A Model of Tidal Rectification by Potential Vorticity Mixing.  
Part I: Homogeneous Ocean*

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(Manuscript received 19 November 1996, in final form 4 October 1998)

ABSTRACT
In previous studies of tidal generation of mean flow over varying topography, the rectification mechanism has generally invoked bottom friction as a source of tidal flux of momentum and vorticity (hence referred as “friction” mechanism). The author proposes a different mechanism based on horizontal mixing of potential vorticity. Drawing analogy from tidal dispersion of passive tracers, this mixing is parameterized through a diffusivity (hence called “diffusivity” mechanism) that is quadratic in the tidal amplitude. In this, Part 1, the mean along-isobath flow near a shelf break is determined for a homogeneous ocean and contrasted with that induced by friction mechanism. In Part 2, the effect of a front will be considered.

It is found that although the mean flow is pointing in the same direction as that induced by friction mechanism (i.e., to the right when facing deep water in the Northern Hemisphere), it varies more slowly with the tidal amplitude. In the typical situation when the mean shear is small compared with the Coriolis parameter, this dependence is linear rather than quadratic, as is the case for the friction mechanism. This linear dependence compares more favorably with observations over Georges Bank.

1. Introduction
There have been many theoretical studies of tidal generation of mean flow over varying topography. Earlier models are formulated in momentum balance (Huthnance 1973; Loder 1980, henceforth Loder80), and two mechanisms have been identified, both invoking bottom friction as the source of the tidal (Reynolds) stress. One mechanism stems from direct frictional drag on the tidal velocity, and the resulting mean flow may point in either direction, depending on properties of the tidal ellipse in the far field. The other is due to frictional retardation of the Coriolis acceleration, and the mean flow is always directed to the right when facing deep water (in the Northern Hemisphere). Loder80 argued that this latter mechanism—called “friction” mechanism here—is the dominant one over Georges Bank, which can explain the observed clockwise circulation.

This friction mechanism has also been elucidated through vorticity balance (Zimmerman 1978; Robinson 1981; Young 1982; Maas and Zimmerman 1987), which has provided additional insight. In essence, because of conservation of potential vorticity (PV), fluctuating PV is generated as tidal current crosses depth contours. Without friction, this fluctuating PV lags the tidal current by exactly a quarter cycle, thus producing no PV flux when averaged over tidal cycle. But with friction, the phase of PV is slightly advanced so that a flood (ebb) tide is partly in phase with a negative (positive) PV, resulting in a net offshore flux of PV. For a submarine bank, for example, this outward flux of PV would induce an anticyclonic circulation around the bank (i.e., clockwise in the Northern Hemisphere) so as to be balanced by the cyclonic frictional torque acting on the circulation.

Besides theoretical studies, primitive equation numerical models have been used to simulate flow over Georges Bank (Greenberg 1983; Lynch and Naime 1993; Chen and Beardsley 1995). Forced only by tides, these models have nevertheless produced a mean clockwise circulation consistent with observation. While the friction mechanism is obviously operative in these models, whether it is the dominant mechanism in driving the mean flow is more difficult to establish. This is because these models do contain horizontal dimensions and, as we shall demonstrate, there is an additional mechanism stemming from horizontal advective processes that may contribute to tidal rectification.

In fact, the previous reliance on bottom friction as the only source of PV flux seems not fully consistent with what we know about a different but nonetheless

* Lamont–Doherty Earth Observatory Contribution Number 5779.

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related problem: the tidal dispersion of tracers. In this latter problem, as discussed further below, it is generally the horizontal advective processes, rather than vertical friction, that dominate the dispersal of tracers. Since PV is conserved like tracers, it should be subject to similar mixing; since such mixing invariably leads to nonzero mean flow when the water depth is not uniform, one wonders if PV mixing may constitute a viable mechanism for tidal rectification. It is this question that motivates the present study.

For the organization of the paper, section 2 contains the hypothesis of PV mixing, followed in section 3 by derivation of model equations. The solution is discussed in section 4 and compared in section 5 with observations over Georges Bank. In section 6, the model results are summarized and additional discussion is provided.

2. Diffusivity hypothesis

Tidal dispersion of passive tracers is a subject that has been extensively studied in the past, as reviewed recently by Geyer and Signell (1992). While bottom friction may contribute to horizontal mixing through shear dispersion, horizontal advective processes are generally needed to account for the observed efficiency of dispersion. As an example of these processes, Zimmerman (1976) considered the interaction of tidal current with residual eddies—the latter presumably the product of irregular topography—and found that, even with weak eddies, particle trajectories are nevertheless chaotic, thus highly effective in mixing water mass properties. Approximating particle trajectories over individual tidal cycles as consisting of “random” walks, Zimmerman was able to derive a tidal diffusivity \( \kappa \) of the form

\[
\kappa = b U l,
\]  

where \( U \) is the amplitude of the tidal current, \( l \) the tidal excursion (\( \sim U/\omega \) with \( \omega \) being the tidal frequency), and \( b \) a dimensionless “mixing” coefficient signifying effectiveness of tidal mixing. This mixing coefficient increases with strength of the eddies, but it has a maximum when eddy (or topographic) scale is comparable to the tidal excursion (Zimmerman 1986). Zimmerman (1981) argued in a separate paper that maximization of the tidal power may very well equilibrate the topographic scale to tidal excursion over geological times, thus maximizing the mixing coefficient. Using typical strength for residual eddies, he estimated a mixing coefficient of order 0.1, a value large enough to account for tidal dispersion observed in the western Dutch Wadden Sea.

While preexisting residual eddies provide a useful tool for formulating the tidal dispersion, it may not be assumed a priori for study of tidal rectification since it is precisely the residual motion that needs to be explained. On the other hand, Zimmerman (1986) acknowledged that the residual velocity field is not an essential ingredient of the dispersion process since there are many other factors that may randomize Lagrangian trajectories and enhance mixing. Examples include spatial variation of the tidal current (Ridderinkhof and Zimmerman 1992) and small-scale turbulence (Awaji 1982). In fact, since the essence of Lagrangian chaos lies in the nonlinearity of Eulerian to Lagrangian transformation, any complication in forcing field and/or external condition would enhance the horizontal mixing. In a coastal environment, there are many such complications, including wind-driven motion, density stratification, and coastal trapped waves; we shall assume, therefore, that the mixing coefficient \( b \) in (2.1) is “saturated” in that it no longer depends on the tidal amplitude. This translates to a tidal diffusivity \( \kappa \) that is quadratic in the tidal amplitude, a dependence which has been widely used in the past (Geyer and Signell 1992) but is more firmly justified by Zimmerman’s theory.

Properties that are mixed are the ones that are conserved with fluid motion, the randomness of which smooths their distribution over larger scale. But in addition to water mass properties, there is a dynamical property, namely PV, that is conserved and hence mixed. In the case of PV, however, one perceives possible complications because the fluid motion that does the mixing is coupled to the property being mixed. This coupling however is weak since there is a scale separation of the two: the mixed PV, by the definition used here, is referring to a spatial average over randomized fluid motions. In fact, it is this same scale separation that underlies the concept of tidal diffusivity of tracers: That is, over the scale of tidal excursion, the tracer concentration may remain highly heterogeneous, being dominated by conservation, and only its spatial average over a distance of many tidal excursions can be said to be homogenized. With this definition of mixing, it is obvious that diffusivity is not a useful concept in addressing property distribution over the scale of tidal excursions, as noted by Geyer and Signell (1992).

Young (1987) has discussed the theoretical basis for PV mixing and its parameterization through diffusivity. In deriving homogenization of PV, he has stipulated, however, that PV be conserved with mean fluid motion, a condition that is unnecessarily restrictive. Numerical calculations, on the other hand, have shown PV homogenization under more general conditions (e.g., Rhines and Young 1982), which provides a support for PV mixing hypothesized here. Moreover, since it is the same advective process that mixes both tracers and PV, it is assumed that the same tidal diffusivity as given by (2.1) applies to the PV field. The obvious questions are then: What is the mean flow resulting from such diffusion of PV? How does it differ from the friction-induced mean flow? And, can the two be differentiated from observation? These are the questions that will be addressed from this study.
3. Model formulation

To elucidate the basic mechanism, I shall consider a homogeneous ocean overlying a straight topography representative of a shelf break, as sketched in Fig. 1. A right-handed Cartesian coordinate system is used with x, y, and z axes directed offshore, along-isobath, and upward, respectively, and the origin is set at the shelf break.

Since the fluid is homogeneous, one may assume the flow—outside a thin bottom boundary layer—to be vertically uniform so that a potential vorticity q may be defined as

$$q = \frac{1}{h} \cdot (f + \mathbf{k} \cdot \nabla \times \mathbf{v}),$$

where h is the water depth, f the Coriolis parameter (taken to be positive), and v the velocity. Since we are concerned with mean flow driven by tides, a tidal current imposed in the far field provides the only forcing of the model. For simplicity, we assume that the condition is uniform along isobaths and the tidal current is normal component with local amplitude U.

Averaging over a tidal cycle and neglecting free-surface displacement, this equation becomes

$$\frac{dq}{dt} + \nabla \cdot (hvq) = -\frac{k}{h} \cdot \nabla \times \left(C_p U \frac{v}{h}\right).$$

Eq. (3.6) can be integrated once to yield

$$huq' = -C_p U \frac{\bar{\nabla}}{h},$$

or, there is a local balance between PV flux and bottom stress divided by the water depth.

Equation (3.8) also contains the essential balance of the friction mechanism, and the difference of the two lies only in the next step when one considers the PV flux on the lhs. For the friction mechanism, it is the bottom friction that alters the quadrature phase relation between u' and q' to facilitate this flux; in the diffusivity mechanism, on the other hand, it is the randomness of the Lagrangian trajectories that causes this flux. As discussed in section 2, this flux can be parameterized in terms of the mean field as

$$u'q' = -\kappa \frac{dq}{dx},$$

where \(\kappa\) is the tidal diffusivity given by (2.1) and \(\bar{\nabla}\) is the mean potential vorticity given by [from Eq. (3.1)]

$$\bar{\nabla} = \frac{1}{h} \left(f + \frac{d\bar{\nabla}}{dx}\right).$$

To isolate the controlling parameter, let us nondimensionalize variables in accordance with following scaling rules: the horizontal distance x by L (the width of the slope region), depth h by its maximum value H, mean...
velocity $\vec{v}$ by $fL$, and the mean potential vorticity $\overline{\vec{q}}$ by $fH$. To simplify expressions, we shall henceforth drop overbars and use subscript $x$ to denote derivative. In their nondimensional form, (3.8)–(3.10) combine to [using also (2.1) and (3.3)]

$$q_s = \alpha v, \quad \text{(3.11)}$$
$$q = h^{-1}(1 + v_s), \quad \text{(3.12)}$$

where

$$\alpha = \frac{C_{D}L}{bH} \frac{L}{\ell}, \quad \text{(3.13)}$$

with $\ell$ being the tidal excursion in the far field, $\alpha$ is a friction parameter that measures the importance of bottom friction relative to horizontal mixing. The two equations, (3.11) and (3.12), can be combined to yield a single equation governing the mean flow $v$:

$$v_{xx} - h^{-1}h v_s - \alpha v = h^{-1}h_s, \quad \text{(3.14)}$$

which is to be solved subjected to the boundary condition (3.7) that

$$v \to 0 \quad \text{as} \quad x \to \pm \infty. \quad \text{(3.15)}$$

It is seen that besides the topography, the frictional parameter $\alpha$ is the only parameter that governs the solution behavior, and it is through this parameter that tidal amplitude enters the problem.

One notes the similarity of (3.14) to Eq. (31) of Loder80 although they are based on very different mechanisms. This difference is reflected most prominently in how tidal amplitude enters the equation: the coefficient preceding the mean flow (the third term on the l.h.s) varies as $-1$ power of the tidal amplitude, while in Loder80, it varies as $-2$ power. One expects therefore that the resulting mean flow has different functional dependence on the tidal amplitude, as will be discussed later. Another difference is the simpler matter through which topography enters the equation, which facilitates analytical solution as derived later.

4. Solution

To understand the solution behavior, I shall first consider the asymptotic limits of strong and weak friction before presenting the general solution.

a. Strong-friction regime ($\alpha \gg 1$)

In this limit, one seeks a solution to (3.14) of the form

$$v \approx -\alpha^{-1}h^{-1}h_s. \quad \text{(4.1)}$$

The approximation is valid if, when substituted into (3.14), the neglected terms (the first two terms) are indeed small. This would be the case if topography is sufficiently smooth and the water depth is not too shallow, which are assumed to be the case. Since water deepens offshore (i.e., $dh/dx > 0$), the mean flow is negative, or directed to the right when facing deep water. While this is in the same direction as that predicted by the friction mechanism, the underlying cause is different. To explain the diffusivity mechanism, one notes first of all that this is the limit when mean shear is small compared with the Coriolis parameter so that PV decreases offshore with increasing water depth. The horizontal mixing of PV by tides thus tends to decrease PV of the shallower water, or there is effectively an offshore flux of PV, the same sign as that induced by friction mechanism (see section 1). With the PV flux balanced by the same bottom stress (3.8), the mean flow is thus directed in the same direction.

Written in dimensional units, (4.1) becomes

$$v^* = -\frac{b}{C_{D}L \frac{dx^*}{\ell}} \frac{dh^*}{dx^*}, \quad \text{(4.2)}$$

where the asterisk symbols indicate dimensional variables. In this strong-friction regime, the mean flow is thus proportional to the local topographic slope, as that induced by friction mechanism. In contrast, however, its dependence on the tidal amplitude—through the tidal excursion $l^*$—is linear rather than quadratic [see Eq. (29) of Loder80]. This difference can be understood by a closer examination of (3.8): With the friction mechanism, the tidal flux (l.h.s) is caused by bottom friction so that the resistance coefficient $C_{D}U$ enters linearly both sides of the equation and cancels out; since aside from this factor the tidal flux is quadratic in the tidal amplitude, it thus drives a mean flow of the same dependence. In the present formulation, on the other hand, tides enter the l.h.s only through a diffusivity that is quadratic in the tidal amplitude [see Eqs. (3.9) and (2.1)]. This functional dependence is thus partly canceled by the tidal amplitude appearing on the r.h.s, resulting in a mean flow that is linear in the tidal amplitude. As will be discussed in section 5, this qualitative difference between the two predictions may be used to differentiate the two mechanisms from observational data.

b. Weak-friction regime ($\alpha \ll 1$)

As $\alpha$ decreases, the mean flow obviously may not increase indefinitely since the first two terms in (3.14) eventually become important. To assess their effect, let us consider the asymptotic limit of $\alpha \ll 1$. Since the width of the slope enters the expression of $\alpha$, this limit corresponds to the step topography considered in Loder80. The finite width of the slope under the present scaling, however, renders the following derivation slightly more complicated. For simplicity, let us assume that the bottom is level outside the slope, as sketched in Fig. 1.

Over the flat regions, Eq. (3.14) becomes

$$v_{ss} - \alpha v = 0, \quad \text{(4.3)}$$

which has exponential solution with $e$-folding distance
of $\alpha^{-1/2}$. Subjected to boundary conditions (3.15), it is easy to see that

$$v_x = \alpha^{1/2}v \quad \text{for} \quad x \leq 0,$$  \hspace{1cm} (4.4)

and

$$v_x = -\alpha^{1/2}v \quad \text{for} \quad x \geq 1.$$  \hspace{1cm} (4.5)

Over the slope, the smallness of $\alpha$ renders $q$ uniform, or

$$q = C,$$  \hspace{1cm} (4.6)

a constant yet to be determined. Using definition (3.12), one has then

$$v_x = C h_0 - 1 \quad \text{at} \quad x = 0,$$  \hspace{1cm} (4.7)

with $h_0$ being the depth of the shelf, and

$$v_x = C - 1 \quad \text{at} \quad x = 1.$$  \hspace{1cm} (4.8)

Eliminating $v_x$ from (4.4), (4.5), (4.7), and (4.8) yield

$$\alpha^{1/2}[u]_x = C(1 + h_0) - 2,$$  \hspace{1cm} (4.9)

where the bracket indicates the difference across the slope region. There is another constraint on $[u]_x$ that can be derived by multiplying (4.6) with $h$ and integrate, which results in

$$[u]_x = C \int_0^1 h \, dx - 1.$$  \hspace{1cm} (4.10)

Comparing (4.9) and (4.10) and realizing that $\alpha$ is small, one obtains, to the lowest order,

$$C = 2(1 + h_0)^{-1},$$  \hspace{1cm} (4.11)

which gives the value of PV over the slope region. Since $C$ is $O(1)$, one infers from (4.7) and (4.8) that $v_x$ is of $O(1)$ over the slope as well. Since $v$ and $u$ are continuous across the edge of the slope, Eqs. (4.4) and (4.5) then imply that $v$ is of $O(\alpha^{-1/2})$ over the slope. More precisely, using (4.11) one derives from (4.4) and (4.7) that

$$v_{\text{max}} = -\alpha^{-1/2}(1 + h_0)^{-1}(1 - h_0) + O(1).$$  \hspace{1cm} (4.12)

When compared with the asymptotic solution for strong friction (4.1), one sees that the dependence on the tidal amplitude has reduced from being linear to one-half power. That is, as tide intensifies, the rate of increase of the rectified flow slows. Since Loder80’s solution for a step topography shows that the mean flow varies linearly in the tidal amplitude, the present power dependence thus is half that for the frictional mechanism as for both asymptotic limits.

c. General solution

Having considered the asymptotic limits of large and small $\alpha$, the general solution to (3.14) that is uniformly valid in $\alpha$ will now be considered. To facilitate analytical solution, let us consider a slope flanked by flat regions and the depth profile in the slope region is exponential

$$h = e^{s(x-1)} \quad \text{for} \quad 0 < x < 1,$$  \hspace{1cm} (4.13)

where $s$ is related to the shelf depth $h_0$ by

$$s = -\ln h_0.$$  \hspace{1cm} (4.14)

Substituting (4.13) into (3.14), one obtains, over the slope region,

$$v_{ss} - sv_x - \alpha v = s$$  \hspace{1cm} (4.15)

and, over the flat regions, Eq. (4.3). These are ordinary differential equations with constant coefficients and hence can be easily solved, subject to boundary conditions (3.15) and the matching of $v$ and $v_x$ across edges of the slope region. The solution is presented in the appendix and is plotted in Fig. 2 for $h_0 = 0.2$ and a few selected values of $\alpha$. Also plotted are the asymptotic solution (4.1) for $\alpha = 10$ (dashed line) and the asymptotic value (4.12) for $\alpha = 0.1$ (the dash-dotted line).

For $\alpha = 10$, the mean flow, as expected, approaches the asymptotic solution of the strong-friction regime but with the discontinuity at the breaks smoothed by diffusivity over a distance of $O(\alpha^{-1/2}) \approx O(0.3)$. Reducing $\alpha$ by half (e.g., tide is twice as strong) yields a mean flow that is intensified by about the same factor, and the mean flow is further smoothed out around the topographic breaks. As $\alpha$ is further reduced, the above tendency continues, but the rate of increase for the mean flow slows. When $\alpha$ is reduced to 0.1, one approaches the asymptotic solution of the weak-friction regime.

5. Comparison with observations

There are extensive current measurements over Georges Bank off the northeastern United States. The flow there is characterized by strong tides, as well as a well-documented clockwise gyre around the bank. Although this gyre exhibits seasonal variation, it is a year-round feature that has been attributed as tidally driven (Loder80; Greenberg 1983). Furthermore, from current meter records over the northwestern slope of the bank, Magnell et al. (1980) found a strong correlation between the subtidal current and tidal amplitude. Since it is diff-
ficult to conceive of an external forcing that elicits such a common response from both tidal amplitude and mean flow—for one thing, the latter is direction-specific while the former is not—this observation provides a strong support for the tidal origin of the subtidal flow.

To compare the model solution with observation, one needs to estimate the value of \( \alpha \), which contains an uncertain mixing coefficient \( b \) [Eq. (3.13)]. Zimmerman (1986) estimated it to be of \( O(0.1) \), a value large enough to account for the observed tracer dispersion in Dutch Wadden Sea. While the topography is smoother over the open shelf of the Georges Bank, there are also other forcings, such as wind-driven motion, stratification, and coastal waves that may enhance horizontal mixing. For lack of relevant studies, we shall therefore use the same value of 0.1 for the mixing coefficient. As we shall see, qualitative features discussed below do not depend on a precise value of \( b \), and only its order of magnitude estimate is needed. While Georges Bank has a curved boundary, its radius of curvature is large compared with width of the slope; neglect of the curvature effect in the model may thus be justified. Based on Loder80 (his Table 1, the northwestern side), I shall use 10 km for the width of the slope, and 200 and 40 m for the depth of the two flat regions flanking the slope (hence the nondimensionalized shelf depth is 0.2). From Fig. 3b of Magnell et al. (1980), one sees that the tidal amplitude (half of crest to trough) varies between 20 and 40 cm s\(^{-1}\). Since water depth at the measuring site is 85 m, this translates through Eq. (3.3) to a tidal amplitude in the far field (of depth 200 m) that varies between about 10 and 20 cm s\(^{-1}\), or a tidal excursion between 1 and 2 km (for the semidiurnal tide). Additionally, using \( C_p = 2 \times 10^{-3} \), one arrives at a value of \( \alpha \) between 10 and 5 over the range of the observed tidal amplitude. The mean flow for this tidal range thus should fall between the upper two curves of Fig. 2.

One notes first of all that although tides are quite strong, the model solution still falls in the strong-friction regime, for which the asymptotic solution (4.1) approximately holds. The maximum mean flow has a non-dimensional speed of 0.1 and 0.2, which translates to a dimensional speed between 10 and 20 cm s\(^{-1}\), which is of the right order of the observed mean flow. Since the mean flow induced by friction mechanism can have similar magnitude (Loder80) and since both formulations contain parameters that can be tuned (perhaps more so for the diffusivity mechanism), this agreement with observation does not provide a critical test of the two mechanisms.

On the other hand, the functional dependence of the mean flow on the tidal amplitude is less sensitive to the value of these uncertain parameters. Based on our scaling discussed above, the mean flow over Georges Bank should fall in the strong-friction regime, so that this dependence is linear. For Loder80’s solution based on frictional mechanism, the dependence decreases from being quadratic to linear when the slope becomes a step. Although it is difficult to be more precise without detailed calculation, the parameter values Loder80 has used (his Table 1) certainly do not qualify the slope as a step, and a cursory examination of his coefficients [his Eq. (35)] suggests that this functional dependence should be closer to being quadratic than linear. What is observational evidence?

Magnell et al. (1980) have examined the correlation between subtidal flow and tidal amplitude, and noted their linear dependence, as clearly shown in their Fig. 3. In fact, this linear relation holds so well that the observed along-isobath current—the sum of the subtidal and tidal components—returns to almost exactly zero at the ebb tide, regardless of the tidal amplitude. This intriguing feature has not been previously explained but appears to be consistent with the diffusivity mechanism proposed here.

6. Summary and discussion

I have proposed here a mechanism of tidal rectification based on horizontal mixing of potential vorticity. Drawing analogy from tidal dispersion of tracers, this mixing is parameterized through a diffusivity that is quadratic in the tidal amplitude. The mean flow produced by this diffusivity mechanism is pointed in the same direction as that caused by friction mechanism, but its power dependence on the tidal amplitude is about half that of the latter. For the tidal regime over Georges Bank, in particular, this dependence is linear rather than quadratic (the case for friction mechanism), which seems to compare more favorably with observation.

While the present mechanism is likely to be operative in the complicated coastal environment, whether it is contained in numerical models mentioned in section 1 is less clear. As the diffusivity mechanism is premised on tidal random walk, the models need to at least resolve the scale of tidal excursion, which is only marginally met in Lynch and Naimie (1993). Despite this limitation, it is nevertheless interesting to note that, when they reduce the grid size by half (from 6 to 3 km), there is a 40% strengthening of the mean flow. Since the tidal current—having much larger spatial scale—is already well resolved by their coarser grids, one does not expect the mean flow induced by the friction mechanism to be changed much. On the other hand, such an increase of resolution would enhance tidal random walk and hence the diffusivity mechanism, which provides a plausible explanation of their results.

When they further reduce the grid size to 2 km (Lynch et al. 1995), the solution seems to have converged and there is no additional strengthening of the mean flow, which may indicate saturation of the mixing as hypothesized in section 2. One way to check this interpretation is to incorporate random topography of typical amplitude to see if indeed the rectified flow remains unchanged. If one is assured of this incorporation of the diffusivity mechanism in the model, then the relative
importance of the two mechanisms in the model may be assessed. This can be done by examining the spatial structure of the mean flow and tidal amplitude to determine their functional relation. Another simple test is to vary the bottom drag coefficient and see how the mean flow varies. From the discussion in section 4, the mean flow induced by a friction mechanism is relatively insensitive to such variation, but it varies inversely with the drag coefficient according to the diffusivity mechanism.

Acknowledgments. This work is supported by the National Science Foundation under Grants OCE97-24697 and OPP93-13700. I want to thank anonymous reviewers for their helpful comments that have improved the substance and presentation of the paper.

APPENDIX

General Solution

The equation to be solved is (4.15):

\[ v_{xx} - s v_x - \alpha v = s, \]  

(A.1)

with \( s \) being a nonzero constant over the sloping region and \( s = 0 \) over the flat regions. Subject to the boundary conditions (3.15), the general solutions are given by

\[
\begin{align*}
A \exp(\sqrt{\alpha} x), & \quad x < 0 \\
-\frac{s}{\alpha} + B_1 \exp(m_1 x) + B_2 \exp(m_2 x), & \quad 1 > x > 0 \\
C \exp[-\sqrt{\alpha}(x - 1)], & \quad x > 1,
\end{align*}
\]

(A.2)

where

\[ m_{1,2} = \frac{s}{2} \left( 1 \pm \left( 1 + 4\alpha/s^2 \right)^{1/2} \right). \]  

(A.3)

The four unknown constants \( A, B_1, B_2, \) and \( C \) are determined by matching \( v \) and \( v_x \) across \( x = 0 \) and 1. Defining

\[
\begin{align*}
p_1 &= (\sqrt{\alpha} - m_1) \\
q_1 &= (\sqrt{\alpha} - m_2) \\
p_2 &= (\sqrt{\alpha} - m_1) \exp(m_1) \\
q_2 &= (\sqrt{\alpha} - m_2) \exp(m_2) \\
r &= s/\sqrt{\alpha} \\
W &= p_1 q_2 - p_2 q_1,
\end{align*}
\]

(A.4)

the above unknown constants are then given by

\[
\begin{align*}
B_1 &= W^{-1} r(q_2 - q_1) \\
B_2 &= -W^{-1} r(p_2 - p_1) \\
A &= -s/\alpha + B_1 + B_2 \\
C &= -s/\alpha + B_1 \exp(m_1) + B_2 \exp(m_2),
\end{align*}
\]

which uniquely specify the solution.

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