

## Lagrangian Spectra and Diapycnal Mixing in Stratified Flow

REN-CHIEH LIEN AND ERIC A. D'ASARO

*Applied Physics Laboratory, University of Washington, Seattle, Washington*

25 April 2003 and 11 September 2003

### ABSTRACT

Taylor's single-particle dispersion model is revisited and applied to unstratified and density stratified flows using observationally based and theoretical models of the Lagrangian velocity and density spectra, which are compared with existing parameterizations of diapycnal diffusion in these flows. For unstratified homogeneous turbulence, the vertical particle dispersion coefficient  $K_z$  computed from model Lagrangian velocity spectra agrees well with contemporary estimates of the diffusivity. For internal waves with no mixing, a large apparent dispersion occurs for times somewhat larger than the inverse buoyancy frequency  $1/N$ . No dispersion occurs at long times. For stratified homogeneous turbulence with energy dissipation rate  $\varepsilon$ ,  $K_z = \Gamma_d \varepsilon N^{-2}$ , the same form as Osborn, but with  $\Gamma_d$  of about 2.5. This high value is attributed to apparent dispersion due to internal waves and an improper form of the model spectra that allows internal waves to exist at low frequencies. A diapycnal dispersion coefficient  $K_*$  is formulated based on a white spectrum of Lagrangian density change  $D\rho/Dt$  with level  $\beta_p \chi$ , where  $\chi$  is the rate of dissipation of density variance. This yields  $K_* = (\pi/2)\beta_p \chi / \bar{\rho}_z^2$ , where  $\bar{\rho}_z$  is the mean vertical density gradient. This has the same form as the Osborn and Cox model for diapycnal diffusivity if  $\beta_p = 1/\pi$ .

### 1. Introduction

Measurements of ocean turbulence using neutrally buoyant floats have produced empirical forms for the spectra of velocity (Lien et al. 1998) and density (Lien et al. 2002) following a Lagrangian trajectory in stratified or unstratified turbulence. These spectra predict the dispersion of particles and thus imply mixing rates. In this paper, we compare these mixing rates to those derived by other methods and thus attempt to clarify the relationship between Lagrangian and Eulerian turbulence measurements.

Taylor (1921) pioneered the study of turbulent mixing through the examination of the statistics of single particle dispersion. In a stationary and shear-free flow, the dispersion coefficient  $K_z$  is defined as

$$K_z = \frac{1}{2} \frac{d\langle z^2 \rangle}{dt} = \int_0^t R_w(\tau) d\tau, \quad (1)$$

where  $R_w(\tau) = \langle w(t)w(t + \tau) \rangle$  is the Lagrangian velocity covariance function at a time lag  $\tau$ ,  $z$  is the displacement of a particle from its initial position, and angle brackets denote an average over an ensemble of particles. Two fundamental results were obtained by Taylor without

invoking explicit properties of turbulence. First, at time scales much smaller than the integral time scale of velocity  $T = \int_0^\infty R_w(\tau) d\tau / R_w(0)$ ,  $K_z$  grows linearly with time. Second, for  $t \gg T$ ,  $K_z$  converges to a constant value. The temporal evolution of  $K_z$  depends on the details of  $R_w(\tau)$ .

The turbulent transport of a scalar  $c$ —for example, temperature or density—is often parameterized by an eddy diffusivity defined as the ratio of the flux  $F_c$  to its mean gradient  $\partial_z \bar{c}$ ; that is,  $K_e = -F_c / \partial_z \bar{c}$ . Dispersion predicts the flux of particles, and so, if  $c$  is the particle concentration and  $K_z$  is the eddy diffusivity for particles, then  $K_e$  should equal  $K_z$ . Here, we will compare estimates of  $K_z$  from Lagrangian models with estimates of  $K_e$  from Eulerian models.

For stratified turbulence, two parameterizations of  $K_e$  are commonly used, particularly in oceanic measurements. The Osborn and Cox (1972) method is based on measuring  $\chi_c$ , the dissipation rate of scalar variance, and a mean scalar gradient ( $\partial_z \bar{c}$ ); that is,

$$K_h = \frac{1}{2} \frac{\chi_c}{(\partial_z \bar{c})^2}. \quad (2)$$

This expression was originally derived from the scalar variance equation by assuming a balance between the turbulence flux and dissipation. In this derivation,  $K_h$  is the vertical eddy diffusivity and ( $\partial_z \bar{c}$ ) is the average vertical gradient. Winters and D'Asaro (1996) derive

*Corresponding author address:* Dr. Ren-Chieh Lien, Applied Physics Laboratory, University of Washington, 1013 NE 40th Street, Seattle, WA 98105-6698.  
E-mail: lien@apl.washington.edu

(2) exactly, but with  $\chi_c$  describing the diascalar flux, that is, the flux of scalar across isoscalar surfaces, and  $(\partial_z \bar{c})$  is the gradient of the profile of  $c$  after it has been monotonically sorted. For a well-stratified fluid, the difference between these definitions does not create large differences in the computed eddy coefficient (D'Asaro et al. 2004). For the rest of this paper, (2) will be used for density flux, that is,  $c = \rho$ .

The Osborn (1980) method is based on  $\varepsilon$ , the dissipation rate of turbulent kinetic energy,

$$K_\rho = \Gamma \frac{\varepsilon}{N^2}. \tag{3}$$

Here  $\Gamma$  is the mixing efficiency, closely related to the flux Richardson number, and  $N$  is the buoyancy frequency. The value of  $\Gamma$  is typically 0.2 or less in the stratified ocean (Osborn 1980). The values of  $K_\rho$  and  $K_h$ , when sufficiently averaged, generally agree with each other and with other estimates of diapycnal diffusivity (Gregg 1998).

This paper will 1) revisit the dispersion coefficient  $K_z$  using up-to-date Lagrangian velocity spectra in unstratified turbulence (section 2), internal waves (section 3), and stratified turbulence (sections 4 and 5); 2) investigate diapycnal dispersion by defining a diapycnal dispersion coefficient  $K_*$  based on Lagrangian scalar spectra (section 5); and 3) reconcile the dispersion coefficients  $K_z$  and  $K_*$  with the diapycnal eddy diffusivity coefficients  $K_h$  and  $K_\rho$ .

### 2. Unstratified turbulence

Taylor's dispersion coefficient  $K_z$  (1) can be expressed in terms of Lagrangian velocity spectrum  $\Phi_w(\omega)$  or structure function  $D(\tau) = \langle [w(t + \tau) - w(t)]^2 \rangle$ ; for example,

$$\begin{aligned} K_z(t) &= \int_0^t \left[ R_w(0) - \frac{1}{2} D(\tau) \right] d\tau \\ &= \int_0^\infty \Phi_w(\omega) \frac{\sin(\omega t)}{\omega} d\omega. \end{aligned} \tag{4}$$

In the inertial subrange, Kolmogorov scaling suggests forms for both the structure function  $D(\tau) = C_0 \varepsilon \tau$  and the spectrum  $\Phi_w(\omega) = \beta \varepsilon \omega^{-2}$ . Monin and Yaglom (1975) show that the two constants are related as  $C_0 = \beta \pi$ . Lien and D'Asaro (2002) find that the value  $C_0 = 6 \pm 0.5$  and  $\beta = 2 \pm 0.2$  are consistent with existing data at high Reynolds number.

Three Lagrangian velocity spectra (Fig. 1) are chosen to study the behavior of  $K_z$ . The first spectrum is proposed by Tennekes and Lumley (1972):

$$\Phi_{TL}(\omega) = \begin{cases} \beta_{TL} \varepsilon / (\omega_o^{TL})^2 & \text{for } \omega \leq \omega_o^{TL}, \\ \beta_{TL} \varepsilon / \omega^2 & \text{for } \omega > \omega_o^{TL}, \end{cases} \tag{5}$$

The second form (Mordant et al. 2001) assumes an

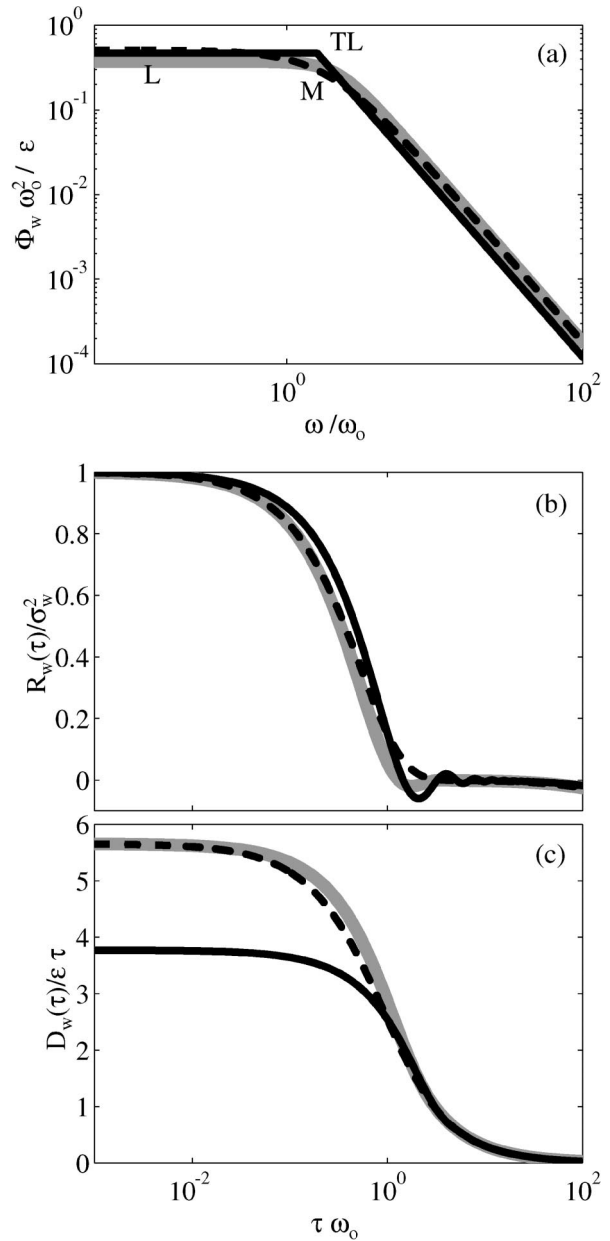


FIG. 1. (a) Lagrangian spectra, (b) correlation functions, and (c) structure functions for three model spectra: gray, L for Lien et al. (1998); black, TL for Tennekes and Lumley (1972); and dashed, M for Mordant et al. (2001).

exponential form  $e^{-\tau T}$  for the Lagrangian velocity covariance function; the velocity spectrum has a Lorentzian form,

$$\Phi_M(\omega) = \beta_M \frac{\varepsilon}{(\omega_o^M)^2 + \omega^2}. \tag{6}$$

The third form is constructed empirically using observations taken in the oceanic surface mixed layer (Lien et al. 1998):

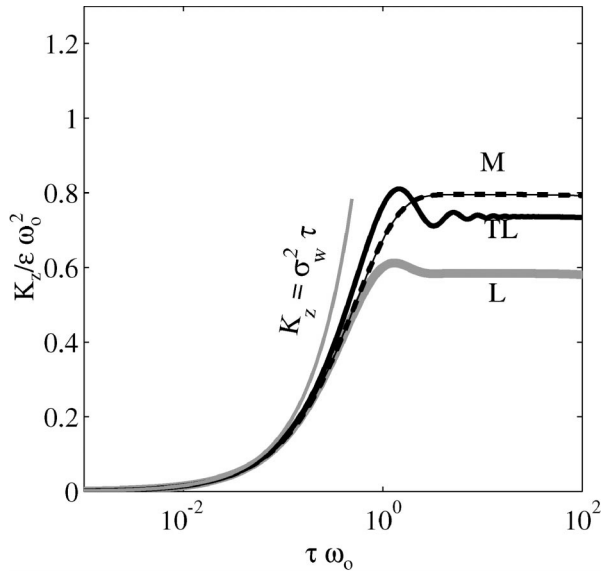


FIG. 2. Dispersion coefficients computed using Lagrangian spectra shown in Fig. 1.

$$\Phi_L(\omega) = \beta_L \frac{\epsilon}{[\omega^4 + (2.2\omega_o)^4]^{1/2}}. \quad (7)$$

The three spectral forms are plotted in Fig. 1a. In each, the spectrum transitions from a flat slope at low frequencies to an inertial subrange with a  $-2$  slope. The transition occurs at a large-eddy frequency  $\omega_o^{TL}$ ,  $\omega_o^M$ , or  $\omega_o$ . The large-eddy frequency is defined as  $\omega_o = \epsilon^{1/3} k_o^{2/3}$ , where  $k_o$  is the large-eddy wavenumber (Lien et al. 1998). The transition is most abrupt for  $\Phi_{TL}$  and the smoothest for  $\Phi_L$ . The inertial subrange Kolmogorov constants are  $\beta_{TL} = 1.2$ ,  $\beta_M = 1.8$  and  $\beta_L = 1.8$ . Given these values and requiring all three forms to have the same velocity variance,  $\sigma_w^2 = 1.6\epsilon/\omega_o$ , requires  $\omega_o^{TL} = 1.6\omega_o$ , and  $\omega_o^M = 1.9\omega_o$ .

Correlation and structure functions are computed from the spectrum and plotted in Figs. 1b and 1c, respectively. For these plots, the inertial subrange has been extended to a Nyquist frequency  $10^6 \omega_o$ ; our results are

insensitive to this factor as long as the inertial subrange is sufficiently wide. The three correlation functions have a similar exponential form. The three structure functions normalized by  $\epsilon\tau$  have a plateau at  $\tau\omega_o \ll O(1)$ . The level of the plateaus gives  $C_0(=\pi\beta)$ , yielding 3.8 for  $\Phi_{TL}$  and 5.7 for  $\Phi_M$  and  $\Phi_L$ , consistent with the results of previous work (Yeung 2002). Figure 2 shows the particle dispersion coefficients  $K_z(t)$  computed from the three Lagrangian spectra using (4). The curves are consistent with Taylor’s results.  $K_z(\tau)$  increases linearly as  $\sigma_w^2\tau$  at  $\tau \ll \omega_o^{-1}$  and reaches a constant level  $K_z(\infty) = (\pi/2)\alpha$  at  $\tau > \omega_o^{-1}$ , where  $\alpha$  is the spectral level at  $\omega \ll \omega_o$ . Thus we scale  $K_z(\infty)$  with  $\epsilon\omega_o^{-2}$ , the spectral level at low frequencies;  $K_z(\infty)\omega_o^2\epsilon^{-1}$  equals 0.73, 0.78, and 0.58 for  $\Phi_{TL}$ ,  $\Phi_M$ , and  $\Phi_L$ , respectively.

The model of Mellor and Yamada (1974) will be used as a summary of the rates of mixing in unstratified turbulence. They parameterized the vertical eddy diffusivity  $K_M = 0.4ql$ , where  $q^2 = 3\sigma_w^2$  and  $l$  is the master length scale. Assuming  $l = k_o^{-1}$  and using  $\sigma_w^2 = 1.6\epsilon\omega_o^{-1}$  and  $\omega_o = \epsilon^{1/3}k_o^{2/3}$ , predicts  $K_M = 0.88\epsilon\omega_o^{-2}$ . Thus for unstratified homogeneous turbulence the coefficients for dispersion  $K_z$  and diffusion  $K_M$  are very close.

### 3. Oceanic internal waves—Vertical dispersion

Internal gravity waves are the dominant cause of vertical motion in the ocean for frequencies greater than  $f$ , the inertial frequency. A large body of literature starting with Fofonoff (1969) and reviewed by Garrett and Munk (1979) finds the vertical displacements due to these motions to be well represented by linear wave theory with a “Garrett–Munk” (henceforth GM) vertical velocity spectrum, which can be approximated as

$$\Phi_{GM}(\omega) = \phi_0 \quad (f < \omega < N) \quad \text{and} \quad (8)$$

$$\Phi_{GM}(\omega) = 0 \quad (\omega > N \text{ or } \omega < f), \quad (9)$$

where  $\phi_0$  sets the internal wave energy level. Figures 3a and 3b show  $\Phi_{GM}$  and the associated dispersion function  $K_{GM}(t)$  for a typical value of  $N/f = 100$ . Since

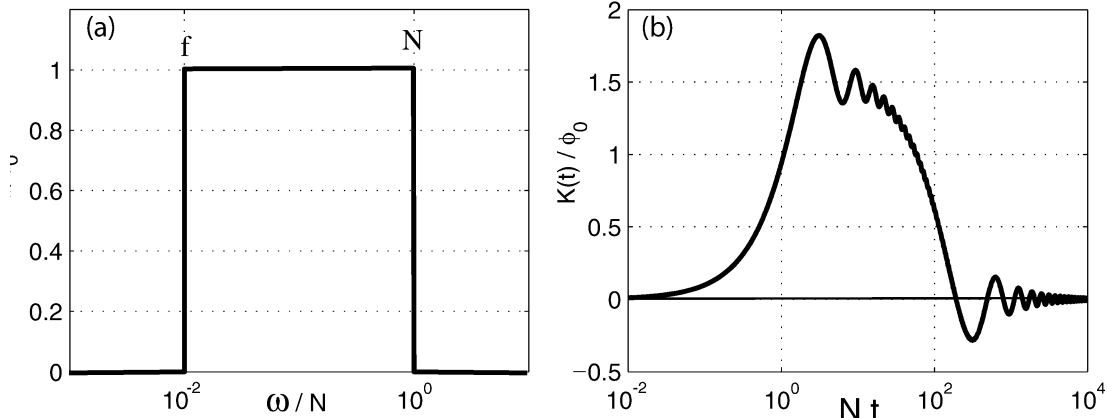


FIG. 3. (a) Spectrum and (b) dispersion coefficient computed for an internal wave spectrum similar to that of Garrett and Munk (1979).

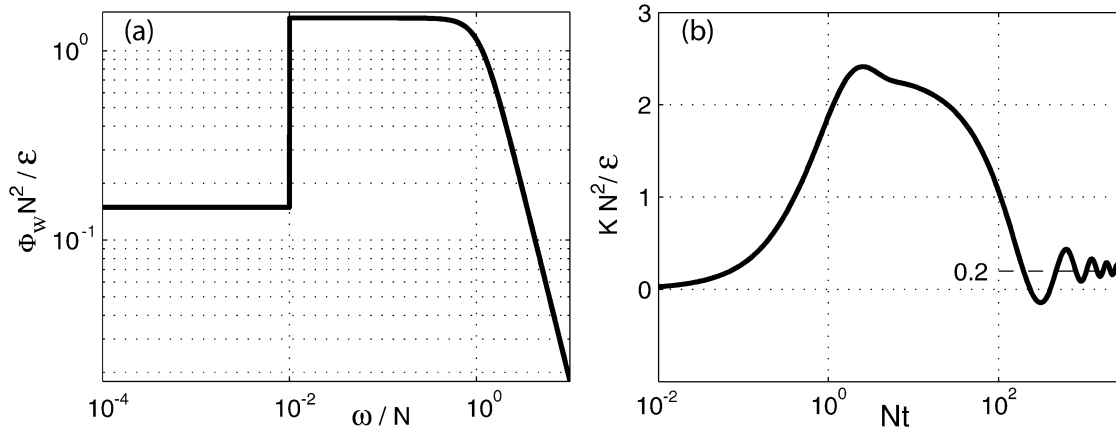


FIG. 4. (a) Spectrum and (b) dispersion coefficient for  $\Phi_L(\omega)$  modified with low-frequency wave cutoff at  $\omega = f$  with  $N/f = 100$ .

internal waves are adiabatic, they cannot cause mixing and  $K_{GM}(\infty) = 0$ . However, they do cause a net displacement of water from its initial position, resulting in an apparent vertical dispersion with a maximum value of about  $(\pi/2)\phi_0$  near  $Nt = 2$ . The value decreases to zero for  $Nt = 200$  and then oscillates around zero with decreasing amplitude. Smoothing  $\Phi_{GM}$  near  $f$  and  $N$  results in a decreased amplitude of oscillations and a more rapid decay of  $K_{GM}(t)$  for  $Nt \gg 100$ .

**4. Stratified turbulence—Vertical dispersion**

D’Asaro and Lien (2000a) argue that in stratified turbulence the large-eddy frequency  $\omega_o$  should be approximately  $N$ . They support this by oceanic observations in which Lagrangian vertical velocity spectra exhibit a form nearly indistinguishable from (7) and  $N = 2(\pm 0.3)\omega_o$ . The spectra are isotropic for  $\omega \gg N$ , supporting the idea that these are turbulent motions. The spectra are highly anisotropic in the same sense as internal waves for  $\omega \ll N$ , supporting the idea that these motions are due to internal waves. These results should apply for flows with gradient Richardson numbers of order 1; for sufficiently larger Richardson numbers the spectral form near  $N$  changes (D’Asaro and Lien 2000b); for sufficiently smaller Richardson numbers the flow geometry determines  $\omega_o$ .

These results can be used to compute the dispersion coefficient for stratified turbulence. Applying  $N = 2(\pm 0.3)\omega_o$  to the result in the previous section, that is,  $K_z(\infty)\omega_o^2\epsilon^{-1}$  is equal to 0.73, 0.78, and 0.58 for  $\Phi_{TL}$ ,  $\Phi_M$ , and  $\Phi_L$ , respectively, we find

$$K_z(\infty) = \Gamma_d \epsilon N^{-2}, \tag{10}$$

where  $\Gamma_d = 3.0(\pm 0.9)$ ,  $3.2(\pm 0.9)$ , and  $2.4(\pm 0.7)$  for  $\Phi_{TL}$ ,  $\Phi_M$ , and  $\Phi_L$ , respectively. Expression (10) is identical to (3) except that  $\Gamma_d$  is an order of magnitude greater than  $\Gamma$  and thus does not properly model diapycnal mixing.

The large value of  $K_z(\infty)$  is explained by adiabatic

internal waves being responsible for most of the vertical excursions. As shown in Fig. 3 internal waves produce a large apparent vertical dispersion for  $1 < Nt < N/f$ . However, if the vertical motions of stratified turbulence are dominated by internal waves for  $\omega < N$ , then the spectrum must also reflect the low frequency limit of internal waves. In the open ocean, this is due to  $f$ . In a confined sea or a laboratory vessel it might be set by the lowest mode of oscillation. In either case, the vertical velocity spectrum cannot remain white at low frequency as in (5), (6), and (7); instead, it must decrease by a factor  $\Gamma/\Gamma_d$  so as to bring  $K_z(\infty)$  into agreement with  $K_\rho$ .

An example of a Lagrangian spectrum with a low frequency cutoff is shown in Fig. 4. The dispersion function  $K_z(t)$  is similar to those in Fig. 2 for  $Nt \leq 10$  but decreases as in Fig. 3 for large  $Nt$ , ultimately reaching a value of  $\Gamma\epsilon/N^2$  at long times. The initial large apparent dispersion is due to the wave part of the flow; the final smaller value is the true dispersion due to diapycnal mixing.

**5. Stratified turbulence—Diapycnal dispersion**

A more direct connection between dispersion and diffusion is obtained by examining dispersion in an isopycnal coordinate system (Pearson et al. 1983). Define an isopycnal displacement  $z_*$  as the displacement of a particle away from its initial isopycnal. For a flow with a well-defined vertical gradient in density  $\rho_z$ ,  $z_*$  can be approximated by

$$z_* = \rho'/\rho_z, \tag{11}$$

where  $\rho'$  is the deviation of the density from its initial value.

Following Taylor’s dispersion theory, as in (1), we define a diapycnal dispersion coefficient as

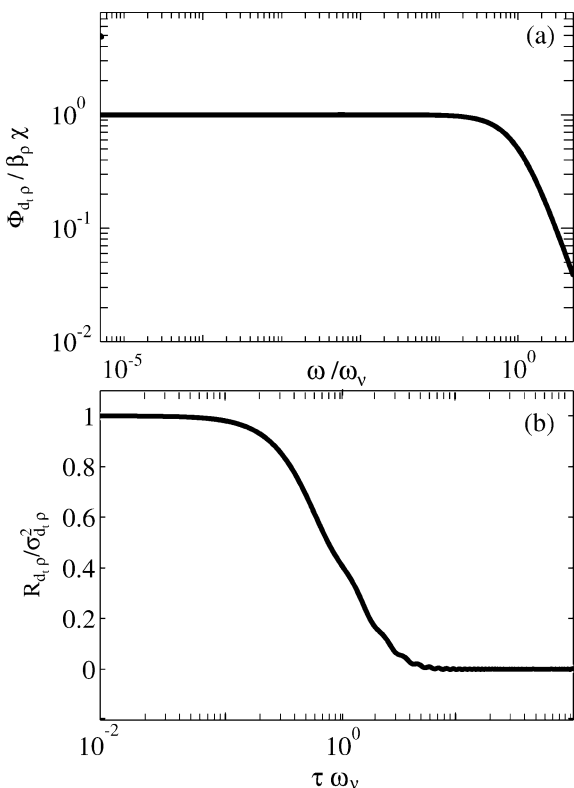


FIG. 5. (a) Lagrangian spectra and (b) Lagrangian correlation function of the time rate change of density.

$$\begin{aligned}
 K_*(t) &= \frac{1}{2} \frac{d\langle z_*^2 \rangle}{dt} = \int_0^t R_{w_*}(\tau) d\tau \\
 &= \int_0^t \left[ R_{w_*}(0) - \frac{1}{2} D_{w_*}(\tau) \right] d\tau \\
 &= \int_0^\infty \Phi_{w_*} \frac{\sin(\omega t)}{\omega} d\omega = \rho_z^{-2} \int_0^\infty \Phi_\rho \frac{\sin(\omega t)}{\omega} d\omega.
 \end{aligned}
 \tag{12}$$

Here,  $w_*$  is the diapycnal velocity,  $R_{w_*}$  is the diapycnal velocity covariance function,  $D_{w_*}$  is the corresponding structure function,  $\Phi_{w_*}$  is the diapycnal velocity spectrum, and  $\Phi_\rho$  is the spectrum of  $D\rho/Dt$ , the Lagrangian rate of change of density.

To evaluate (12), Lagrangian spectra of density change are needed. There is only scant theoretical or measurement guidance available. Kolmogorov scaling suggests  $\Phi_\rho(\omega) = \beta_\rho \chi$  in an inertial subrange extending from  $\omega > \omega_o \approx N/2$  to  $\omega < \omega_\nu = \epsilon^{1/2} \nu^{-1/2}$ . Here  $\beta_\rho$  is a Kolmogorov constant,  $\omega_\nu$  is the Kolmogorov frequency at which viscous effects become important, and  $\nu$  is the kinematic viscosity. Lagrangian observations in an oceanic shear-stratified flow provide vague evidence of white  $\Phi_\rho$  spectrum in the inertial subrange (Lien et al. 2002). Yeung (2001) performed direct numerical

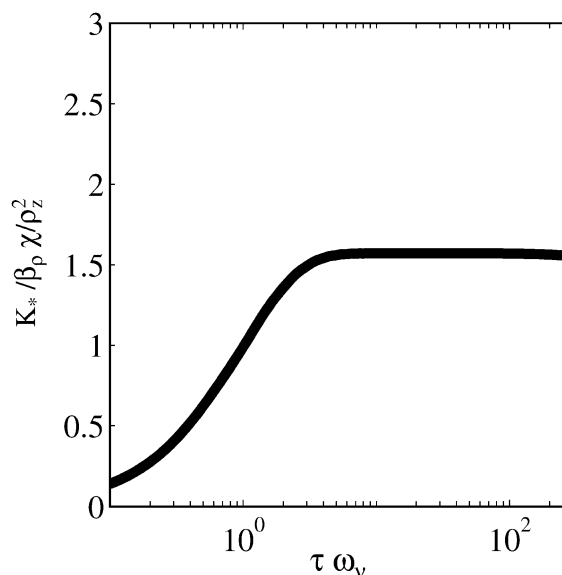


FIG. 6. Diapycnal dispersion coefficient for the spectrum in Fig. 5.

simulations but did not observe a white  $\Phi_\rho$  in the inertial subrange.

We will assume a white spectrum  $\Phi_\rho = \beta_\rho \chi$  and apply a cutoff of  $\omega^{-2}$  for  $\omega > \omega_\nu$  (Fig. 5). Figure 6 shows that  $K_*(t)$  attains a constant value within the inertial subrange:

$$K_* = \frac{\pi}{2} \frac{\beta_\rho \chi}{\rho_z^2}.
 \tag{13}$$

The true diapycnal dispersion rate is given by  $K_*(\infty)$ , which depends on the spectral level of  $\Phi_\rho$  at low frequency. All contributions to  $D\rho/Dt$  and thus  $\Phi_\rho$  must occur at diffusive scales since adiabatic motions cannot affect  $D\rho/Dt$ . Diffusive motions have frequencies near  $\omega_\nu$  and thus contribute to the increase in  $K_*$  for small times. Motions within the inertial subrange are adiabatic and do not contribute to  $K_*$ . Motions at lower frequencies should also be adiabatic and will also not contribute to  $K_*$ . Thus we expect (12) to be accurate at all frequencies and  $K_*(\infty)$  to be given by (13).

Expression (13) is identical to (2), except for the numerical factor. Equating  $K_*$  and  $K_h$  implies  $\beta_\rho = 1/\pi$ . This is similar to the previous estimates of Yeung (2001) ( $\beta_\rho \approx 0.5$ ) and Lien et al. (2002) ( $\beta_\rho \approx 1$ ).

### 6. Discussion

Figure 7 helps to illustrate the above ideas. Consider the motion of an isopycnal surface within a given volume in a turbulent stratified fluid. The volume is chosen to be somewhat larger than the overturning scale of the turbulence. The average position of the surface (black) moves vertically in response to internal waves. If there is no mixing, the internal waves move dye vertically from its initial position, causing apparent vertical dis-

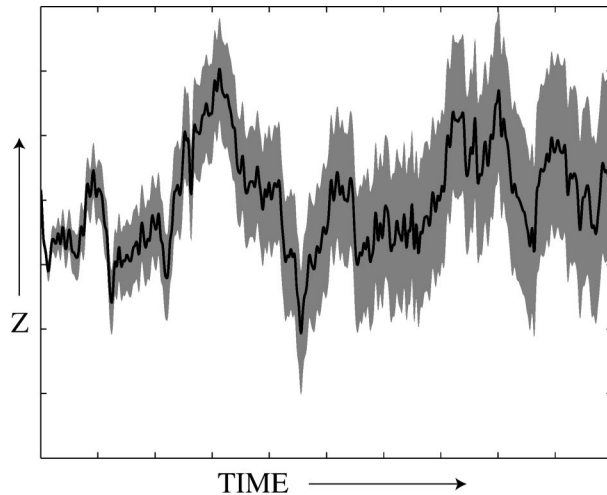


FIG. 7. Cartoon of dye (gray) spreading from an isopycnal surface (black line). Initially, vertical displacements of dye are due to the adiabatic motion of the density surface. Later, as the diapycnal spreading becomes larger, it becomes the dominant cause of net dye motion.

persion at short times. The motion is bounded by the wave amplitude so that the dispersion decreases to zero for long times (Fig. 3).

Suppose dye is placed on the surface at time  $t = 0$  and spreads away from the isopycnal by mixing. The gray shading indicates the spread of the dye. At short times, waves are still the dominant cause of vertical motion of the dye. At longer times, the spread of the dye exceeds the amplitude of the waves, resulting in a finite dispersion that is smaller than the initial wave-induced apparent dispersion. This sequence is described by Fig. 4. The transition time between these two regimes depends on the amplitude and frequency of the internal waves, even if these play no role in the mixing.

The Lagrangian vertical velocity spectrum, or equivalently the dispersion function  $K_z(t)$ , contains information on both diabatic and adiabatic motions. The diabatic processes can be isolated by taking a sufficiently long time average, that is, finding  $K_z(\infty)$ . In some systems this may not be possible. For example, we have several measurements of  $\Phi_w$  in stratified fluids, but none extend to low enough frequency to see the expected decrease in level. This issue is not unique to Lagrangian spectra, but is common to many mixing diagnoses that use an Eulerian coordinate system (Winters and D'Asaro 1996).

These problems can be mostly overcome by working in an isopycnal coordinate system. Obviously, the mixing rate is more easily determined by measuring the diapycnal spreading of the dye around the isopycnal than by measuring its vertical spreading. Thus it is not surprising that the Lagrangian spectrum of density, which measures the motion of particles across isopycnals, gives a measurement of diapycnal diffusivity (13) that is completely insensitive to adiabatic motions of

the isopycnal and therefore does not need to be averaged over long time scales to provide useful information.

Measurements of  $\Phi_w$  and  $\Phi_\rho$  can, in principle, be made using water-following floats (D'Asaro 2003). Practically, however, these floats only follow water at high frequencies. At low frequencies they are usually isopycnal; that is, they return to a given density surface. Isopycnal floats would follow the black line in Fig. 7, while truly Lagrangian floats, like dye, would undergo random walks filling the shaded region. This distinction is critical for the estimation of  $K_z(\infty)$  from  $\Phi_w$  since it is the low frequency value of  $\Phi_w$  that determines  $K_z(\infty)$ . As discussed in D'Asaro (2003), it becomes increasingly difficult to design floats that are accurately Lagrangian at frequencies very much smaller than  $N$ . Float technology may be a better match for estimation of  $K_*$  from  $\Phi_\rho$ . If, as hypothesized here, the spectrum is indeed white for all frequencies below the Kolmogorov frequency, then measurements of  $\Phi_\rho$  within the inertial subrange will be sufficient. Existing floats can be accurately Lagrangian for these frequencies. However, a better understanding of the form and scaling of  $\Phi_\rho$ , gained by both theory and observations, will be required before diapycnal mixing rates can be reliably computed using Lagrangian measurements of high frequency density fluctuations.

## 7. Conclusions

This study has examined the relationship between diapycnal mixing and Lagrangian spectra of vertical velocity and temperature change through the concept of single particle dispersion [(1)]. This is motivated by recent measurements that define empirical forms for these spectra in turbulent flows.

- In unstratified homogeneous turbulence, Lagrangian velocity spectra predict dispersion coefficients that closely match known diffusion coefficients.
- Adiabatic internal gravity waves in density stratified flows produce large apparent dispersions at time lags somewhat larger than  $1/N$ , but no average dispersion at long times.
- In density stratified turbulence with Richardson numbers of  $O(1)$ , the existing empirical Lagrangian velocity spectra predict vertical dispersion coefficients with the Osborn (1980) [(3)] form but with a mixing efficiency  $\Gamma_d$  much larger than observed. This is explained by the presence of a large adiabatic gravity wave component in these flows that leads to a large apparent dispersion at short times. If gravity waves and other adiabatic vertical motions become sufficiently weak at low frequency, then the Lagrangian velocity spectrum can be used to measure diapycnal mixing. The existing spectral forms must be modified to represent these low frequency changes. Similarly, measurements of the Lagrangian velocity spectrum

must extend to sufficiently low frequency to be useful for the estimation of diapycnal mixing.

- Inertial subrange theory and limited observations predict a white Lagrangian inertial subrange spectrum for density change with a level  $\beta_\rho \chi$ , where  $\chi$  is the dissipation rate for density. This form gives a diapycnal dispersion coefficient with the Osborn and Cox (1972) [(2)] form and thus predicts a value  $\beta_\rho = 1/\pi$  for the Kolomogorov constant. This formulation is insensitive to adiabatic motions such as gravity waves and is thus a promising approach for estimation of diapycnal mixing from neutrally buoyant floats.

*Acknowledgments.* This work was supported by NSF Grants OCE9617671 and OCE0117411.

#### REFERENCES

- D'Asaro, E. A., 2003: Performance of autonomous Lagrangian floats. *J. Atmos. Oceanic Technol.*, **20**, 896–911.
- , and R.-C. Lien, 2000a: Lagrangian measurements of waves and turbulence in stratified flows. *J. Phys. Oceanogr.*, **30**, 641–655.
- , and —, 2000b: The wave–turbulence transition in stratified flows. *J. Phys. Oceanogr.*, **30**, 1669–1678.
- , K. B. Winters, and R.-C. Lien, 2004: Lagrangian estimates of diapycnal mixing in a simulated K–H instability. *J. Atmos. Oceanic Technol.*, **21**, 799–809.
- Fofonoff, N. P., 1969: Spectral characteristics of internal waves in the ocean. *Deep-Sea Res.*, **16** (Suppl.), 59–71.
- Garrett, C. J. R., and W. H. Munk, 1979: Internal waves in the ocean. *Annu. Rev. Fluid Mech.*, **11**, 339–369.
- Gregg, M. C., 1998: Estimation and geography of diapycnal mixing in the stratified ocean. *Physical Processes in Lakes and Oceans*, J. Imberger, Ed., Coastal and Estuarine Studies, Vol. 54, Amer. Geophys. Union, 305–338.
- Lien, R.-C., and E. A. D'Asaro, 2002: The Kolmogorov constant for the Lagrangian velocity spectrum and structure function. *Phys. Fluids*, **14**, 4456–4459.
- , —, and G. Dairiki, 1998: Lagrangian frequency spectra of vertical velocity and vorticity in high-Reynolds-number oceanic turbulence. *J. Fluid Mech.*, **362**, 177–198.
- , —, and M. J. McPhaden, 2002: Internal waves and turbulence in the upper central equatorial Pacific: Lagrangian and Eulerian observations. *J. Phys. Oceanogr.*, **32**, 2619–2639.
- Mellor, G. L., and T. Yamada, 1974: A hierarchy of turbulence closure models for planetary boundary layers. *J. Atmos. Sci.*, **31**, 1791–1806.
- Monin, A. S., and A. M. Yaglom, 1975: *Statistical Fluid Mechanics: Mechanics of Turbulence*. 2d ed., Vol. 2, The MIT Press, 874 pp.
- Mordant, N., P. Metz, O. Michel, and J.-F. Pinton, 2001: Measurement of Lagrangian velocity in fully developed turbulence. *Phys. Rev. Lett.*, **87**, doi:10.1103/PhysRevLett.87.214501.
- Osborn, T. R., 1980: Estimates of the local rate of vertical diffusion from dissipation measurements. *J. Phys. Oceanogr.*, **10**, 83–89.
- , and C. S. Cox, 1972: Oceanic fine structure. *Geophys. Fluid Dyn.*, **3**, 321–345.
- Pearson, H., J. Puttock, and J. Hunt, 1983: A statistical model of particle motions and vertical diffusion in a homogeneous stratified turbulent flow. *J. Fluid Mech.*, **129**, 219–249.
- Taylor, G. I., 1921: Diffusion by continuous motion. *Proc. London Math. Soc.*, **20**, 196–211.
- Tennekes, H., and J. L. Lumley, 1972: *A First Course in Turbulence*. The MIT Press, 300 pp.
- Winters, K. B., and E. A. D'Asaro, 1996: Diapycnal fluxes in density stratified flows. *J. Fluid Mech.*, **317**, 179–193.
- Yeung, P., 2001: Lagrangian characteristics of turbulence and scalar transport in direct numerical simulations. *J. Fluid Mech.*, **427**, 241–274.
- , 2002: Lagrangian investigations of turbulence. *Annu. Rev. Fluid Mech.*, **34**, 115–142.