An Oceanic Wake in the Equatorial Undercurrent Downstream from the Galapagos Archipelago

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ABSTRACT

In the Pacific Equatorial Undercurrent downstream (east) from the Galapagos archipelago, an unusual meander pattern was observed in the spring of 1967. Two separate hypotheses present themselves as explanations for the observed wake phenomenon. The wake may have been a variation of the familiar von Kármán wake, or it may have been a form of the Rossby wake, only recently discussed by White. Through a scale analysis, both hypotheses are found to be reasonable, and both give characteristic length scales (500 km) that agree well with the observed wavelengths. A fundamental difference between the two hypotheses is that the Rossby wake is stationary, while the von Kármán wake is time-dependent. However, the time scale for eddy shedding in a von Kármán wake is found to be on the same scale (2 months) as the length of the cruise that observed the wake phenomenon. Therefore, it appears that the observed oceanic wake may have had characteristics of both the von Kármán and Rossby wakes.

1. Introduction

In the investigation of the distribution of low oxyty water in the eastern equatorial Pacific Ocean by White (1971c), the eastward extension of the Equatorial Undercurrent around the south side of the Galapagos archipelago can be observed to have formed a curious meander pattern downstream (eastward) of the archipelago. The existence of a meander pattern in this situation suggests two hypotheses of formation. First, it may have been associated with the formation of a von Kármán wake (von Kármán, 1911) in the Equatorial Undercurrent downstream from the Galapagos archipelago which straddles the equator near 90W. Second, it may have been a Rossby wake of the type discussed by White (1971a), wherein it was shown that a train of stationary Rossby waves can form in the lee of a cylindrical island placed in a uniform eastward flow on the beta plane.

Concerning the first hypothesis, at least two previous descriptive studies have observed von Kármán wakes in the lee of oceanic islands. The circulation patterns downstream from two small Pacific islands (10 km diameter) were investigated by Van Dorn *et al.* (1967) with the use of both Lagrangian and Eulerian direct current measurements. He observed what appeared to be a stable von Kármán wake in the lee of these islands, but his results were inconclusive. More concretely, Barkley (1972) has recently observed by direct means the existence of a stable von Kármán wake downstream from Johnston Atoll (26 km diameter) in the central North Pacific. With regard to somewhat larger islands, on the scale of the Galapagos archipelago, Patzert (1969) has observed a number of mesoscale eddies in the lee of the Hawaiian Islands. However, Patzert believed the eddies to have been generated by a wind field of unusual vorticity resulting from the orography of the Hawaiian Islands, and not a part of any von Kármán wake.

Of course, if the von Kármán hypothesis were used to account for the meanderings observed downstream from the Galapagos archipelago, then most certainly they would not take the form of the traditional wake pattern observed in the laboratory since the meridional velocity profile of the undercurrent upstream from the archipelago is non-uniform.

Concerning the second hypothesis, to date no one has observed a Rossby wake in the lee of any oceanic islands. However, the unique position of the Galapagos archipelago situated in the path of the eastward flowing Equatorial Undercurrent strongly suggests that the observed oceanic wake was of the Rossby-wake type. As with the von Kármán phenomenon, the non-uniformity of the upstream ambient flow makes it difficult to specify the exact nature of the wake pattern.

The apparent controversy that has arisen here concerning the origins of the wake meander pattern is similar to one concerning the meander patterns contained in the subarctic frontal zone east of Japan. In a descriptive study by Barkley (1968) the subarctic frontal zone, bounded by the eastward extensions of both the Kuroshio and Oyashio currents, is shown on occasion to conform to an oceanic von Kármán com-

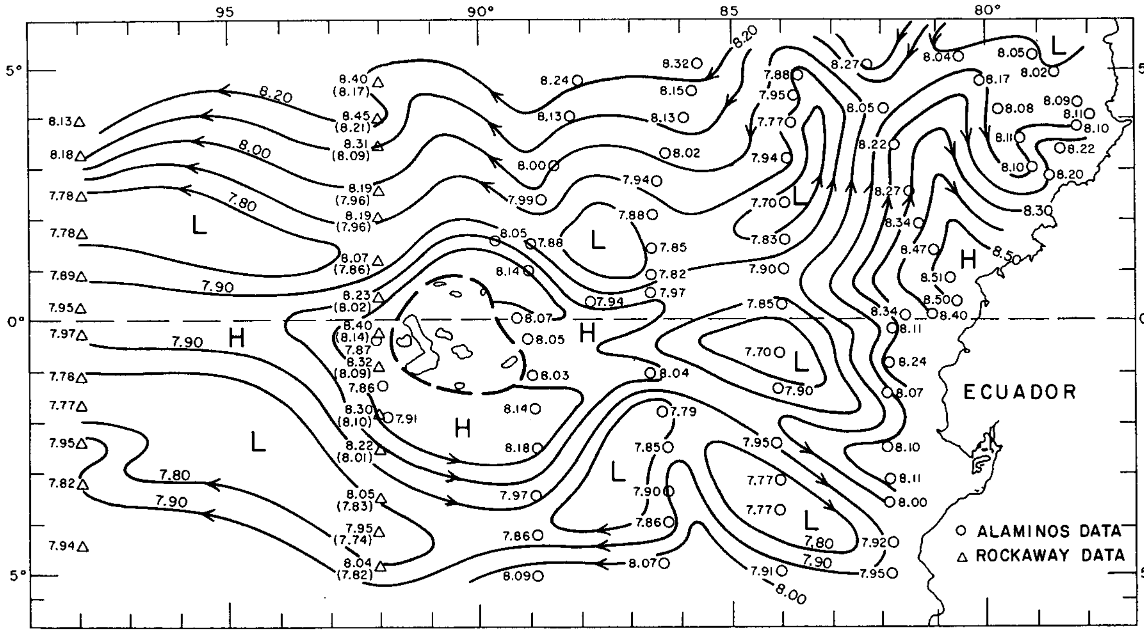

FIG. 1. Acceleration potential (dynamic decimeters) relative to 500 db on the surface where the thermosteric anomaly equals 300 cl ton^{-1} in the vicinity of the Galapagos archipelago.

pound vortex street (Fig. 2), the dynamics in this case remaining obscure. On the other hand, Moore (1963) has constructed a mathematical model on the beta plane that develops a stationary train of damped Rossby waves superimposed upon the eastward current extending across the ocean from the western boundary. Moore believes that the meander pattern found in the Kuroshio-Oyashio extensions east of Japan may be a quasi-steady-state phenomenon of this type.

In the present study, the description of the oceanic wake phenomenon in the region of the eastern equatorial Pacific Ocean by White (1971c) is improved upon by considering EASTROPAC data upstream (west) from the Galapagos archipelago as well as downstream. Then, based upon parameters contained in the descriptive results, a scale analysis is applied to show that both types of wakes, von Kármân and Rossby, are theoretically possible, in which case the oceanic wake observed in the Undercurrent downstream from the Galapagos archipelago can be thought of as containing characteristics of both wake phenomena.

2. Description of the oceanic wake pattern

The data necessary to fully describe the circulation pattern in the vicinity of the Galapagos archipelago was collected by two ships, the *R/V Alaminos* and the USCGC *Rockaway*, during the EASTROPAC studies in the spring of 1967. The positions of their respective stations are shown in Fig. 1 with the *Alaminos* stations lying east of the Galapagos archipelago and the *Rockaway* stations located to the west. Both ships were in the vicinity of the archipelago at about the same time with three stations, located near the equator along 92W, overlapping spatially and differing in time by only two weeks.

From these data, the acceleration potential (Montgomery and Stroup, 1962) relative to 500 db has been computed and its distribution at an isanosteric surface of 300 cl ton^{-1} is contoured in Fig. 1. The 300 cl ton^{-1} surface was chosen because White (1969) found the highest eastward geostrophic speeds of the Equatorial Undercurrent on this surface. The geostrophic velocity is proportional to the gradient of the acceleration potential for a particular latitude, although the geostrophic relationship within 2° latitude of the equator becomes tenuous. Even so, the eastward geostrophic speeds on the south side of the Galapagos archipelago at 3N can be observed to have been approximately 50 cm sec^{-1}. This value compares well with other evaluations observed by direct means. As an example Knauss (1966) found maximum eastward speeds of 70 cm sec^{-1} just west of the Galapagos archipelago at 94W on the equator. More recently, Stevenson and Taft (1971) found to the east of the archipelago at 84W eastward speeds of $\sim$40 cm sec^{-1}.

The initial values of *Rockaway* acceleration potential along the section at 92W appear to have been unusually higher ($\sim$0.30 dynamic decimeters) than corresponding values either east or west of that section, as can be seen in Fig. 1. From the temperature-depth relationships (not shown here) for each of the three overlapping stations along 92W, the high values of the *Rockaway* acceleration potential can be attributed to the greater depth of the thermocline compared to that given by the *Alaminos* data. To bring the values of acceleration potential along the 92W section back into line with those of the other meridional sections a sequence of corrective measures were taken. First, the two values of acceleration potential at each of the three overlapping stations were averaged; next, the averaged values were divided by the *Rockaway* acceleration for those three stations, from which an averaged quotient was obtained; then, the remaining values of *Rockaway* acceleration potential over the rest of the section were multiplied by the averaged quotient to bring them into line with those of the other meridional sections. The resulting values of acceleration potential are contained in parentheses in Fig. 1 and were used in contouring the field of acceleration potential.

In Fig. 1, the Galapagos archipelago is enclosed by a dashed line which approximates the 200 m isobath. The archipelago therefore can be observed to have an effective radius of ~ 120 km and appears to be displaced ~ 30 km south of the equator.

At the westernmost meridional section (98W) in Fig. 1, the eastward flow of the Equatorial Undercurrent straddled the equator with a half-width of ~ 225 km and was clearly surrounded on both the north and the south sides by the westward flow of the South Equatorial Current. East of the Galapagos archipelago, the presence of the meridional boundary of South America caused the eastward flow of the Undercurrent to form into two extensions that turned poleward near 84W and returned much of the water of the Undercurrent to the west at 3N and 5S as the South Equatorial Current. Furthermore, in the region downstream from the Galapagos archipelago, a wake formed consisting of a train of eddies that lay between the eastward extension of the Undercurrent (on both the north and south sides of the equator) and the South Equatorial Current. The wavelength of the meanders associated with these two eddy trains were in the range 400–600 km.

Separating the two eastward extensions of the Equatorial Undercurrent was a low region that nearly straddled the equator. Because of its proximity to the equator, its presence is difficult to interpret. However, suffice it to say that it was in association with the fact that the Galapagos archipelago bifurcated the Equatorial Undercurrent into two separate branches, one north and the other south of the equator. This result is corroborated by the findings of Cochrane and Zuta (1969), who observed the high oxyty core of the Undercurrent on an isanosteric surface of 180 cl ton^{-1} to bifurcate and follow the paths indicated by the acceleration potential shown in Fig. 1.

Of particular interest in the descriptive analysis is the degree to which the circulation pattern was in steady state. In constructing this circulation pattern an assumption is made that the lack of synopticity in the data does not interfere with obtaining a reasonable circulation pattern. The *R/V Alaminos* cruise east of the archipelago lasted for approximately three months in 1967 from 21 January to 10 April, during which time the USCGC *Rockaway* completed its measurements west of the archipelago. Therefore, the circulation pattern can be considered to be quasi-stationary over at least three months. Even so, the possibility that the circulation pattern changed significantly during these cruises cannot be ruled out completely.

3. Testing the von Kármán wake hypothesis

To see if the von Kármán wake hypothesis is a reasonable one for the formation of the meander patterns downstream from the Galapagos archipelago, the eddy Reynolds number in the vicinity of the archipelago is compared to known laboratory results. Rouse (1963) reported from the laboratory that for uniform flow U_0 streaming past a cylindrical object of radius a, a stable von Kármán wake consisting of a regular vortex street formed downstream from the object if the Reynolds number,

$$\mathrm{Re}=\frac{U_0 2a}{\nu}, \tag{1}$$

was in the range $50<\mathrm{Re}<2500$, where ν is the molecular kinematic viscosity. For flow conditions where $10<\mathrm{Re}<50$ the wake pattern did form but it consisted of a stationary eddy pair located directly in the lee of the obstacle. For flow conditions where $\mathrm{Re}<10$ a wake pattern does not form, approximating the potential flow situation. For flow conditions where $\mathrm{Re}>2500$, the von Kármán wake became unstable and no discernible vortices could be found in the turbulent wake.

To compare the laboratory results of Rouse (1963) with the flow around islands on a planetary scale, the molecular kinematic viscosity ν in (1) is related to the other laboratory-scale parameters as the eddy kinematic viscosity (ν') is related to the planetary-scale parameters. The validity of this analogy is in doubt, but Fultz (1961) showed that the molecular viscosity in the laboratory plays the role of small-scale eddy viscosity in planetary situations.

In addition to the criterion given by Rouse for the existence of a stable von Kármán wake, Lin (1959) found an additional criterion for stability, i.e., whenever

$$\gamma=\frac{\nu N}{U_0^2} \tag{2}$$

fell within the range $10^{-3}<\gamma<2.5\times10^{-3}$, where N is the frequency of eddy shedding and γ is the ratio of the Strouhal number to the Reynolds number. As with the Reynolds number, the molecular viscosity in (2) is replaced by the eddy viscosity for planetary-scale applications.

Application of the planetary-scale analogs of Re and γ to previous observational studies yields values that fall within the proper range for the formation of stable von Kármán wakes. As an example, Van Dorn *et al.* (1967)

observed von Kármán wakes in the lee of two small (10 km diameter) islands imbedded in quasi-uniform currents of approximately 50 cm $\sec^{-1}$. The computed eddy Reynolds number (for $\nu' = 10^6$ cm^2 $\sec^{-1}$) in this situation is $\mathrm{Re} = 100$, which lies in the lower range for the formation of a stable von Kármán wake.

Van Dorn does not have enough information to calculate γ, since he did not observe the frequency of shedding. However, Barkley (1972) has observed eddy shedding in the lee of Johnston Atoll. Like Van Dorn, Barkley obtains an eddy Reynolds number ($\mathrm{Re} = 70$) in the lower range for the formation of a stable von Kármán wake. This eddy Reynolds number is calculated using as the eddy viscosity a value obtained from the use of Lin's criterion for von Kármán wake stability. Barkley calculates ν' to be 2.2×10^6 cm^2 $\sec^{-1}$ (where $U_0 = 60$ cm $\sec^{-1}$, $N = 2.9 \times 10^{-6}$ $\sec^{-1}$), utilizing $\gamma = 1.8 \times 10^{-3}$ which is the central value in the criterion for the formation of a stable von Kármán wake given by Lin (1959). Furthermore, this value of ν' compares well with values similarly estimated by Chopra and Hubert (1965), and Lyons and Fujita (1968) for atmospheric von Kármán wake patterns. The period (N^{-1}) separating the shedding of eddies having similar (positive or negative) circulation was observed by Barkley (1972) to have been approximately 4 days.

To calculate the eddy Reynolds number and eddy γ in the case where the Equatorial Undercurrent streams past the Galapagos archipelago, a flow speed that has been both meridionally and vertically averaged will be used. In Section 2, the width of the Undercurrent west of the archipelago was found to be approximately 450 km, nearly twice as wide as the effective diameter of the archipelago, 240 km. Although the maximum speed of the Undercurrent upstream from the archipelago cannot be obtained from the data given, the geostrophic speed measured south of the archipelago suggests the maximum speed to have been approximately 50 cm $\sec^{-1}$. If the Undercurrent can be considered to have a parabolic meridional distribution of velocity and to have a similar profile with depth, then it can be shown that the effective mean speed intercepting the Galapagos archipelago lies near 15 cm $\sec^{-1}$.

Considering the previously obtained effective mean speed and allowing $\nu' = 10^6$ cm^2 $\sec^{-1}$, the eddy Reynolds number for the present situation is approximately 360. In this situation, the Reynolds number is certainly in the range for the formation of a stable von Kármán wake given by Rouse (1963).

In addition, if it is assumed that γ exists within the proper range of values given by Lin (1959), then the longest period (T) between successive eddies is approximately 50 days (for $\gamma = 10^{-3}$, $\nu' = 10^6$ cm^2 $\sec^{-1}$). This value of T is on the same order of magnitude as the length of observation (70 days). Therefore, if the mean upstream velocity can be considered to have been approximately 15 cm $\sec^{-1}$, then this time period result supports the idea that the oceanic wake in the lee of the Galapagos archipelago may have been a von Kármán wake.

To further confirm the von Kármán wake hypothesis, appeal is made to some laboratory experiments by Birkhoff and Zarantonello (1957) who observed the ratio of the upstream flow (U_0) to the drift (U_e) of the von Kármán vortices within the wake to be

$$\frac{U_0}{U_e} = 1.2. \tag{3}$$

From this result, the characteristic spacing between eddies can be calculated as

$$L = U_e T \approx 500 \text{ km}, \tag{4}$$

for $T = 50$ days and $U_0 = 15$ cm $\sec^{-1}$. This value of vortex spacing is well within the range ($400 < L < 600$ km) observed in Fig. 1.

In Section 2, it was noted that the Galapagos archipelago bifurcated the Equatorial Undercurrent into two branches, one north and the other south of the equator. As such, the type of wake formed in such a flow situation might be the compound vortex street previously discussed by Barkley (1968) in connection with the wake pattern observed in the Kuroshio and Oyashio extensions. Fig. 2 gives a schematic representation of the compound vortex street (from Barkley, 1968) and, by comparing this to the oceanic wake observed east of the Galapagos archipelago in Fig. 1, a good deal of similarity may be seen.

In addition, Barkley (personal communication, 1972) has suggested the possibility that the anticyclonic eddy pair, located adjacent and slightly downstream from the Galapagos archipelago (Fig. 1), may be the manifestation of a stationary eddy pair that has been observed to form in the laboratory for $10 < \mathrm{Re} < 50$ (Rouse, 1963). Barkley speculates that the cyclonic vorticity contained

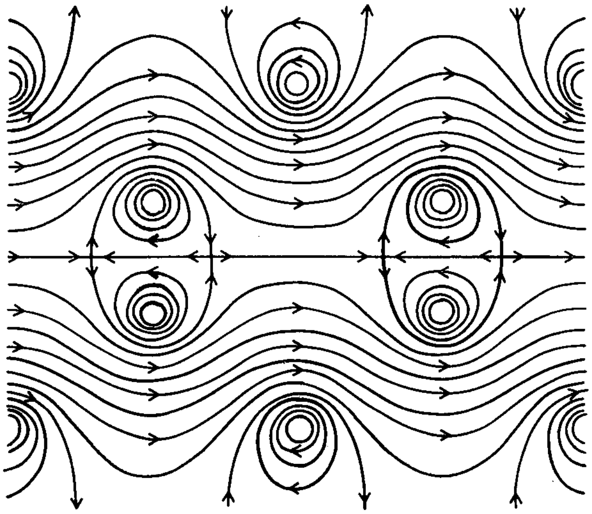

FIG. 2. Von Kármán compound vortex street.

in the upstream ambient flow of the Undercurrent might suppress the formation of anticyclonic von Kármán vortices, so that the effective eddy Reynolds number would be lowered from its calculated value of 360 to a value that would be commensurate with the formation of a stationary eddy pair. Extending this hypothesis, Barkley further contends that the streamline enclosing the archipelago and the stationary eddy pair could be treated, in the ideal case, as a solid boundary. In this case, a Rossby wake might be expected to form farther downstream in response to this virtual obstacle.

4. Testing the Rossby wake hypothesis

Earlier White (1971a) found that a circular cylinder of radius a, positioned on the earth's surface in a uniform eastward flow U_0, would produce a downstream wake, called a Rossby wake, due to the spherical shape of the rotating earth (approximated by the beta plane). White established that for Island numbers

$$\mathrm{Is}=(\beta a^2/U_0)^{\frac{1}{2}} \tag{5}$$

less than unity the wake would not appear, but that a potential flow situation would prevail. In a later paper (White, 1971b), the stationary Rossby wake was found to be unstable for $\mathrm{Is}>1.5$. Therefore, the criterion for a stable stationary Rossby wake is that the Island number be within the range $1.0<\mathrm{Is}<1.5$.

If, as in the previous section, the effective mean speed of the Undercurrent can be considered to be 15 cm sec^{-1}, then the Island number for the Galapagos archipelago would be $\mathrm{Is}=1.4$. In this situation a stable stationary Rossby wake would be expected to form in the Equatorial Undercurrent downstream from the Galapagos archipelago. For uniform flow past a cylindrical island, Fig. 3 (taken from White, 1971b) displays the streamfunction distribution for the situation where $\mathrm{Is}=1.4$.

To further confirm the credibility of the Rossby wake hypothesis in this situation, the wavelength (L) of the meander contained in the wake can be calculated from Rossby wake theory (White, 1971a). Formally L is given by

$$L=2\pi\left(\frac{U_0}{\beta}\right)^{\frac{1}{2}}, \tag{6}$$

which yields a wavelength of 500 km. This value for L is close to what is actually observed in Section 2 and is nearly the same as the eddy separation distance obtained in the von Kármán wake theory for $U_0=15$ cm sec^{-1}.

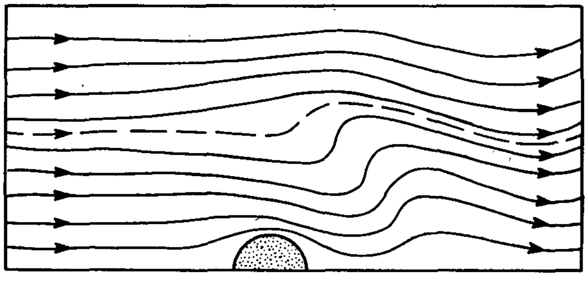

FIG. 3. Rossby wake meander pattern ($\mathrm{Is}=1.40$).

5. Discussion

If the effective speed of the Equatorial Undercurrent, averaged vertically and meridionally, can be considered to be ~15 cm sec^{-1}, then both the von Kármán wake and the Rossby wake have meanders with similar wavelengths (~500 km). Moreover, this value for the wavelength matches that observed in the oceanic wake east of the Galapagos archipelago. If the effective speed of the Undercurrent were greater than 15 cm sec^{-1}, then it might be possible to make a determination as to whether one kind of wake dominates the other. The reason for this is that as the ambient upstream flow U_0 increases, the Rossby wake contains larger wavelengths but the von Kármán wake contains shorter ones.

The Rossby wake is stationary, but the von Kármán wake is time-dependent. If the time period with which eddy shedding of von Kármán vortices took place was much shorter than the length of time the cruise used to investigate the wake pattern, then the observed circulation pattern might be expected to smooth out the wake, unless the sampling scheme (cruise transect) happened to alias the rapidly moving eddy trains to longer periods. However, for ambient flow speeds of 15 cm sec^{-1}, the time scale (50 days) for eddy shedding is nearly the same as that (70 days) of the length of the cruise. Therefore, the cruise might be expected to have been carried out rapidly enough so that the oceanic wake observed can be considered as a quasi-steady von Kármán wake. Moreover, there is nothing to guarantee that the circulation pattern in Fig. 1 is from a quasi-steady ocean; although the different kinds of data (i.e., oxyty, salinity, acceleration potential) support each other in defining a reasonably coherent picture of the circulation pattern.

As a result of the scale analysis, both the Rossby and von Kármán wake hypotheses seem equally possible. If the two phenomena can be considered to be linearly superimposed upon one another, then a picture forms of a stationary Rossby wake with a train of von Kármán eddies propagating downstream with a speed, given in (6), slightly less than the ambient speed U_0. Such a picture would fit in nicely with what is observed in Fig. 1; however, such a hypothetical picture ignores some of the problems inherent in a complex situation of this kind.

A major problem concerns the fact that the Equatorial Undercurrent is not a uniform current extending to infinity in the meridional direction, as is true in both the von Kármán and Rossby wake theories. This nonuniformity of the ambient flow most certainly leads to some interesting variations of the wakes observed in the laboratory. As an example, Barkley (personal communication, 1972) has suggested that the cyclonic flow of the Undercurrent might inhibit the development of anticyclonic von Kármán vortices. The non-uniformity of the ambient flow is particularly critical for the Rossby wake, wherein stationary waves cannot exist in the westward flow of the South Equatorial Current that borders the Undercurrent.

Another aspect of the problems that need attention is the difficulty with working so close to the equator, wherein the isopleths of acceleration potential do not approximate streamlines. As a result, the circulation pattern within 1°–2° of the equator is obscure and can lead to false concepts about the real nature of the wake patterns.

Due to the complexity of the situation, one possible approach to resolve the basic physics of the oceanic wake in the Equatorial Undercurrent east of the Galapagos archipelago is to use a numerical simulation that includes the effects that contribute to the formation of both kinds of wakes. In addition, a descriptive approach to determine whether the wake system is time-variant or stationary would do much toward establishing the physics important to the formation of the oceanic wake.

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