Spiral Eddies

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ABSTRACT

Small cyclonic spiral eddies with a scale of 10 km are very frequently observed, both from satellites and space shuttles, at the ocean surface. The authors suggest that they are the sea surface signature of ageostrophic baroclinic instabilities. The spiral-like cyclones generated and evolving in this baroclinic model regime are found to be consistent with the observed spatial and temporal scales. Both modeled and observed spiral eddies are associated with streaks of strong cyclonic shear and convergence. The numerical experiments presented indicate that spiral eddies are restricted to the very upper ocean, and that they are a source of kinetic energy for the larger scale flow.

1. Introduction

a. Spiral eddies

Spiral eddies are a distinct sea surface pattern. Up to the 1980s, they were considered to be rare dynamic features in the ocean. Photographs of the world’s oceans from space shuttles (Scully-Power 1986; Munk et al. 2000), and images of Norwegian coastal waters from radar satellites (Dokken and Wahl 1996), have shown that these submesoscale eddies are indeed common. A spiral eddy street made visible by the sun’s reflection from the surface of the sea is shown in Fig. 1.

Surface convergence compacts naturally occurring surface active material into slicks. Both Synthetic Aperture Radar (SAR) and sun-glitter images of the ocean surface have a rather broad distribution of sea slicks, or streaks, with a spacing of the order of 1 km and a width of the order of 100 m (cf. Fig. 1). The streaks are domains of reduced surface roughness, making them visible to the remote instruments. It is the larger-scale arrangement of these streaks that forms the images of the spiral eddies. A small minority of the streaks are shear lines. On the SAR and glitter images they can be identified when there is a crossing ship wake. The wake is then dislocated across the shear line consistent with a large cyclonic shear. In several cases (e.g., Scully-Power 1986; Munk et al. 2000) the shear has been estimated to be of the order $10^{-3}$ s$^{-1}$, ten times the magnitude of the planetary vorticity $\Omega$ at midlatitude. Munk et al. (2000) find the shear lines to be exclusive to the spirals: “. . . spiral features, unlike other surface features, reveal strong cyclonic displacements of ship tracks.” The lines seem to persist, still tracing out spirals, in SAR images at somewhat greater wind speeds when the regular slicks have been suppressed (e.g., Dokken and Wahl 1996; Johannessen et al. 1996). We thus associate these shear lines with the eddies.

Spiral eddies have been observed in many oceans in both hemispheres and are found both in coastal areas and in the open ocean. They generally appear in an interconnected pattern, but it should be noted that observed patterns are frequently somewhat more random than that of Fig. 1 (cf. Scully-Power 1986; Munk et al. 2000). Scully-Power (1986) describes spiral eddies to be always cyclonic, with a diameter of 12–15 km, while in the European Remote Sensing Satellite SAR observations of Dokken and Wahl (1996) from the Norwegian Coast, 85% of the eddies are cyclones and 15% anticyclones. The average diameter of the anticyclonic eddies was found to be more than three times that of the cyclones, which was estimated to be approximately 7 km. Munk et al. (2000) collected more than 400 photographs taken by astronauts containing spiral eddies, and found them to be “10–25 km in size and overwhelmingly cyclonic.” Still, there seem to exist no in situ measurements of the spirals, and only one observation of temporal evolution. Flament and Armi (1985) show spirals to evolve with a timescale of $O(f^{-1})$ in a series of three infrared (AVHRR) images of upwelling filaments off Monterey. The reader is referred to Munk et al. (2000) for further observational characteristics and an historic account on the discovery of spirals.

In summary, spiral eddies are submesoscale cyclonic eddies. The slicks that trace them out are associated both

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Fig. 1. Photograph of a spiral eddy street in the Mediterranean Sea off the coast of the Egyptian/Libyan border (32.0°N, 26.0°E, north is upward), from Scully-Power (1986). The diameter of the eddies is roughly 10 km.

with convergence and large cyclonic shears. Observations from space platforms suggest spiral eddies to be plentiful in the world’s oceans.

b. Spiral models

The horizontal scale of spiral eddies is in the low range of the internal radius of deformation in the ocean. Adding to this their cyclonic appearance makes a compelling argument that the phenomenon is influenced by the rotation of the earth, baroclinic and ageostrophic. Ageostrophy is consistent with the timescale of Flament and Armi (1985), and will be argued to account for the spirals’ cyclonic preference.

Density fronts are associated with both baroclinicity and shear lines of relatively strong cyclonic vorticity. The generation of spiral eddies through cyclogenesis at oceanic density fronts could be a common denominator of the observations. Both Scully-Power (1986) and Munk et al. (2000) believe strong cyclonic shears to be a characteristic of the spirals. Dokken and Wahl (1996) find eddies to be “situated exactly at the border between, in all probability, two different water masses” in several of their images. A large fraction of the space shuttle observations is from the Mediterranean. A special case is found in the Alboran Sea between Spain and Algeria. At the same location where spiral eddies were observed from the shuttle, the later in situ measurements of Tintore et al. (1988) found there to be an intense density front, the Almeria–Oran Front. In laboratory experiments on the circulation of the western Alboran Sea gyre, Gleizon et al. (1996) find similar spirals to develop along the frontal limit of the Atlantic jet. The horizontal density stratification between the Norwegian Coastal Current and the North Atlantic Current does in general display a wide range of mesoscale activity (e.g., Sætre and Mork 1981). Even though spiral eddies are frequently observed, it seems that surprisingly little effort has been put into finding a model for their generation and life. We are aware only of the contributions of Eldevik and Dysthe (1999) and Munk et al. (2000). In their nonlinear, three-dimensional numerical experiment, Eldevik and Dysthe (1999) use as the initial condition a slightly disturbed Margules-type front. Initially, small-scale frontal disturbances grow according to the linear ageostrophic predictions of Iga (1993). In the nonlinear stage of development, the frontal wave winds up cyclonically into spirals [cf. Fig. 5 of Eldevik and Dysthe (1999)]. It has a clear similarity to the eddy street in Fig. 1. Associated with the curl-up is a relatively strong convergence that accumulates passive surface floats in the spirals. The instability is found to be predominantly baroclinic and the dimension of the spirals is of the order of 10 km. The simulation demonstrated how a front with cyclonic shear will curl up into cyclonic spiral eddies having the right dimension. The question of why cyclonic shear seems to dominate in ocean fronts is not addressed, and is simply taken as a fact as static stability requires the shear across a Margules-type front to be cyclonic.

Munk et al. (2000) offer a model for the generation of spiral eddies in addition to their observational material. The preference for cyclonic shears is addressed. They present a sequential process consisting of two essentially two-dimensional stages. First there is an ageostrophic frontal preconditioning phase, producing strong cyclonic shears. Then they assume a shear (i.e., barotropic) instability to wind up spirals in the established shear zone. The vertical frontal preconditioning follows the semigeostrophic model of Hoskins and Bretherton (1972), and the horizontal windup the “cat’s eyes” vortex solution of Stuart (1967).

Separating the problem into these two distinct phases, Munk et al. (2000) give the reader an appealing image of the generation of ocean spirals. What is excluded, however, are the details on the transition between, and the possible coexistence of, the two phases. In meteorology, sharp fronts and cyclones are understood to be generated in the same baroclinic instability process (e.g., Garnier et al. 1998). As an unstable frontal wave evolves, there is both sharpening of horizontal gradients perpendicular to the wave and nonlinear windup of the wave to produce cyclones. An estimate of the preferred length scale of the corresponding oceanic frontal waves is required in order to apply such a model to the spiral eddies.
c. This contribution

In this paper the whole process is modeled with a nonlinear, three-dimensional numerical ocean model. A variety of analogous experiments were conducted. The initial geostrophic flow of the reference experiment has a weak horizontal stratification with a corresponding wide surface velocity jet. There is no external forcing present. This system is unstable, yielding a growing wave. The preferred wavelength of the instability is investigated in particular and is, indeed, found suitable as a horizontal restriction for the spiral eddies. This unstable wave is responsible both for frontogenesis and the development of spirals. In agreement with Eldevik and Dysthe (1999), but contrary to the instability model of Munk et al. (2000), the generation and life of spiral eddies are found to be of baroclinic nature. For large periods of the simulations, the kinetic energy of the larger scale flow is found to feed on that of the eddies.

This paper is organized as follows: in section 2, ageostrophic baroclinic instability and frontogenesis are argued to be likely to set the scene for the generation of spiral eddies. The expected horizontal and temporal scales of the linear instability problem are addressed in particular. Numerical experiments that are set up accordingly, are described in section 3. In section 4 the results of the experiments are discussed, and a possible life cycle for spiral eddies is described. The concluding remarks of section 5 sum up the paper.

2. Ageostrophic baroclinic instability

Frontal shear lines were associated with the spiral eddies in the previous section. The existing models (i.e., Eldevik and Dysthe 1999; Munk et al. 2000) suggest that an instability winds up a density front to produce the spirals. This line of research is continued herein. A model of spiral eddies as the sea surface signature of ageostrophic baroclinic instability is sketched below.

a. Cyclonic fronts

Going back to the classical papers of Charney (1947) and Eady (1949), frontogenesis is understood to take place through baroclinic instability. Their linear stability analyses, performed within a quasigeostrophic (QG) framework, predict unstable frontal waves on a scale consistent with the synoptic weather. It is well known that the strong horizontal velocity shear associated with fronts is predominantly cyclonic. This is not captured by a QG formulation due to its restriction on scales; QG requires that the characteristic timescale of the dynamics is much larger than $f^{-1}$, or equivalently that the internal radius of deformation is much larger than the inertial length scale. Ageostrophy must be taken into account when modeling the cyclonic preference of frontogenesis. This is a main point in our model for the spiral eddies.

The ageostrophic asymmetry between cyclonic and anticyclonic shear is often explained using the conservation

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \mathbf{u} \cdot \nabla Q = 0$$

of the Ertel potential vorticity (PV)

$$Q = -\frac{1}{\rho_0} (\xi + f k) \cdot \nabla \rho.$$

Prior to any frontogenesis, $Q = -(f/\rho_0) \partial \rho / \partial z > 0$ is a good approximation. Using the thermal wind relation, we then obtain

$$\rho_0 \partial Q = -g \rho_0 f |\nabla_h \rho|^2 - (f + \xi) \frac{\partial \rho}{\partial z} \geq 0.$$  

In the above, $\xi = (\xi, \eta, \zeta)$ is the curl of the velocity field $\mathbf{u} = (u, v, w)$, $\rho$ is the density field, $\rho_0$ is a constant reference density, and $g$ is gravity. Subscript $H$ refers to the horizontal part of the vector quantity in question. It then follows from Eq. (3) that

$$f + \xi > -\frac{g}{\rho_0 f} |\nabla_h \rho|^2 \left( \frac{\partial \rho}{\partial z} \right)^{-1} \geq 0,$$

provided static stability. This gives a lower bound $(\zeta = -f)$, but no upper bound for the vorticity $\zeta$. Large horizontal density gradients are thus expected to produce large cyclonic shear. See also Gill (1982, p. 575) for a two-dimensional and semigeostrophic argument similar to the above.

b. Submesoscale instability

The scale of spiral eddies is submesoscale, the very low range of the oceanic dynamical equivalent to the atmospheric synoptic weather. We will in the following infer that unstable frontal waves emerging from buoyant geostrophic flows are prone to produce small-scale “weather systems” in the ocean. This gives estimates for the length and growth rate of the most unstable frontal wave compatible with the spiral observations.

As a basic model we take an outcropping, rather weak, stratification in geostrophic equilibrium with a wide surface velocity jet flowing in the $x$ direction. See Fig. 2 for an example. No external forcing is assumed. Instability can only take place through the net release of kinetic or potential energy of the initial flow to unstable motion. Let $\lambda$ be the wavelength of the most unstable perturbation growing along the jet. The instability leads to frontogenesis, and the front may eventually break and curl up into eddies whose horizontal dimension $D$ should be restricted by $\lambda$. As baroclinicity has been argued to be important, a reasonable estimate would be

$$R < D < \lambda,$$

where $R = \sqrt{g H f}$ is the internal radius of deformation,
FIG. 2. The initial geostrophic flow. Solid lines are isopycnals given in \( \sigma \) units. The broken contours are lines of constant velocity in units of m s\(^{-1}\). The fundamental length scale of baroclinic processes. Quantities \( H \) and \( g' = g \Delta \rho / \rho_0 \) are the depth of the initial jet and the reduced gravity, respectively, with \( \Delta \rho \) being the density difference over \( H \). Other dimensional parameters characterizing the jet are its half-width \( L \) and its maximum velocity \( U \) found at the surface center.

For the initial configuration of the numerical experiments described in section 3 [Eqs. (14)–(16); see also Fukamachi et al. (1995)] and illustrated in Fig. 2, it is readily shown that on a suitable nondimensional form, the only independent dimensionless parameters governing the linear stability are the Burger number

\[
Bu = \frac{R^2}{L^2},
\]

and the Richardson number

\[
Ri = \frac{R^2}{L_0^2},
\]

where \( L_0 = U/f \) is the inertial length scale. The so-called \( \pi \) theorem of dimensional analysis (e.g., Barenblatt 1996) then gives a relation of the form

\[
\frac{\lambda}{R} = \phi(Bu, Ri)
\]

for the most unstable frontal wave. This is equivalent with assuming that \( \lambda \) depends only on the three other horizontal length scales at hand, \( \lambda = \lambda(L, R, L_0) \).

In the appendix we take into account some relevant analytical and numerical findings of several authors on ageostrophic (i.e., 1/Ri nonnegligible) baroclinic instability. The results of their linear analyses can also be described using \( Bu \) and \( Ri \) only. This provides a basis for an explicit estimate to the relation (8). An equivalent exercise is done for the growth rate of the most unstable wave, \( \gamma \). The explicit formulas thus found predict \( \lambda \sim 25 \) km and \( \gamma^{-1} \sim 24 \) h at midlatitude.

This section adds up to the following model for the generation of spiral eddies: buoyant geostrophic flows as seen in Fig. 2 are prone to submesoscale, ageostrophic baroclinic instabilities. A process of frontogenesis and cyclogenesis is expected to take place when the unstable wave evolves. As it enters the nonlinear regime, ageostrophy should assure a preference for strong cyclonic shears that on a timescale \( \gamma^{-1} \) wind up spiral eddies whose dimensions is restricted by \( \lambda \). The numerical experiments of the next sections are found to support this conceptual model.

3. The numerical experiments

In a previous paper (Eldevik and Dysthe 1999), we investigated numerically the nonlinear windup of a sharp Margules-type front from an unstable frontal wave. We demonstrated how cyclonic spiral eddies were formed that had the right horizontal dimension. Thus, starting from a Margules-type front, the dominance of cyclonic shear in sharp fronts was taken as a fact and used in the initial configuration. In the present paper we investigate the instability of an initial situation where a front has not yet been formed and where no asymmetry between cyclonic and anticyclonic shear has been ini-
tialized. A model for the generation of spiral eddies from this instability was put forward in the previous section. Describing frontogenesis and cyclonic eddies evolving from such initial conditions is not a novelty in numerical ocean modeling (see, e.g., Wang 1993; Samelson and Chapman 1995), but neither of these authors describe actual spirals and instabilities at the small scale sought herein.

a. The numerical model

The primitive equations, \( \sigma \)-coordinate numerical model of Berntsen et al. (1996) was used for the experiments. Only a few comments will be made on the ocean model at hand. There is no splitting between barotropic and baroclinic modes. Only one time step, \( \Delta t \), is used. The evolution of the free surface is solved fully implicit, and is therefore not restricted by the CFL criterion. Surface gravity waves are filtered out by the solver for the chosen \( \Delta t \), thus the surface elevation corresponds to the pressure anomalies that would be present at that boundary if replaced by a rigid lid. The fields are advected with a so-called “superbee” flux limiter. Yang and Przekwas (1992) show in their review paper that this scheme does an accurate job in generating and maintaining sharp gradients. This property is much desired as the spiral eddy model suggested depends on the presence of distinct density fronts. The use of such total variation diminishing (TVD) schemes have become quite common in ocean models describing baroclinic dynamics and fronts (e.g., James 1996; Pietrzak 1998; Berntsen and Svendsen 1999).

b. Governing equations; initial and boundary conditions

The computational domain is a channel, periodic in the \( x \) direction, with walls at \( y = \pm L_y \), and of constant depth \( H_0 \). No external forcing is present. As the depth is constant, the \( \sigma \)-coordinate description is essentially Cartesian. The governing equations are the continuity equation

\[ \nabla \cdot u = 0, \]  

the conservation of mass

\[ \frac{Dp}{Dt} = \frac{\partial}{\partial z} \left( \frac{\partial p}{\partial z} \right), \]  

the horizontal momentum equations under the Boussinesq approximation

\[ \frac{Du}{Dt} - f v = - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( K_M \frac{\partial u}{\partial z} \right), \]  

\[ \frac{Dv}{Dt} + f u = - \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( K_M \frac{\partial v}{\partial z} \right), \]

and the hydrostatic balance

\[ 0 = -\frac{\partial p}{\partial z} - \rho g. \]  

The motion is restricted to an \( f \) plane. The notation above is standard. The quantities not previously defined are \( p \), the pressure, and \( K_H \) and \( K_M \), the vertical diffusivity and viscosity, respectively, estimated according to Munk and Anderson (1948). The horizontal equivalents are set to zero, as the goal is to resolve rather finescale horizontal motion. The effect of physical and numerical diffusion and dissipation will be commented on.

The initial configuration, taken from Fukamachi et al. (1995), is an upwelling stratification with a rather weak horizontal density gradient and a corresponding geostrophic current jet:

\[ \rho(y, z) = \rho_0 \left[ 1 + \frac{f U}{g H} \int_0^y Y(y') \, dy' \right] - \Delta \rho \frac{z + H}{H} \]  

\[ u(y, z) = U Y(y) \frac{z + H}{H} \]  

for \( z \geq -H \), where

\[ Y(y) = \begin{cases} \frac{1}{2} \left[ 1 + \cos \left( \frac{\pi y}{L_y} \right) \right], & |y| \leq L_y \\ 0, & |y| > L_y \end{cases} \]

gives the cross-channel variation. The quantities \( U \), \( H \), \( L_y \), and \( \Delta \rho \) are the characteristic dimensional scales of this flow as described and defined in section 2, and \( \rho(0, -H) = \rho_0 \) is the density at the base of the jet. The surface elevation is initialized consistently. For \( z < -H \), the fluid is motionless of constant density \( \rho_0 \). The current jet is symmetric about a centerline (\( y = 0 \)) with equal amounts of cyclonic and anticyclonic shear (of magnitude \( 0.1 f \)). No external deformation field is present to compress the shear region into a front. Frontogenesis can only take place through the net release of kinetic or potential energy of the initial flow to unstable motion.

This initial configuration may seem somewhat special as the vertical variation is taken to be linear, and the pycnoclines do connect to the thermocline at \( z = -100 \) m in a discontinuous fashion. On the other hand, the previous dimensional analysis suggests that the onset of instability is rather insensitive to the details of the initial geometry. This is confirmed, also with regard to the windup of spirals, through experiments with other geometries, briefly described in the next section and summed up in Table 3. The above initial flow was chosen because Eqs. (14) and (15) averaged over \(-H \leq z \leq 0\) give a geostrophically balanced solution of the \( z \)-independent “variable-temperature layer” (VTL) model as shown by Fukamachi et al. (1995). Geostrophic flow as described by the primitive equations will in general not average to geostrophic flow in the VTL model.
linear instability of the initial flow is well documented by these authors for both models. A comparison between the nonlinear submesoscale eddies generated on this background in the three-dimensional model described here and those generated from equivalent initial conditions in a VTL model has been done by Eldøen (2000). It is found that the instabilities and vortices generated in the VTL model do not show the expected cyclone/anticyclone asymmetry. This is understood from the fact that no parallel to the conservation of Ertel PV, given by Eqs. (1) and (2), seems to exist for the VTL model.

c. The experiments

Tables 1 and 2 list the values of the different dimensional parameters for the varying experiments performed. The nondimensional parameters Bu and Ri argued to govern the instability, defined by Eqs. (6) and (7), and the estimated dimensionless most unstable wavelength and growth rate, \( \hat{\lambda} \) and \( \hat{\psi} \) determined from Eqs. (A.2) and (A.5), as well as the more empirical fit \( \hat{\psi} \) of Eq. (A.7), are also given. The geostrophic flow of the reference experiment, shown in Fig. 2, is not intended to depict any specific location, but should be representative for a rather wide, midlatitude, buoyant current. The two variations from the reference experiment were set up by varying \( \text{Bu} \) and \( \text{Ri} \) through varying \( L \) and \( \Delta \rho \). Doubling the vertical density difference constitutes the “Well strat.” experiment, while reducing the jet width by 75% the “Narrow”.

The length of the channel was set to the predicted most unstable wavelength \( \hat{\lambda} = \hat{\phi}R \), where \( R \) is the internal radius of deformation as previously defined, \( R = \sqrt{(\Delta \rho \rho_0)gH}/f = \sqrt{g H/f} \). The flow was exposed to small perturbations of this wavelength initially. Instability, frontogenesis, and spiral eddies were the result as described in the following section. The estimate \( \hat{\lambda} \) was checked by increasing and decreasing the channel length by 15%, the fraction required for noticeable change in growth rate. The most unstable wavelength found through this procedure \( \lambda/R \), to the above accuracy, its growth rate \( \gamma/f \), and the dimension of spirals \( D/R \) are listed in the three rightmost columns of Table 2 for the different experiments. As expected from the appendix, and in particular from the data of Fukamachi et al. (1995), \( \hat{\lambda} \) is found to give a good estimate of \( \lambda \), while \( \hat{\gamma} = \hat{\psi}f \) is only in qualitative agreement with \( \gamma \). See also Table 3 for the results found for other geometries.

In all of the experiments of Table 2, the horizontal grid consisted of uniform square cells with 32 cells to the length of the channel. There were 41 uniformly spaced \( \sigma \) layers. The growth rates are to a certain degree dependent on the temporal resolution \( \Delta t \). Varying \( \Delta t \), keeping all other physical and numerical parameters fixed, gave that convergence to a certain growth rate was assured for \( \Delta t = 60 \) s. This time step was used. This is only an issue for the estimate of \( \gamma \). Spiral eddies were generated irrespective of \( \Delta t \) (the poorest try was \( \Delta t = 450 \) s). For coarser spatial resolution than 16 cells to \( \lambda \) and 21 \( \sigma \) layers, the representation of the spiral signature, if present at all, was found to be poor. Setting \( L \), to 40 and 60 km showed that the channel walls were of no influence.

4. The birth and life of spiral eddies

Details of the instability and the spiral eddies generated in the reference experiment are presented in this section. The other experiments of Table 2 ran for a period of ten days and gave similar results. The evolution of the surface relative vorticity is shown in Fig. 3 for nine consecutive days (day 2 to 10). (One day is denoted by the unit d.) The reader should also glance at Fig. 10, displaying the patterns traced out by surface floats. Spiral eddies do unfold from the unstable initial current.

a. Frontogenesis and cyclonic shear lines

In the early stages, Figs. 3a,b, the initial perturbation is seen to amplify, but there is no preference for cyclonic vorticity. A day later, Fig. 3c, there is the predicted sharpening of gradients at the cyclonic side of the jet through frontogenesis and the frontal wave is about to break. In the following snapshot, where nonlinear effects are clearly present, cyclonic vorticity is concentrated in a shear line of vorticity several times \( f \). The spiral-to-be is seen where the frontal wave broke. In Fig. 3e the shear line curls into a cyclonic spiral. It is suggested that this line corresponds to the shear lines

| Table 1. Dimensional parameters kept fixed. |
|---|---|---|---|---|---|---|
| \( f \) (s\(^{-1}\)) | \( H \) (m) | \( H_0 \) (m) | \( L \) (km) | \( U \) (m s\(^{-1}\)) | \( \rho_0 \) (kg m\(^{-3}\)) | \( \rho_1 \) (kg m\(^{-3}\)) |
| 10\(^{-4}\) | 100 | 200 | 50 | 0.30 | 27.0 | 26.0 |

| Table 2. The dimensional and nondimensional parameters of the initial geostrophic flow and the characteristics of the instability for the different experiments. |
|---|---|---|---|---|---|---|---|---|---|
| Expt | \( \Delta \rho \) (kg m\(^{-3}\)) | \( L \) (km) | \( R \) (km) | \( \text{Bu} \) | \( \text{Ri} \) | \( \hat{\phi} \) | \( \hat{\psi} \) | \( \hat{\psi} \) | \( \lambda/R \) | \( \gamma/f \) | \( D/R \) |
| Reference | 0.19 | 20.0 | 4.2 | 0.05 | 2.0 | 4.9 | 0.17 | 0.15 | 4.9 | 0.14 | 2 |
| Well strat. | 0.38 | 20.0 | 6.0 | 0.09 | 4.0 | 4.5 | 0.13 | 0.10 | 4.5 | 0.10 | 2 |
| Narrow | 0.19 | 5.0 | 4.2 | 0.72 | 2.0 | 5.4 | 0.14 | 0.07 | 5.4 | 0.10 | 2 |
associated with the spirals in the observational material. The width of the latter is \(O(100 \text{ m})\), and not resolved in the present experiment. Still, it is possible that this would be the actual finescale structure if resolved. The narrow concentration of passive surface floats in Fig. 10 lends credibility to this. The further evolution seen in Fig. 3 will be commented on in relation with the energy diagnostics performed below. Cyclonic (anticyclonic) regions are as one expects regions of low (high) pressure anomalies. This is shown with regards to the surface elevation in Fig. 4. The ageostrophic part of the velocity field is also shown, and the ageostrophy is substantial. The timescale of the events is of \(O(f^{-1})\), consistent both with the predictions of section 2, and the observations of Flament and Armi (1985).

The development of the along-channel averaged stratification in the period when the unstable wave leaves the linear regime, breaks and curls up is shown in Fig. 5. A near-surface layer, sharp density front has been generated at the cyclonic side of the jet to produce large cyclonic shears. The actual sharp density front associated with the shear line describing the cyclone of Fig. 3e, may be seen in the two transects constituting Fig. 6.

When dissipation is neglected and the Boussinesq approximation is assumed, the evolution of vertical vorticity \(\zeta\) is given by

\[
\frac{D\zeta}{Dt} = (f k + \xi) \cdot \nabla w. \tag{17}
\]

This implies that the circulation in any horizontal plane, say \(z = z_o\), of the computational domain is constant in time, and with our initial conditions

\[
\int \zeta(x, y, z_o, t) \, dx \, dy = \int \zeta(x, y, z_o, 0) \, dx \, dy = 0. \tag{18}
\]

Thus cyclone/anticyclone asymmetry takes place through the predominantly nonlinear compression of the cyclonic region. The temporal evolution of the vorticity production, \(D\zeta/Dt\), averaged over the cyclonic and anticyclonic regions at each vertical level, is displayed in Fig. 7. Positive shears are mainly produced near the surface, the area of the strongest horizontal gradients in Figs. 5 and 6, while negative shears are produced more evenly over the water columns of the active layer. The homogeneous lower fluid remains practically motionless throughout the event. From the vorticity Eq. (17), \(D\zeta/Dt\) may be separated into the contribution from horizontal vorticity tilting, \(\xi_n \cdot \nabla w\), and the contribution from vertical vorticity stretching, \((\zeta + f)\partial w/\partial z\), caused by the horizontal convergence. The influence of the different terms in the upper 25 m is given in Fig. 8a. The production is clearly dominated by vortex stretching. This is expected, as the factor \((f + \zeta)\) is responsible for the asymmetric evolution. The net cyclonic and anticyclonic production is shown in Fig. 8b. Integrating over the total depth does produce the same pictures qualitatively, but the asymmetry is less pronounced as the production of negative vorticity is more vertically homogeneous. The relative vorticity estimated in two vertical transects through the eddy at day 6 may be seen in Fig. 9. The eddy is intensified toward the surface.

### b. Spiral streaks

In Fig. 3, the onset of the instability and the birth of the spiral eddies are displayed in terms of relative vorticity. The relative vorticity is not a conserved quantity of the flow, but the same evolution was observed in the surface density distribution (not shown). The spiral is then described by a core of relatively heavy water.

In order to look at the details of the surface patterns generated, passive floats were initially distributed uniformly in the surface layer. The floats were advected with the surface velocity field, and local concentration changed due to the surface convergence and divergence of the evolving flow. This accumulation of floats into streaks corresponds to the shear lines that have been argued to be partly responsible for the spiral signatures in the observations. In Fig. 10, where the arrows are the surface velocity field, floats are displayed in the areas where the concentration is found to be more than five times the initial. Thus streaks are established that at \(t = 6\) d trace out spiral eddies at the scale of 10 km. It is seen that the streaks to a large extent form where there are large velocity shears, the frontal area. The frontal convergence is found to be large, \(O(10^{-4} \text{ s}^{-1})\), hence floats are effectively accumulated. This magnitude is consistent with the cross-frontal convergence estimated by Flament and Armi (2000) for the narrow accumulation of debris coinciding with the strong cyclonic front of a seaward filament rooted off central California. Such surface convergence implies a significant horizontal confluence and steepening of pycnoclines as seen in Figs. 5 and 6 that are accompanying the transformation from the (quasi-)linear regime displayed in Fig. 3c, to the fully developed cyclone of Figs. 3e and 10b. The non-dimensional size of eddies, \(D/R\), is estimated at this stage.

#### Table 3. Characteristics for spiral eddies generated in other geometries.

<table>
<thead>
<tr>
<th>Expt</th>
<th>(R) (km)</th>
<th>(B )</th>
<th>(R_i)</th>
<th>(\phi)</th>
<th>(\phi)</th>
<th>(\psi)</th>
<th>(\lambda/R)</th>
<th>(\gamma/f)</th>
<th>(D/R)</th>
</tr>
</thead>
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<tr>
<td>Margules</td>
<td>6.9</td>
<td>0.33</td>
<td>3.0</td>
<td>4.9</td>
<td>0.14</td>
<td>0.08</td>
<td>4.6</td>
<td>0.14</td>
<td>3/2</td>
</tr>
<tr>
<td>Cont. strat.</td>
<td>5.4</td>
<td>0.18</td>
<td>1.3</td>
<td>5.1</td>
<td>0.19</td>
<td>0.14</td>
<td>5.1</td>
<td>0.10</td>
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</tbody>
</table>

This table lists the characteristics for spiral eddies generated in other geometries, with specific parameters such as the radius of the eddies, Buoyancy number, Richardson number, relative vorticity, and other relevant ratios.
Fig. 3. The onset of instability and the generation of a spiral eddy as seen in the surface relative vorticity. The colour scale is related to the planetary vorticity $f$. Note that its range varies. The spatial resolution of the figures is that of the experiment.
Spiralling structures are seen in the floats all through to day 10. Several new features have appeared by then. There are essentially two jets. In the area around $y = 0$, where the spiral first appeared, there is the cyclonic side of a jet where the floats appear as rather elongated features resembling cat’s eyes circulation. Around $y = 15$ km, there is a meandering jet with incipient spirals at its cyclonic side.

c. Energy diagnostics

The primitive equations (9)–(13), assuming nonviscous flow, implies the conservation of energy

$$\frac{d}{dt}(\text{KE} + \text{PE}) = 0,$$  \hspace{1cm} (19)

where $\text{KE} = \rho_0 \int (u^2 + v^2) \, dV/2$ is the total (horizontal) kinetic energy and $\text{PE} = \int \rho g \, dV$ is the total potential energy of the fluid in the channel. Let $\bar{\chi}$ be the along-channel average,

$$\bar{\chi}(y, z, t) = \frac{1}{\lambda} \int_0^\lambda \chi(x, y, z, t) \, dx,$$  \hspace{1cm} (20)

$\chi$ being any quantity of the flow. Then $\chi = \bar{\chi} + \chi'$, where $\chi' = \chi - \bar{\chi}$ ($\chi'' = 0$) represents the residual, or eddy, part of $\chi$. The kinetic energy may be separated into one contribution from the mean motion, mean kinetic energy (MKE), and one from the eddy motion, eddy kinetic energy (EKE),

$$\text{KE} = \text{MKE} + \text{EKE}$$  \hspace{1cm} (21)

$$\text{MKE} = \frac{\lambda}{\rho_0} \int (\bar{u}^2 + \bar{v}^2) \, dy \, dz$$  \hspace{1cm} (22)

$$\text{EKE} = \frac{\lambda}{2 \rho_0} \int (\bar{u}^2 + \bar{v}^2) \, dy \, dz.$$  \hspace{1cm} (23)

The eddies must draw their energy from either the MKE or the (available) potential energy, restricted by $d(\text{MKE} + \text{EKE} + \text{PE})/dt = 0$. Orlanski and Cox (1973) show that the energy budget is governed by

$$\frac{d}{dt}\text{MKE} = C(\text{PE}, \text{MKE}) - C(\text{MKE}, \text{EKE})$$  \hspace{1cm} (24)

$$\frac{d}{dt}\text{EKE} = C(\text{PE}, \text{EKE}) + C(\text{MKE}, \text{EKE})$$  \hspace{1cm} (25)

$$\frac{d}{dt}\text{PE} = -C(\text{PE}, \text{MKE}) - C(\text{PE}, \text{EKE}),$$  \hspace{1cm} (26)
Fig. 5. The frontogenesis as seen in the along-channel averaged stratification. Units are $\sigma_r$.

(a) $t=4.0\,d.$

(b) $t=5.0\,d.$

(c) $t=6.0\,d.$

Fig. 6. Transects (a) $x = 17.5\,km$ and (b) $y = 0$, of isopycnals going through the eddy at $t = 6\,d.$

\[ C(\text{MKE, EKE}) = -\lambda \int \rho_o u_{vi} v' \frac{\partial u_i}{\partial y} \,dy \,dz \]
- $\lambda \int \rho_o u_{vi} w' \cdot \frac{\partial u_i}{\partial z} \,dy \,dz \] (27)

\[ C(\text{PE, MKE}) = -\lambda \int \rho w' g \,dy \,dz \] (28)

\[ C(\text{PE, EKE}) = -\lambda \int \rho w' g \,dy \,dz. \] (29)

The energy transfer terms, $C(\cdot, \cdot)$, are such that $C(A, B) = -C(B, A) > 0$ represents the positive conversion of energy of type $A$ into type $B$. For $C(\text{PE, EKE}) > 0$ there is buoyant production of eddy kinetic energy that is, baroclinic instability. For $C(\text{MKE, EKE}) > 0$ there is Reynolds shear production of EKE, that is, barotropic instability. The first term on the right-hand side of Eq.
Fig. 7. The production of cyclonic (solid contours) and anticyclonic (broken contours) vorticity averaged over areas of respective vorticity at each vertical level. The unit is $f^2$. The cyclonic contour interval is five times that of the anticyclonic.

(27) is the contribution from horizontal shear and the second is that from vertical shear, that is from Kelvin–Helmholtz instabilities.

In Fig. 11 the evolution of EKE in the reference experiment is seen, and the evolution of the energy transfer terms responsible for generating EKE are plotted in Fig. 12. The units are joule (J) and watt (W), respectively. The values may be related to respective densities through dividing by $(20 \times 10^3)^2 \times 100 \text{ m}^3 = 4 \times 10^{10} \text{ m}^3$, an estimate of the fluid volume in motion. From these figures, the flow, as seen in Figs. 3 and 10, may be understood to go through four phases. The growth of EKE is exponential, indicating linear instability up to about 4.5 days. Then nonlinearity sets in. The EKE $\varepsilon$-folds in 0.4 days through the linear instability. Thus the $\varepsilon$-folding time of the amplifying wave is $T_\varepsilon = 0.8$ d, or equivalently $\gamma = 0.14f$. For the next 48 hours, the growth is weaker and nonlinear. This is when the frontal wave breaks and reveals spiral structure. All through these two first phases, the eddies feed on the potential energy of the stratification. Then for the next two days or so, up to $t = 8.25$ d, the EKE decays. It is interesting to see that most of this energy goes into mean motion (cf. Fig. 12). This explains the weakening of the eddy and the appearance of more along-channel elongated structures in its place (Figs. 3g, 10d). Then again there is growth for a day. This is when secondary instabilities set in, and it is the only period when the horizontal Reynolds shear is seen to produce EKE in Fig. 12. By looking at the spatial structure of the integrands defining the transfer coefficients of interest, it was found that horizontal shear instability is responsible for the reappearing eddies at $y = 0$, while the incipient cyclone seen at $y = 15$ km is generated by baroclinic instability. Thus a new spiral may be expected to develop there. A cartoon of the energy flow for this first 10-day period of linear instability, spiral windup, and eventual secondary instabilities is drawn in Fig. 13. The indicated average transfer rates are normalized by the average $C(\text{PE}, \text{EKE})$ in the period. The average of the transfer $C(\text{PE}, \text{MKE})$, not directly involved in the instability, is included for completeness. This interchange is not associated with any distinct events, but gives a continuous flow of energy from PE to MKE.

Consider for a moment a numerical model with a horizontal spatial resolution that does not resolve spiral eddies. The influence of spirals is parameterized through a horizontal harmonic eddy viscosity, $A_M$, dissipating the mean flow. Munk et al. (2000), based on their assumption that the instability winding up the eddies is barotropic, provide the estimate $A_M = \frac{3}{10^3} \text{ m}^2 \text{ s}^{-1}$. This is representative of the viscosities used in larger-scale ocean models.

For the above eddy parameterization to be consistent with our resolving experiment, the equality $-\bar{u} \bar{u}' = A_M \frac{\partial \bar{u}}{\partial y}$ should hold (e.g., Kundu 1990), implying $C(\text{MKE}, \text{EKE}) = \lambda \int \rho_A A_M |\partial \bar{u}/\partial y|^2 \text{ dy dz} + \cdots$ in the definition (27). The up-gradient flow of kinetic energy displayed in Fig. 12 then requires a negative eddy
viscosity. Thus spiral eddies as described in the present experiment cannot be parameterized in the traditional way. The classification of the instability as baroclinic may therefore be argued to be essential when accounting for the influence of spiral eddies in the larger-scale dynamics. Orlanski and Cox (1973), Qiu et al. (1988), and Wang (1993), in their numerical studies of baroclinic instabilities of ocean currents, all find that the eddies may indeed accelerate the mean flow.

In the Narrow jet experiment of relatively high Burger number, \( Bu = 0.72 \), the barotropic contribution to the linear instability and the first breaking of the front was nonnegligible and positive. The dynamics were still predominantly baroclinic, and negative \( C(\text{MKE, EKE}) \) characterized the further nonlinear development. Horizontal shear production/dissipation of EKE constituted the barotropic transfer of energy at all times.

d. The further development of the flow

The reference experiment was terminated at \( t = 20 \) d. Eddies with spiral structure persist throughout. The vorticity and floats, at days 13 and 16, are shown in Figs. 14 and 15, respectively. The patterns have become more complex, as could be expected from the last frames of Figs. 3 and 10. There is the vanishing signature of the cat’s eyes, and the new spiral-like eddies akin to the original spirals. This could account for the observations of less organized spiral eddy fields than displayed in Fig. 1. A new feature is the appearance of closed, but weak, anticyclonic circulation. The accumulation of floats is still related to areas of positive shear. As far as this tracer is concerned, anticyclonic eddies will not turn...
Fig. 10. The evolution of surface patterns traced out by the passive floats and the surface velocity field. The periodic domain is shown twice.
up in observations. Although the spatial resolution allows for it, the flow does not seem to produce eddies of decreasing scale. The increasing numbers of eddies are of the same size as the first.

Compared to the first period, the development of EKE is much less dramatic during the last ten days; see Fig. 16. The energy flow cartoon for this period is displayed in Fig. 17. Numbers are given relative to the scale of Fig. 13. As before, potential energy is continuously converted to eddy kinetic energy and the barotropic conversion is mostly negative in the period. With no dissipation, a further growth in EKE is predicted. This is seen not to be the case. As there is no external forcing present, numerical diffusion and dissipation will sooner or later get the better of the experiment. It is reasonable to link the systematic decay of EKE in Fig. 16 to this.

The observed evolution of eddy kinetic energy in time may be approximated by \( \text{EKE} \sim \exp(-0.1\tau) \sim 1 - 0.1\tau \), where \( \tau = td^{-1} \) (cf. Fig. 16). The details of the transfers \( C(\cdot, \text{EKE}) \) give \( \text{EKE} \sim 1 + 0.1\tau \) in the absence of dissipation. An estimate of the numerical dissipation of EKE readily follows, \( \text{diss} \sim \exp(-0.2\tau) \). If this is parameterized through eddy viscosity, \( \text{diss} \sim \exp[-2A_M(2\pi/l)^2\tau] \), then \( A_M \sim 1 \times 10^{-2} \text{ m}^2 \text{s}^{-1} \). Diffusion and dissipation of this magnitude have negligible influence on the distinct instabilities and eddies seen in the first half of the experiment, but will eventually be the death of (numerical) spiral eddies. The timescale for the decay is 10 days. One can only speculate whether actual eddies generated through the given dynamics will be dissipated at this rate. A continuous flow of energy toward smaller and smaller scales is often expected, but as seen in the present experiment this is not always the case. In addition, it may be argued that the influence from actual, time-dependent, external forcing on the buoyant flow may be expected to have a dominant influence on a timescale of \( O(10 \text{ d}) \).

The vertical eddy coefficients, determined from Munk and Anderson (1948), had a neutral value of \( 2 \times 10^{-3} \text{ m}^2 \text{s}^{-1} \). We found no significant effect of this prescribed mixing. Increasing this value to \( 2 \times 10^{-2} \text{ m}^2 \text{s}^{-1} \), gave noticeable change at the later stages. The linear growth was reduced by 10%, and the eventual secondary structures were significantly weaker, but still spiraling. Around day 15, the flow, with diffuse vortices of either vorticity of \( |\zeta| < f \), did no longer resemble that of the reference experiment.

e. Spirals generated from other initial geometries

Spiral eddies generated through baroclinic instabilities are not limited to initial conditions of the type described here. As already mentioned, the results of Eldvik and Dysthe (1999) are consistent with the findings herein.

Fig. 13. The average energy flow diagram for the first ten days. Arrows give the flow direction, and the numbers the relative size of average energy transfer terms.
An experiment with continuous stratification over the total depth, as opposed to the motionless, homogeneous lower layer of Fig. 2, was also performed. For this experiment, the initial stratification was described by a horizontal tanh profile of exponential decay vertically, and geostrophically balanced by a surface velocity jet as described by Wang (1993). When perturbed with the wavelength predicted from Eq. (A.2), baroclinic instability produced spiral eddies. This wavelength is slightly less than half the wavelength used by Wang (1993) to describe ocean frontogenesis through baroclinic instability, and was found to amplify twice as fast. The findings from these two different initial geometries are listed in Table 3, labeled “Margules” and “Cont. strat.,” respectively.

5. Concluding remarks

Examining a variety of relevant previous research on baroclinic instability, and performing the numerical experiments of Tables 2 and 3, we have shown that spiral eddies may be understood to be the sea surface signature of ageostrophic baroclinic instabilities: Buoyant geostrophic flows as seen in Fig. 2 are prone to submesoscale instabilities. As the most unstable wave amplifies, ageostrophic frontogenesis takes place to produce
a narrow frontal zone of strong cyclonic shear and convergence that intensifies toward the surface. Entering this nonlinear regime, the wave winds up to produce spiral eddies (Figs. 3, 10) with a corresponding stretching of the frontal zone. The most unstable wavelength, $\lambda$, and the diameter of the eddies, $D$, scale as the internal radius of deformation, $R$, roughly $\lambda \sim 5R$ and $D \sim 2R$. The flow evolves with a characteristic timescale of $O(f^{-1})$. These scales are in agreement with the observations. Due to the convergence, surface floats accumulate at the front. Both slicks and shear lines are readily observed from space platforms and are associated with spiral observations. No external forcing is required to drive the frontogenesis, and the evolving three-dimensional dynamics seem to be rather insensitive to the initial geometry.

The linear instability, as shown by Fukamachi et al. (1995) for the initial configuration (14)–(16), and the following nonlinear windup are clearly baroclinic processes (Fig. 12). This was also the case for the spirals generated from a Margules-type front in the experiment of Eldevik and Dysthe (1999) and in the Cont. strat. experiment. It seems to contradict the shear instability model suggested by Munk et al. (2000). Shear instability is only found to be a component in the establishment of secondary circulations. Then it does produce circulation akin to the cat’s eyes assumed by Munk et al. (2000).

When secondary instabilities set in, a more complex pattern of cyclones is produced (Figs. 14 and 15). Their dimension is still about 10 km, their signature spiraling, and they are associated with convergence and relative strong shear as well. More random distributions of spiral eddies than seen in Fig. 1 may be due to such secondary instabilities.

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APPENDIX

Estimates of $\lambda/R$ and $\gamma/f$

For a flow geometry as in Fig. 2 with $\text{Ri} \geq 1$, negligible $B_u$, and the homogeneous fluid below replaced by a rigid boundary, Stone (1966) found analytically

$$\frac{\lambda}{R} = \phi_0 \sqrt{1 + \text{Ri}^{-1}} \quad (A1)$$

with $\phi_0 = 4\pi\sqrt{10} \approx 4.0$.

Iga (1993), Barth (1994), and Fukamachi et al. (1995) all solve the instability problem for the linearized primitive equations for several combinations of $B_u$ and $\text{Ri}$ numerically. Eldevik (2000) gives a detailed interpretation of the different papers in the present context. It should be noted that only the initial conditions of Fukamachi et al. (1995) have a strict geometric similarity to Fig. 2. The $\lambda/R$ data deduced from the above papers are displayed in Fig. 18. Although the similarity formally required by the $\pi$ theorem is not present, the relation

![Fig. 16. The EKE (ln scale) for days 10 to 20. The broken line of slope $-0.13 \quad d^{-1}$ gives an estimate of the numerical dissipation.](image1)

![Fig. 17. The average energy flow diagram for days 10 to 20.](image2)
Fig. 18. Values of $\lambda/R$ vs $Bu$ and $Ri$ from data. The corresponding values of the suggested approximation (A2) are given by the stars and contour lines.

\[ \frac{\lambda}{R} = \psi(Bu, Ri) = \frac{\psi_0}{2} \sqrt{1 + Ri^{-1} + \frac{Bu}{2}}, \quad (A2) \]

given by the asterisks and the isolines, is seen to give a reasonable fit to the data. For negligible $Bu$, it reduces to Stone’s result. The particular form of the added $Bu$-dependent term is not purely ad hoc. It may be shown (cf. Eldevik 2000) that Eq. (A2) in the QG limit is consistent with estimating $\lambda/R$ from the Eady (1949) model. In section 3, the most unstable wavelength of our different numerical experiments are found to support the suggested $\psi$ (see Tables 2 and 3).

It follows from Eq. (A2) and the data of Fig. 18 that

\[ 4R \leq \lambda \leq 6R, \quad (A3) \]

provided $Bu \leq 1 \leq Ri$. The parameteres of geostrophically balanced oceanic flow would in general be within these limits, that is, $L_0 \leq R \leq L$. The deformation radius $R$, defined from the upwelling initial stratification, should be rather small, typically 5 km at midlatitudes. The same stratification would give $R = 14$ km at $15^\circ$N/S. Together with the estimate (A3), this implies a range of submesoscale wavelengths ($\lambda \sim 25$ km at midlatitudes) suitable for creating spiral eddies.

An equivalent analysis and comparison may be done with regard to growth rates. Let $\gamma$ be the growth rate of the most unstable wave $\lambda$. Then $\gamma/f$, as $\lambda/R$, may be sought as a nondimensional function of $Bu$ and $Ri$,

\[ \frac{\gamma}{f} = \psi(Bu, Ri). \quad (A4) \]

An estimate similar to (A2) is the relation

\[ \frac{\gamma}{f} = \psi(Bu, Ri) = \frac{\psi_0}{\sqrt{Ri(1 + Ri^{-1} + Bu)}} \quad (A5) \]

\[ \sim \frac{U}{f\lambda} \quad (A6) \]

with $\psi_0 = \sqrt{5/54} \approx 0.3$. It is found by requiring that $\psi(0, Ri)$ corresponds to the finding of Stone (1966) and that the QG limit of $\psi$, or rather $\psi\sqrt{Ri}$, is consistent with the Eady (1949) model. The rough approximation (A6) is found by combining Eqs. (A2) and (A5). The values of $\gamma/f$ from the different linear analyses, as well as $\psi$ are shown in Fig. 19. The symbols and lines correspond to the same experiments as in Fig. 18, and the asterisks and contour lines show the values of $\psi$ given by Eq. (A5). The estimate (A5) of the growth rate, is not as good as the estimate (A2) of the wavelength, but it does increase (decrease) with $Bu$ and $Ri$ as data increases (decreases) and covers to some extent the data range.

For moderate Burger and Richardson numbers, say $Bu = 0.5$, $Ri = 4$, $\psi = 0.1$, which is a representative value for the data of Fig. 19 as well. This corresponds to $e$-folding in a day at midlatitudes. Rosenhead (1932) predicts a timescale $\tau = \lambda/U$ for the nonlinear windup of a vortex sheet, which is roughly the same as $1/\gamma$ [see Eq. (A6)]. This indicates that the subsequent nonlinear flow will evolve on the timescale of the linear instability, and is supported by our numerical experiments.

As already mentioned, an implicit assumption in using the $\pi$ theorem in the present context is that there exists a geometric similarity among the geostrophic initial flows considered. This is not the case for the papers...
Fig. 19. Values of $\gamma/f$ vs $Bu$ and $Ri$ from data. The corresponding values of the suggested approximation (A5) are given by the stars and contour lines. The crosses give the values of the alternative approximation (A7).

included in the analyses above. Thus the relations $\phi$ and $\psi$ should not be expected to be the same for the different papers. In fact, it came as a surprise that the rather simple relations (A2) and (A5) could be found that gave reasonable fits to the heterogeneous group of initial and boundary conditions considered. The nondimensional growth rate $\hat{\psi}$, given by Eq. (A5), seems to be the poorest estimate. One can only speculate that $g$ is more sensitive to the details of the initial configuration, and in particular the horizontal velocity shear within the jet. The models of Stone (1966) and Eady (1949), that are at the base of the predictions, neglect this shear. If the consistency with the Eady model is dismissed, the relation given by the crosses in Fig. 19,

$$\frac{\gamma}{f} = \hat{\psi}(Bu, Ri) = \frac{\psi_0}{\sqrt{Ri(1 + Ri^{-1} + 10Bu)}}. \quad (A7)$$

models the data better, Iga's (1993) analysis aside.

REFERENCES


