate for similar fluctuations in density inside the meddy, the mean background APE\(_V\) is subtracted from the meddy's APE\(_V\). Since any errors in isopycnal displacement will overestimate the energy, this approximation is used in an attempt to reduce the overestimate, and results in a 20% reduction in total APE in 1984 and 30% in 1985. To calculate the total energy possessed by the lens, integration was then performed radially, over 60 km for the 1984 data and 40 km for the 1985 data.

Table 2 lists the amount of energy for each year. Over the one year interval, the meddy lost 80% of its total energy (KE + APE), an e-folding rate of 9 months. The average loss rate of APE was \( 1.3 \times 10^6 \) J s\(^{-1}\), and the average dissipation rate of KE was \( 1.6 \times 10^6 \) J s\(^{-1}\). This loss of APE of 0.02% per day is in line with estimates of conversion rates for a salt lens off the Bahamas (McDowell 1982) and for Gulf Stream rings (Olson 1980). The ratio of KE/APE decreased from 1.1 to 0.6 between the two velocity surveys. Hebert et al. (1990) have used simple models to discuss this ratio and its change in comparison with other aspect ratios. It is apparent that oceanic lenses have a complicated history, making it difficult to interpret this ratio.

### 5. Discussion

With the PEGASUS measurements, we are able to distinguish details of this meddy's velocity structure

<table>
<thead>
<tr>
<th>Year</th>
<th>APE (J)</th>
<th>KE (J)</th>
<th>KE/APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>( 75 \times 10^{12} )</td>
<td>( 79 \times 10^{12} )</td>
<td>1.1</td>
</tr>
<tr>
<td>1985</td>
<td>( 21 \times 10^{12} )</td>
<td>( 12 \times 10^{12} )</td>
<td>.6</td>
</tr>
</tbody>
</table>

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unattainable with the hydrographic data alone. In both surveys, one year apart, it consisted of a solid-body core surrounded by a region in which velocity had an exponential form. A dominant characteristic of the radial velocity profiles is the abrupt transition from strong anticyclonic relative vorticity to weak cyclonic at the radius of maximum velocity. This sharp transition is consistent with the core being abraded from the outer edges by intrusions, rather than simple horizontal diffusion, which would result in a much smoother change in sign of relative vorticity. We are unable to resolve the details of the transition with the PEGASUS dataset.

We can use the fact that the transition remains unresolvedly sharp, as it migrates inward over the one year period, to obtain an upper boundary on the eddy diffusivity just outside the velocity maximum. Since the radius of the velocity maximum decreases from 24 to 16 km in one year, there is, in the framework of the velocity maximum, a steady outward flow, $u_0$. This outward advection of undiluted waters with salinity $S$ is mixed with the surrounding, fresher waters. On average the salinity decays from core values to the outside approximately exponentially on a scale of 12 km, estimated from the radial salinity distribution at 1000 m in 1984. The simplest steady-state salt budget balance for this scenario is approximately

$$u_0 dS/dr = A d^2 S/d^2 r$$

(6)

where $A$ is the eddy diffusivity to be estimated. Solving for exponential decay starting at $r = r_0$ and fitting the solution

$$S = S_0 \exp[-(u_0/A)r]$$

(7)

to the salt distribution in Fig. 9 gives $A/u_0 = 12$ km. Since $u_0 = (24 \text{ km} - 16 \text{ km})/400 \text{ days} = 2.3 \times 10^{-4}$ m s$^{-1}$, we obtain $A = 2.8$ m$^2$ s$^{-1}$. This is comparable (actually smaller by a factor 2 to 3) to that estimated by Schmitt and Georgi (1982) for mixing at the 5 km scale along the edge of the Gulf Stream.

In 1984 the radius of transition ($r_0$) was nearly constant with depth above 1000 m and decreased slightly below this depth. The salinity front, defined by the maximum salinity variability, was also at a constant radius, $1.3r_0$, above 1000 m and at a slightly smaller radius below 1000 m. In contrast, the 1985 data show a large decrease in $r_0$ both above and below 800 m. The salinity front was at its maximum distance beyond the vorticity front at mid-depth, and occurred at a smaller relative radius at depths above and below. The larger decrease in $r_0$ and the salinity front with depth in 1985 than in 1984 is evidence of greater erosion occurring beneath the lens than above it. When this erosion is combined with the horizontal decay, the result is preferential chiseling of the lower outer edges of the core. Hebert (1988a) has shown this erosion underneath the eddy is due to extensive salt fingering; the horizontal decay is through lateral intrusions (Ruddick and Hebert 1988).

A curious finding from the PEGASUS data is that the anticyclonic vorticity of the core may have actually increased (more negative) over the survey period between 1000 and 1100 m. The spinup in this limited depth range can be considered in terms of the gradient wind equation [Eq. (1)]. As the lens moved south, $f$ decreased by 15%. For a given pressure (or dynamic height) gradient, the azimuthal velocity would increase. The pressure gradient, however, may or may not have been constant. The inherent scatter in the density data make it difficult to evaluate $\partial \Delta \phi/\partial r$ independent of the velocity measurements, so its local value is not well known. To test whether the observed increase in angular velocity can be seen in the density field, the magnitude of $\partial \Delta \phi/\partial r$ was estimated using the velocity measurements and the gradient wind equation for each year at $r = 15$ km. The resulting dynamic height gradient was $1.27 \times 10^{-8}$ dyn m/m in 1984, and was $1.19 \times 10^{5}$ dyn m/m in 1985, a decrease of 6%. The 15% decrease in $f$, offset by a 6% decrease in dynamic height gradient could still support a 9% increase in velocity (or rotation rate). The observed increase in anticyclonic rotation rate was on the order of 4% and is not inconsistent with these estimates of $\partial \Delta \phi/\partial r$.

The low, constant potential vorticity within the core of the lens both years, particularly above 1000 m, suggests the core remained isolated from the surrounding waters. Even in the lower portion, where PV was not conserved between 1984 and 1985, its changes were governed by variations in the local stratification resulting from decay of the lens, not by a decrease in the background field. These changes suggest that fluid parcels within the upper portion of the core remained isolated, whereas those in the lower portion did not. This is consistent with Hebert's (1988a) finding that double diffusive processes are more effective beneath the eddy core than above it.

The coincidence of the potential vorticity front with the radial limit of the high salinity waters in the core is consistent with nondissipative flow in which fluid parcels must conserve both potential vorticity and water mass properties. The strong potential vorticity front at the radius of maximum velocity indicates that there was little exchange between the outer region and the core waters, otherwise the potential vorticity field would have been substantially smoothed or homogenized. This is also illustrated in the salinity field. At $r < r_0$ at mid-depths, the salinity is nearly constant, indicating that there is little stirring of fresh waters inward of the velocity maximum, i.e. $r_0$ is substantially a water boundary as well. In addition, observations of SOFAR floats (Richardson et al. 1989) show that core waters could be traced for at least two years. (This was fortunate if not remarkable considering the fact that there was very little salt left in the eddy by then!)

Between the two surveys, the front had migrated from 24 to 16 km from the eddy's center as a result of "abrasion" at the edge of the core. Beyond the front,
potential vorticity surfaces leveled out and were dominated by the stratification, allowing isopycnal mixing with the ambient waters.

We know that the lens survived for approximately two years after it was first found. But it is difficult to project the meddy back to its time and place of formation to estimate its total lifespan. If we assume that the lens maintained its average speed of 1.8 cm s\(^{-1}\) (Richardson et al. 1989) and that it was formed off the Iberian Shelf, a formation site proposed by Käse et al. (1989), it would have taken 2 years traveling westward for it to reach 32°N, 22°W, where it was first found in 1984. A mechanism is then necessary to cause a change in mean direction to due south, as was observed for the subsequent 2 years. If this scenario were true, the meddy must either have been much larger than any others that have been observed when it was first formed, or its decay rate was originally much less than it was between 1984 and 1985, or its advection rate changed significantly. Another suggested formation region is in the vicinity of the Azores current (Käse and Zenk 1987) near 35°N, which is much closer to, and to the north of the meddy’s location in October 1984. Assuming the same translation speed would imply an age of 7 months when it was first found.

The dynamics of each of these regions may be conducive to lens formation, but the \(T-S\) properties indicate an alternate source region may be necessary. The Azores current water is not as saline as the meddies observed. Further, the Iberian Shelf Mediterranean Water has a double maximum in salinity which this meddy did not. Thus, meddies with different characteristics are likely formed in different regions.

6. Summary

Absolute velocity measurements in a meddy revealed details of the velocity field and its changes over one year that cannot be resolved from hydrographic surveys alone. The meddy consisted of two velocity regimes: a weakly stratified core in near solid body rotation and a surrounding region in which velocity decayed exponentially with radius.

The meddy core rotated with a depth-dependent rotation period near 6 days at its mid-depth (1000 m). Although horizontal intrusions abraded the core’s radius from 24 to 16 km, there was no decrease in rotation rate over one year at this mid-depth. Spindown was evident above and below the mid-depth, most significantly below where double diffusive processes had their greatest effect (Hebert 1988a). At mid-depth in the lens there was evidence for an increase in negative angular velocity.

The other region, although still anomalously warm and saline, was riddled with intrusions of the background waters, which had also decreased the total radius of the meddy from 60 to 40 km. The PEGASUS measurements showed a sharp transition between the inner core of strong anticyclonic relative vorticity and the outer region of weaker cyclonic velocity shear.

This vorticity front essentially kept the core isolated from the outer region. Potential vorticity was conserved within the uncertainty of the estimate (15%) in the upper confines of the core. In the lower portion of the core, changes in potential vorticity were governed by changes in stratification. Although the background vorticity decreased significantly as the lens moved 500 km to the south, it had little effect on the core of the lens.

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