Near-Inertial Wave Propagation in the Western Arctic

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ABSTRACT

From October 1997 through October 1998, the Surface Heat Budget of the Arctic (SHEBA) ice camp drifted across the western Arctic Ocean, from the central Canada Basin over the Northwind Ridge and across the Chukchi Cap. During much of this period, the velocity and shear fields in the upper ocean were monitored by Doppler sonar. Near-inertial internal waves are found to be the dominant contributors to the superinertial motion field. Typical rms velocities are 1–2 cm s\(^{-1}\). In this work, the velocity and shear variances associated with upward- and downward-propagating wave groups are quantified. Patterns are detected in these variances that correlate with underlying seafloor depth. These are explored with the objective of assessing the role that these extremely low-energy near-inertial waves play in the larger-scale evolution of the Canada Basin. The specific focus is the energy flux delivered to the slopes and shelves of the basin, available for driving mixing at the ocean boundaries. The energy and shear variances associated with downward-propagating waves are relatively uniform over the entire SHEBA drift, independent of the season and depth of the underlying topography. Variances associated with upward-propagating waves follow a (depth)\(^{-1/2}\) dependence. Over the deep slopes, vertical wavenumber spectra of upward-propagating waves are blue-shifted relative to their downward counterparts, perhaps as a result of reflection from a sloping seafloor. To aid in interpretation of the observations, a simple, linear model is used to compare the effects of viscous (volume) versus underice (surface) dissipation for near-inertial waves. The latter is found to be the dominant mechanism. A parallel examination of the topography of the western Arctic shows that much of the continental slope is close to critical for near-inertial wave reflection. The picture that emerges is consistent with “one bounce” rather than trans-Arctic propagation. The dominant surface-generated waves are substantially absorbed in the underice boundary layer following a single roundtrip to the seafloor. However, surface-generated waves can interact strongly with nearby (<300 km) slopes, potentially contributing to dissipation rates of order \(10^{-6}–10^{-7}\) W m\(^{-2}\) in a zone several hundred meters above the bottom. The waves that survive the reflection process (and are not back-reflected) display a measurable blue shift over the slopes and contribute to the observed dependence of energy on seafloor depth that is seen in these upper-ocean observations.

1. Introduction

Near-inertial waves represent a resonant response of the ocean to impulsive forcing. The resonant frequency is \(f = 2\Omega \sin(\Phi)\), where \(\Omega\) is the earth’s rate of rotation and \(\Phi\) is the latitude. On a spherical earth, the waves, once generated, are constrained to refract toward lower latitudes (Munk and Phillips 1968). In the Arctic, near-inertial waves will invariably refract southward toward the ocean boundaries, potentially contributing to diapycnal mixing processes on the slopes and shelves. To estimate the magnitude of this process, we imagine a near-surface wind (wind) forcing over the 10\(^6\) m\(^2\) area of the Arctic Ocean resulting in an average downward near-inertial energy flux of order 0.01–0.1 mW m\(^{-2}\) (D’Asaro and Morehead 1991; Halle and Pinkel 2003; Merrifield and Pinkel 1996). Assuming propagation without significant energy loss, this corresponds to a shoreward flux of 10–100 W m\(^{-2}\), propagating across the 10\(^4\) km circumference of the shelf break. If this flux dissipates in an annular region of 50-km width, 200-m average depth, a mean dissipation of \(10^{-6}\) to \(10^{-5}\) W m\(^{-3}\) results. These mixing rates are comparable to energetic open-ocean and coastal sites. If real, they would have a great impact on property distributions in the Arctic.

This naive, “straw man” scenario posits that near-inertial waves propagate through the Arctic like surface swell propagates across the open sea. Despite the random (in space, time, and direction) midocean forcing of the swell, one always sees a landward energy flux at the borders of the ocean. The arriving flux represents the cumulative effect of past generation events distributed across a large area of the ocean surface, modestly diminished by dissipation and nonlinear transfers.

If dissipative effects are severe and trans-Arctic
propagation is unrealistic, can local near-inertial generation contribute significantly to deep mixing on nearby continental slopes? Consider a second straw man, where the energy input at a single square meter of the sea surface is mapped to a reflection area on the deep slope. For simplicity, a 2D situation is taken, with all of the energy radiated at a single near-inertial frequency. The total downward flux, $F_{dn}$, is delivered to a seafloor area $\sin(\theta)/\sin(\theta + \gamma)$, where $\theta$ and $\gamma$ are the angles of energy propagation and seafloor slope relative to the horizontal. If the full incident flux ($=2F_{dn}$ at the critical frequency, $\theta = \gamma$) is available for mixing an “internal surf zone” (Thorpe 2001) of depth $D$, the dissipation is $\varepsilon = (F_{dn}/D) \sin(\theta + \gamma)/\sin(\theta)$. Taking $F_{dn} = 0.1 \text{ mW m}^{-2}$, $D = 200 \text{ m}$, and $\theta = \gamma$, the resulting dissipation is $\varepsilon = 10^{-6} \text{ W m}^{-3}$.

Timmermans et al. (2003) find that a dissipation rate of $10^{-7} \text{ W m}^{-3}$ along the deep (>2500 m) boundaries of the Canada Basin is sufficient to balance the geothermal heat flux in the interior. Acknowledging that surface-generated energy does not always propagate shoreward, is not completely dissipated on the slope, and so on, this straw man flux estimate can be degraded by a factor of 10 and still contribute significantly to the geothermal flux balance. Much of the incident flux might be left to reflect upslope and shoal.

Given the near-absence of direct dissipation measurements over arctic slopes and shelves, there is some motivation for exploring the details of these straw man scenarios. A key issue is that of “propagation without loss.” Surface attenuation, volume attenuation, back reflection, and dissipation on deep slopes are the principal unknowns. Assessing the relative importance of these various processes is the focus of this paper.

The yearlong Surface Heat Budget of the Arctic (SHEBA) experiment provided an opportunity to investigate both the temporal and geographic variability of the Arctic wave field. The experiment was staged from the Canadian Coast Guard icebreaker des Groseilliers, which was frozen into the southern Canada Basin (75°N, 144°W) in October 1997 (Fig. 1). The ship and a surrounding ice camp subsequently drifted westward across the basin (November 1997–February 1998) over the Northwind Ridge (March–April 1998) and then northward along the Chukchi slope and over the Chukchi Cap (June 1998). The camp was recovered in October 1998 (80.5°N, 165°W).

Downward-looking Doppler sonars documented the internal wave field and lower-frequency current structures through much of the drift. The sonars profiled to 250 m, with a depth resolution of 3 m. Operation was interrupted by the periodic breakup of the camp over the winter and by problems associated with the partial melting of the ice cover in midsummer. Nevertheless, excellent views of the wave field were obtained in the central Canada Basin (December 1997), over the Chukchi Cap (spring–summer 1998), and in the deep ocean to the northwest of the cap (August–September 1998).

The Arctic Ocean is a unique wave propagation environment. Typical wave field energy levels are a factor
of 10–100 less than in the open ocean (Levine et al. 1985; D’Asaro and Morison 1992). Shear variance is also reduced, although to a lesser extent than energy.\(^1\) Nonlinear interactions, which proceed at a rate dependent on wave field energy, must be very weak in the Arctic.

In the deep Arctic Ocean, the waves are generated primarily at the sea surface by ice motion resulting from passing storms (Morison 1986; McPhee and Kantha 1989; Halle and Pinkel 2003). Unlike the open sea, where the horizontal scale of the surface stress is set by the scale of the storm, in the Arctic, the stress scale is also influenced by the morphology and relative motion of the ice. Ice keels and other small-scale features impose a vertical motion in the mixed layer, leading to the generation of relatively small-scale (<1 km), high-frequency waves (McPhee and Kantha 1989). Larger-scale, low-frequency waves result from lateral stresses imposed at the scale of the floe.

D’Asaro (1989) emphasizes that a relatively broad bandwidth of generated motion is needed if energy is to escape from the mixed layer into the thermocline below. There is now evidence (Halle and Pinkel 2003) that the efficiency with which an arctic wind event generates propagating near-inertial motions is inversely related to the horizontal scale of the floes, in accord with D’Asaro’s argument. While both high and low modes are generated, the relative proportion of high modes (\(l_z < 100\) m) is greater in the Arctic than in the open ocean, a consequence of the \(\approx 1–10\)-km scale of typical floes.

To assist in interpreting the SHEBA observations, it is useful to consider several simple propagation scenarios. As propagating near-inertial waves shoal (over the course of multiple surface and seafloor reflections), the energy density, \(E\) (J m\(^{-3}\)), of both upward- and downward-propagating waves should grow, varying inversely with bottom depth \(H\) in the absence of dissipation, generation, or lateral ray divergence (Fig. 2a). If, in contrast, the waves are back-reflected into the deep sea (Fig. 2b), there will be a reduction in upward arriving wave energy over the continental slopes (and a corresponding increase where the packet “resurfaces” offshore). If the waves have a lifetime that is long in comparison with a single round-trip surface to seafloor transit (the straw man scenario; Fig. 2c), the energy density of both upward- and downward-propagating waves will increase as the shelf is approached. Both will experience the \(H^{-1}\) “shoaling” effect. If waves decay after a single round-trip cycle, say because of underice dissipation (Fig. 2d), down-going energy will depend only on local generation conditions and will be independent of the depth of the seafloor below. In this case, the spatial concentration (shoaling) of upward-propagating wave packets over the slopes still occurs, although the magnitude of the effect is reduced relative to the nondissipative case. An examination of the spatial variability of upward- and downward-propagating wave energy, interpreted through simple model studies of propagation, dissipation, and reflection, can advance understanding of this process.\(^2\)

The SHEBA observations are presented in section 2. These include waves that are found in the upper ocean (50–300 m), either immediately following local surface generation/reflection (downward energy propagation) or more distant seafloor generation/reflection (upward energy propagation).

\(^1\) Nevertheless, the Gregg (1989) parameterization relating turbulent dissipation to background finescale shear predicts an eddy diffusivity that is smaller than molecular viscosity at subthermocline depths.

\(^2\) For simplicity, the potentially significant effects of nonlinearity, refraction by large-scale currents, and seafloor scattering are not considered. D’Asaro (1991) includes these processes in his discussion of open-ocean propagation. The failure of the linear model presented here to account for the occurrence of small-scale upward-propagating waves in the deep Arctic Ocean attests to the significance of these neglected processes.
A spatial propagation filter (section 3) is used to separate upward- and downward-propagating waves and to track wave energy and shear as a function of the depth of the underlying seafloor. The observations display a complicated dependence of $E$ on $H$. Temporal variability appears as a significant "noise," complicating the effort to identify geographic patterns. In general, both energy and shear variance associated with downward-propagating waves appear to be nearly independent of seafloor depth. For upward-propagating waves, a significant dependence is seen, with $E \sim 1/H^{1/2}$.

The issue of the reflection of near-inertial waves from a sloping seafloor is explored in section 4. Using a full ocean depth buoyancy profile and a digital bathymetric map, it is found that large regions of the western Arctic slopes are back-reflective or near-critically reflective to near-inertial waves. Deep local mixing along the Arctic slopes is likely, fueled by near-critically reflected waves. The signature of the bottom reflection process in the upper-ocean SHEBA observations is investigated in section 5.

A linear wave propagation model is applied in the appendix to address whether volume, seafloor (D'Asaro 1982), or underice attenuation (Morison et al. 1985) materially affect arctic wave propagation. The conclusion is that underice attenuation is a critical factor. Only near-inertial waves ($\sigma \sim f$), which propagate great distances between successive surface reflections, and high-frequency waves ($\sigma \gg f$), which rapidly traverse the underice turbulent boundary layer, can propagate over significant distances in the Arctic.

2. Observations

The Doppler sonar initially used in SHEBA was a 140-kHz four-beam instrument constructed specifically for Arctic operations. The transducers, each of dimension 12 in. vertical by 8 in. horizontal, had narrower beam widths ($\sim 2^\circ \times \sim 3^\circ$) than typical commercially available systems. These minimized sidelobe contamination from reflections associated with ice keels and other underice structures. Sonar beams were oriented 45° down from horizontal. A 4.5-ms repeat sequence code (Pinkel and Smith 1992) was transmitted with a 4.16-kHz bandwidth, yielding a depth resolution of 3.27 m.

The SHEBA sonar was designed for deployment from a well on the base ship. However, when the vessel ultimately selected for SHEBA proved to not have such a well, the sonar was deployed from a heated tent over a hole in the ice. The initial installation was performed in November 1997. One-minute-average profiles of echo covariance (velocity) and acoustic scattering intensity were recorded on optical disk. The winter record was interrupted in January by storms that broke up the SHEBA floe, temporarily disrupting electrical power. It terminated on 9 February during a period of floe rafting, when the sonar transducers were destroyed.

A replacement 161-kHz sonar was hastily constructed. This device used conventional 6-in. disk transducers angled 60° downward from horizontal. An 18-bit code of 12.5-kHz bandwidth was transmitted, resulting in depth resolution of 3.69 m. The data record resumed in late March and continued through the end of the experiment in early October 1998, with a few gaps over the summer.

During the course of the drift, a daily record of sea-floor depth was maintained using the ship's echo sounder. These data are used in the studies of wave shoaling presented here.

The thermocline can be characterized by the buoyancy frequency $N = \left[\frac{g}{\rho_0} \left(\frac{\partial \sigma}{\partial z}\right)\right]^{1/2}$, where $g$ represents gravitational acceleration, $\rho$ is the potential density of the fluid, and $z$ is positive downward. Profiles of $N$ are presented in Fig. 3 for three representative observational periods. In December in the Central Beaufort Sea, high stratification occurs at the base of the mixed layer ($\sim 40$ m) and at the deeper boundary between the halocline and Atlantic waters, centered at 200 m. In addition, an anomalous fresh (salinity $= 28$ psu) lens of water is found just beneath the ice (McPhee et al. 1998), providing significant stratification to the region. In June, over the Chukchi Cap, the distinct underice lens is no longer present. The seasonal pycnocline is shallower (20 m), with greatly reduced stability. By September, a new, seasonal pycnocline has formed with high stratification within 15 m of the surface. A deep secondary maximum in buoyancy frequency is not seen.

3. Wave field description

a. Depth–time variability

For initial analysis, the 1-min sonar velocity records have been hourly averaged and rotated into earth coordinates. The shear field is approximated as a 1.65-m vertical difference prior to February 1998, 1.85 m thereafter. Representative plots of the zonal component of shear are presented in Fig. 3. The record exhibits the natural tendency of shear to be concentrated in regions of higher buoyancy frequency, a consequence of linear refraction.

The December record shows a low energy wave field under a surface layer whose thickness and stratification vary in time. A storm on 6–8 December generates strong shears under the ice. The storm coincides with the passage of the camp over a coherent baroclinic eddy that extends throughout the low stratified region from 50–250 m. Following the storm and passage of the eddy, the downward propagation (upward-propagating crests) of a near-inertial wave packet is seen. The

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3 CTD data were graciously provided by J. Morison and R. Anderson of the University of Washington.
parent vertical group speed is 30–50 m week\(^{-1}\). Numerous upward-propagating packets are also seen.

The June shear field, over the Chukchi Cap, is much more energetic, with a distinct upward-propagating wave group dominating the first 5 days of the record. There is some hint of a corresponding downward-reflected wave packet on 9–12 June. The picture is obscured by a subinertial feature passing under the camp during 10–12 June, between 75–150 m. The pattern of a shear maximum with a contrasting maximum negative shear below indicates a jetlike velocity field. The feature might be one edge of a baroclinic vortex or an independent frontal structure. The camp passes directly over a small vortex on 15–17 June. This vortex has substantially greater shear than a Beaufort Sea eddy and is confined to a smaller region in depth, 25–150 m.

Fig. 3. Depth–time maps of (left) zonal shear and (right) the corresponding monthly averaged buoyancy frequency for three representative observational periods. The dominant shear is associated with near-inertial internal waves, propagating through a background of intermittent mesoscale activity. For these waves, the upward propagation of crests with time corresponds to the downward propagation of energy. Note the refractive enhancement of the shear field in regions of high stability and the baroclinic eddy that passes under the camp on 8 Dec. Shear levels in the interior of the eddy are very low. The horizontal line at 150 m in the Dec record is an artifact. The broad scar occasionally seen in the Jun data is the echo return from the seafloor associated with a previously transmitted acoustic pulse. Here and in Fig. 4, the “green” color represents offscale values less than \(-0.005 \text{ s}^{-1}\). Offscale values greater than +0.005 are pegged “red.”

4 During the period of observation, 1–25 December, the camp drifted 150 km southwestward. The apparent vertical expansion of the wave packet into the upper ocean is in fact a mix of vertical propagation and lateral variability. Further subinertial activity is seen around 27 June. By open-ocean standards, energy levels over the Chukchi Cap are low. Shear levels, however, are comparable to energetic open-ocean or coastal sites. The small vertical scale of the motions, relative to lower latitude wave fields, leads to the large shears. The depth–time intermittency of the wave groups appears to be greater than in the December deep-sea observations.

The August–September shears are relatively uniform in variance over the upper 150 m. A few distinct downward-propagating wave packets stand out above the background. Subinertial activity is seen on 31 August at 25-m depth and 14–15 September from 20 to 100 m.

While a continuous spectrum of motions is present, analyses to be presented in a subsequent work indicate that the field is primarily near-inertial. The apparently broad range of frequencies is a result of simple Doppler shifting. For near-inertial waves, the vertical direction of propagation is given by the rotation of the wave-current pattern with depth (Leaman and Sanford 1975).
In the Northern Hemisphere, downward-propagating wave energy is associated with a clockwise rotation of horizontal currents with increasing depth.

The rotary polarization of the waves (with depth) can be used to separate the overall wave field into upward- and downward-propagating constituents. Unlike traditional wavenumber–frequency spectral analysis (e.g., Pinkel 1984), continuous time series are not required. Intermittent data gaps associated with camp power failures, for example, do not degrade the quality of the separation. However, higher-frequency internal waves that are not rotationally polarized will contribute variance to both positive and negative wavenumbers in a rotary spectral separation.

To perform the separation, vertical profiles of complex current, \( U(z) = u_{\text{East}} + i u_{\text{North}} \), are high-pass filtered in time to include motions of period 15 h and shorter and are Viisala [Wentzel–Kramers–Brillouin (WKB)] stretched (Pinkel 1984) in both amplitude and vertical phase. The stretching is normalized such that the total kinetic energy and the vertical extent of the observation window are preserved. The stretched profiles are then first differenced in depth and Fourier transformed between depths of 39 and 295 m, below the region of most rapid variation of the buoyancy frequency. Fourier coefficients are then “recolored.” Velocity spectral variance is accumulated in specific vertical wavenumber bands, for both positive and negative wavenumbers. Shear products are obtained by weighting the velocity Fourier coefficients by \( ik \) and processing in parallel with velocity. Daily averaged vertical wavenumber spectra are then archived for the geographic variability studies presented below.\(^5\)

Reconstructed “depth profiles” of strictly upward and strictly downward wave motion can be obtained by inverse transforming “reconstituted” sequences of Fourier coefficients. To produce profiles of purely upward (downward) propagation, the negative (positive) wavenumber coefficients are replaced by the complex conjugate of the corresponding positive (negative) wavenumber coefficients. Normalization of the profiles by \( 2^{-1/2} \) preserves the variance of the overall field.

The quality of this rotary separation can be investigated by comparing the many independent hourly velocity profiles. Downward energy propagation should be associated with the upward propagation of wave phase with time. If the sense of rotation in depth indeed specifies the sense of phase propagation in time, the assumption that the shear is primarily near-inertial is confirmed.

Examination of rotary filtered hourly profiles reveals coherent patterns of vertical phase propagation, with the proper relationship between rotation in depth and propagation in time (Fig. 4). Thus, despite the apparently continuous spectrum of temporal frequencies observed, these arctic waves are highly polarized and must have intrinsic frequencies near inertial. Data from throughout the drift have been rotary filtered, providing a look at wave variability with both season and location.

The daily averaged velocity (kinetic energy) and shear variances of the wave field are presented as a function of time in Fig. 5. Individual points represent an integration over 20–160-m vertical scale of the corresponding vertical wavenumber spectra of velocity and shear. The time history of upward-propagating kinetic energy density (Fig. 5a) shows a variation of about a factor of 5 over the year, with peak values occurring over the Chukchi Cap in midsummer. Smallest values are seen in deep water, in the center of the basin (December) and north of the cap (October 1998). Variability in downward energy density (Fig. 5b) is slightly less. Values center on \( \sim 0.7 \text{ cm}^2 \text{ s}^{-2} \), except during May–June, when enhanced energy is seen. A similar pattern is seen in the daily averaged shear (Figs. 5d,e). Again, slightly greater variability in shear is associated with upward propagation. The seafloor depth, obtained from an echo sounder on the des Groselliers is given in Fig. 5c, with slope angle magnitude given in Fig. 5f.

**b. Wave field variability with seafloor depth**

The dependence of the upper-ocean velocity and shear variance on underlying seafloor depth is presented in Fig. 6. Individual entries represent an integration over 20–160-m vertical scale of the corresponding daily averaged vertical wavenumber spectra\(^6\) of velocity and shear. The vertical extent of each entry indicates the 90% confidence interval, calculated under the assumption that the process is spatially homogenous.

At depths shallower than 500 m, velocity and shear variances are at maximum levels and are similar for both upward and downward propagation. This cluster of energetic values is associated with the June–July Chukchi Cap passage, when subinertial activity is high (Fig. 3), the potential for seafloor generation is greatest, and the travel time between surface and seafloor is the smallest. For the moment, let us exclude these ob-

\(^5\) For the shear variance estimates of Figs. 5 and 6, and the spectral presentations of Figs. 7, 8, 9 and 12, individual hourly spectral estimates are divided by the factor \( \sin(k_\text{res} \Delta z_{\text{res}}) \) to account for the smoothing resulting from the finite duration of the transmitted pulse (vertical extent \( \Delta z_{\text{res}} \)) and subsequent averaging in range. A modeled noise corresponding to 5 mm s\(^{-1}\) rms hourly averaged velocity uncertainty is then removed. The combined effect of these various corrections is minimal at scales greater than 10–15 m. To avoid presenting results that are overly sensitive to the “corrections,” spectral displays are here truncated at 10-m vertical scale.

\(^6\) To assess the statistical stability of the various spectral quantities presented, representative wavenumber frequency spectral estimates of shear are first formed. These are subsequently inverse-transformed in frequency and normalized to form estimates of coherence as a function of time lag and vertical wavenumber. Statistical stability is then estimated using the method of D’Asaro and Perkins (1984).
servations from the discussion and concentrate on a comparison between variance levels over the slopes and over the deep seafloor.

For upward-propagating waves, the depth variability of upper-ocean velocity variance (Fig. 6a) displays a somewhat irregular trend, with $E_{up} \sim H^{-1/2}$. The pattern associated with downward-propagating waves shows no perceptible trend (Fig. 6b). The suggestion is that, in the upper ocean, downward-propagating wave energy density is independent of underlying seafloor depth, at all depths greater than 500 m. Shear variance exhibits a slightly more convincing dependence on ocean depth (Fig. 6c) for upward-propagating waves, with little dependence seen for downward waves (Fig. 6d), provided the Chuckchi Cap data are excluded.

Of particular interest is the disparity between the predicted 80% confidence limits and the observed variability of the signals. For the downward-propagating waves, variability in velocity variance (energy density) presumably results from episodic forcing events. For the upward-propagating waves, the variability over the slopes is much greater than would be predicted for a spatially homogenous process.

One can minimize the influence of temporal variability by averaging the daily velocity and shear spectra from the entire experiment, binned at like values of underlying seafloor depth. The horizontal velocity (kinetic energy) spectra associated with upward-propagating motions (Fig. 7a) are smallest when in the deep sea. Spectral levels increase significantly with decreasing depth over slopes shallower than 2000 m. Spectra associated with downward propagation (Fig. 7b) are essentially independent of the depth of the underlying seafloor. Notable exceptions occur for $H \leq 500$ m, over the Chukchi Cap (not plotted), and for $H > 3000$ m. These deep data are primarily from the central Beaufort Gyre in December 1997.

The up-down ratio (Fig. 7c) indicates a general surplus of downward-propagating energy in the deep sea, with the excess most pronounced in the 20–40-m vertical wavelength band. Over the slopes ($H < 2000$ m) the situation reverses, and upward propagation dominates, again most strongly in the 20–40-m band.

Shear spectra (Figs. 7d, e) are related to the velocity spectra by the deterministic factor $k^2$. As expected, they too show a growth in the variance of upward-propagating waves with decreasing seafloor depth. The downward shear spectra are more nearly uniform in depth, excepting the ranges $H < 500$ m and $H > 3500$ m.

The ratio of upward- to downward-propagating shear (Fig. 7f) is essentially identical to that of velocity, as expected. In the deep sea the variance associated with downward propagation exceeds that of upward. In water depths less than $\sim 2000$ m, the situation reverses, with the change most pronounced for 20–40-m scale motions.

The depth-binned spectra of Fig. 7 are replotted in Fig. 8, in log–log format. Wavenumber spectra associ-
ated with downward-propagating energy and shear (Figs. 8b,d) are substantially independent of seafloor depth. The spectra of upward-propagating waves (Figs. 8a,c) have less variance over the deep sea, but grow to have greater variance at high wavenumber as the water shoals.

If wave reflection from a sloping seafloor is the cause of these changes, some component of the increased energy density might result from the spatial compression of the individual reflected wave packets. It follows that there might be an observable increase in the space–time variability of the vertical wavenumber spectrum as the ice camp drifts over the varied topography of the western Arctic.

From Fig. 6, it is apparent that spatial inhomogeneity, rather than statistical imprecision, sets the variability of the spectral estimates. This variability can be quantified by examining the temporal fluctuation in the daily vertical wavenumber spectra used in the vertical separation.

The daily spectra, $E(k_z; t, H)$, are again sorted into categories based on the underlying seafloor depth. Within each category the variability of the daily spectra relative to the cruise-long mean, $\overline{E}(k_z; H)$, is calculated as a function of wavenumber. Since, formally, all bands of the spectrum are estimated at the same statistical precision, the underlying uncertainty in the estimates should not be a function of wavenumber.\(^7\) Plots of the ratio of the variance in spectral level, $(\langle E(k_z; t, H) - \overline{E}(k_z; H) \rangle^2)/\overline{E}(k_z; H)^2$ between upward and downward waves (positive versus negative wavenumber bands in the same spectral estimate) are presented in Fig. 9 as a

\(^7\) In fact, there is a possibility that the effective number of degrees of freedom does vary as a function of wavenumber. If the depth–time extent of the packets in one wavenumber band is longer than in another, there will be fewer independent observations of these waves per unit depth–time. The effective number of degrees of freedom will be less in this band, with increased spectral variability as a consequence.
function of underlying seafloor depth. As with the level of the spectrum, an increase in the intermittency of the spectrum is seen for upward-propagating waves over topography (depths less than 2000 m). The increase is again most pronounced for the shorter waves (10–40 m). Variability in the spectra at low wavenumbers, both positive and negative, is similar.

4. Reflection from continental boundaries:
A geographic summary of the western Arctic

Surface-generated wave groups that are "within range" will eventually encounter the continental slope. They can then forward reflect into shallow water, back-reflect into the deep sea, or dissipate in a near-bottom boundary layer. Forward reflection leads to increasing energy densities as the waves shoal. To assess the likelihood of these three possibilities, the topography of the western Arctic can be examined from the perspective of wave reflection.

The issue of forward versus back reflection is determined by the angle of the seafloor at the reflection site, $\gamma = \tan^{-1}(|\nabla H|)$, relative to the vertical inclination angle of wave energy propagation, $\Theta = \tan^{-1}(\Delta/N)$ ($\sigma \ll N$), and $\Delta = (\sigma^2 - f^2)^{1/2}$. Formally, pure inertial waves ($\Delta = 0$) are back-reflected by arbitrarily small topographic slopes (Sandstrom 1969; Phillips 1966; Eriksen 1982, 1985; Garrett and Gilbert 1988).

Consider an idealized situation (the Garrett 2001 scenario) where the near-inertial waves arriving at the continental slope are generated as purely inertial waves at higher latitudes. Thus, $\sigma = 2\Omega \sin(\Phi_0)$, where $\Phi_0$ is the latitude of origin of the wave group,

$$\Delta = 2\Omega(\sin^2\Phi_0 - \sin^2\Phi)^{1/2}, \quad (1)$$

and

$$\Theta = \tan^{-1}[(2\Omega/N)(\sin^2\Phi_0 - \sin^2\Phi)^{1/2}]. \quad (2)$$

Forward reflection occurs when $\Theta$ is greater than the encountered topographic slope magnitude, $|\gamma(x)|$,

$$\Theta/|\gamma| = \tan^{-1}[(2\Omega/N_{\text{bottom}})(\sin^2\Phi_0 - \sin^2\Phi)^{1/2}]|\gamma(x)|$$

$$> 1. \quad (3)$$

Fig. 6. Daily averages of velocity and shear variance are presented as a function of underlying seafloor depth for both upward- and downward-propagating waves. Vertical lines give the 80% confidence interval for each estimate. For downward-propagating waves, there is little relationship between variance and depth, except for depths less than 500 m. For upward-propagating waves, there is a clear increase in variance with decreasing depth, although the pattern is contaminated by strong secular variability. The curves give the least squares linear fit of log(depth) to log(variance), for depths below 500 m.
The tendency for forward reflection is enhanced in regions of low $N_{\text{bottom}}$. Inertial waves originating far poleward of the topography are also more likely to forward reflect.

Upon reflection, wave frequency is maintained, but both the cross-slope horizontal wavenumber and the vertical wavenumber change such that both the normal flow and normal energy flux vanish at the boundary. The ratio of reflected to incident vertical wavenumbers, $R_k$, a useful metric of this change, is given by

$$ R_k = \frac{(a^2 + 2Ca + 1)(a^2 - 1)}{a^2}, $$

where $a = \tan(\gamma)\tan(\Theta)$, $C = \cos(\phi)$, and $\phi$ is the azimuth of the incident wave relative to the upslope normal to the isobaths (Eriksen 1982). For forward upslope reflection, $R_k > 1$. The energy density of the reflected wave packet is increased by the factor $R_k^2$.

Using a single buoyancy profile as characteristic of the western Arctic, maps of $\Delta f$ and $\Phi_0$ are presented in Figs. 10a and 10b. The southwestern boundary of the Canada Basin is seen to be back-reflective to locally generated waves over a significant fraction of its slope (Fig. 10b). Waves generated in the high Arctic are more likely to forward reflect if they can reach this section of coastline.

There are a number of possible characterizations of the western Arctic that can be distilled from the geographic map. In Fig. 11a, the average amplification of forward-reflected waves, calculated at five values of $\sigma f$ at each of the geographic points in Fig. 10, is presented as a function of seafloor depth. For simplicity, reflection at a normal incidence angle, $C = 1$, is considered at all locations. The “mean” amplification is significant for...
waves of frequency $<1.1f$. However, it is strongly dependent on those few sites where near critical reflection occurs and the amplification ratio approaches infinity.

Several other statistics are more useful. (The ratio of backscattering to forward scattering topographic area is given in Fig. 11b). Backscattering is significant at depths above 2000 m for waves of frequency $1.03f$ or less. Higher-frequency motions are generally able to progress up-slope in the western Arctic.

A significant concern is the ratio of topographic area where the waves are forward scattered but not significantly amplified/spatially compacted to the total area (Fig. 11c). These regions are associated with the transmission of wave energy to shallower waters, where breaking might subsequently occur. At depths greater than 2000 m, most of the slope reflects with little amplification (for $\sigma > 1.03f$). At shallower depths, the likelihood of significant amplification on reflection increases.

Extreme amplification might lead to local mixing just above the deep slopes (Eriksen 1982; Garrett and Gilbert 1988). In Fig. 11d, the ratio of topographic area with greater than fivefold amplification to the total area is presented. For waves $\sigma = 1.01f$, 20% of the deep arctic slope is associated with extreme amplification and potential deep mixing. This ratio rises to 40% at depths less than 1 km. Higher-frequency ($\sigma \geq 1.03f$) waves display a similar trend, at reduced levels.

5. Upper-ocean estimates of deep seafloor slope

For an individual wave packet, upslope reflection leads to an increase in energy density of $R_z^2$ (Phillips 1966; Eriksen 1982). One factor of $R_z$ results from the vertical compaction of the wave group and one from the horizontal. The signature of this process in a moored measurement can be either an increase or a decrease in energy, depending on whether the sensor is receiving the reflected “glint” off a topographic “facet” or not. For horizontal or vertical profiling measurements (such as these SHEBA acoustic profiles) there should be no net energy change observed, as the increased energy density of the packet is offset by the

**Fig. 8.** Vertical wavenumber spectra of (a), (b) velocity and (c), (d) shear as presented in Fig. 7, obtained in depth intervals of 500–1000 m (green) through 3000–3500 m (blue). While the up-down difference in the mean spectral levels (thick lines) is small, a deterministic trend is seen in the spectra associated with upward (but not downward) propagation. (c) Upward shear spectra increase in level and shift to the right as the water depth shoals. A parallel shift is seen in the velocity spectra in (a), although the net variance increase is significantly reduced relative to the shear. The vertical bar gives the 95% confidence interval for the 500–1000-m “depth interval,” which has the fewest independent data. Variability in the “upward” spectra generally exceeds this limit, a consequence of the spatial inhomogeneity of the wave field.
reduced probability of encountering it. A shift of the packet to smaller vertical scale, with an associated increase in shear variance, is expected, however.

Barring back-reflection and bottom boundary layer dissipation, the spatial density of wave packets will increase with shoaling, leading to an expected factor of $H^{-1}$ increase in the climatological velocity and shear variances.

If upslope reflection is indeed responsible for the increase in observed upward-propagating shear, the product $H(S^2)$ should change only as a consequence of the shift in spectral scale to higher wavenumber by the factor $R_k$. Thus, upper-ocean observations of vertically propagating shear contain information about the characteristic $R_k$ at the seafloor. Can this signal be detected?

To attempt an estimate of $R_k$, the linear propagation/dissipation model (see the appendix) is applied. With the observed kinetic energy spectrum associated with downward propagation taken as the initial state, the model [Eq. (A1)] is applied band by band to propagate the spectrum to the seafloor. At this point, the wavenumber scale of the reflected spectrum is shifted by the factor $R_k$. The spectral density is increased by $R_k$, as well, assuming reflection without loss. This corresponds to an increase in packet energy density by $R_k^2$, spread over the broader spectral bandwidth. The “upward spectrum” is next propagated to the surface, again using Eq. (A1). Overall velocity variance is then reduced by the factor $R_k^2$, accounting for the reduced likelihood of observing the spatially compacted wave groups that comprise the spectrum. The modeled shear spectrum is derived from the velocity spectrum by multiplication by $k^2$.

Plots of the modeled “reflected” spectrum, as it would appear in the upper ocean, are compared with the observed spectrum of upward propagation in Fig. 12. The resemblance is not unreasonable, given that the upward waves have been perhaps generated weeks to months earlier and many kilometers from the site where they are eventually observed. Perhaps not surprisingly, the best agreement is seen for modest assumed values of $R_k$ ($=1.25$). More extreme “blue” shifting is probably associated with local mixing near the seafloor. In the upper ocean, one gets to observe the “survivors” of the reflection process.

The deep-water flat-bottom case (Fig. 12d) is included for contrast. Clearly, simple viscosity is insufficient to explain the change in spectral levels between upward- and downward-propagating waves in these December 1997 observations. Previous observations in the deep Canada Basin (Merrifield and Pinkel 1996; Halle and Pinkel 2003) do not exhibit this degree of vertical anisotropy.

6. Summary, discussion, and speculation

With typical velocities of only 0.01–0.02 m s$^{-1}$, the measurement of arctic internal waves represents a sig-
significant technical challenge. The attempt to monitor these motions over the yearlong SHEBA drift produced a number of surprises. Internal wave shears are so strongly concentrated at near-inertial intrinsic frequency that the sense of shear rotation in depth is an effective index of the (vertical) direction of propagation in time. Velocity and shear variances associated with downward-propagating waves fluctuate irregularly over the course of the drift, except during June and July, when the camp is over the Chukchi Cap. Here, levels are elevated. Variances associated with upward propagation are clearly related to the depth of the underlying seafloor, with $E \sim H^{-0.5}$. Over sloping terrain, upward energy frequently exceeds downward. This requires explanation, given the belief that these motions are generated at the sea surface. The space–time variability of the upward-propagating waves is also much greater than the variability of downward propagation groups, and is consistent with significant spatial nonhomogeneity. Vertical wavenumber spectra of upward-propagating shear are blue-shifted relative to their downward-propagating counterparts. The shift is an upper-ocean signature of the seafloor slope, thousands of meters below.

Vertical anisotropy is reduced at the smallest vertical scales resolvable by the sonars (~5–10 m). A linear propagation model (see the appendix) suggests that even molecular viscosity would be sufficient to inhibit round-trip propagation at these scales, increasing the apparent anisotropy. Presumably, nonlinear transfers supply energy to these scales, reducing the anisotropy. (The effectiveness of the vertical propagation filter is also questionable if the vertical waveform of these small-scale motions is significantly nonsinusoidal.)

The initial “straw man” experiment posed in the Introduction suggested a growth in near-inertial energy density from the center to the edges of the deep Arctic Ocean and an additional $1/H$ increase in wave energy density from the deep sea up over the slope. In fact, no correlation between the downward energy flux and location or seafloor depth is seen (excluding the very energetic downward waves over the Chukchi Cap). This is consistent with signature of “one-bounce propagation” (Fig. 2d; appendix) and suggests that underice dissipation is effective in inhibiting the long-range propagation of Arctic near-inertial waves. The model of Morison et al. (1985) predicts the limiting effect of underice dissipation (although the SHEBA observations are less than a definitive test of their model). If the stability of the underice boundary layer is radically increased, by greater river runoff or summer melt, the dissipative dynamics of the wave field might change significantly. In past or future epochs of an ice-free Arctic, the near-inertial energy delivered to the basin boundaries and available for boundary mixing might be significantly larger than at present.

The one-bounce propagation physics of the Arctic suggests that only energy from atmospheric forcing within 300–500 km of the shelves will contribute to boundary mixing. However, roughly 60% of the Arctic
Ocean is within 300 km of a boundary. A significant concern is with the direction of initial propagation. Generated waves are not constrained to propagate to the nearest coast. Equatorward refraction will be a limited factor over the one-bounce path. Waves propagating parallel to the coast or offshore will not survive the transbasin journey. It is thus probably more realistic to posit a $1-10 \text{ W m}^{-2}$ energy flux impinging on the Arctic slopes, arbitrarily “downsized” by a factor of 10 from the straw man value presented in the introduction.

Large areas of the continental slopes of the western Arctic are near-critical for waves in the near-inertial frequency band. Recently, Cacchione (2002) has found a similar behavior for continental slopes at semidiurnal frequency. It is likely that the back-reflection and near-slope dissipation of the incoming wave energy flux reduces the observed energy enhancement relative to the reference dependence $E = H^{2/3}$. The relative incidence of back reflection and/or extreme amplification is sensitive to small departures, $\Delta$, of the wave intrinsic frequency from $f$. The SHEBA data are not adequate to resolve intrinsic frequency to the required precision. Given that a typical wind generation/ice motion event has a lifetime of $3-6f^{-1}$, one can imagine spectral bandwidths of order $0.16-0.3f$ being energized. Following dispersive propagation to the seafloor, approximately

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**Fig. 11.** Scattering properties for this quadrant of the Arctic are summarized in terms of regions of like water depth by considering waves of frequency $\sigma = (1.01, 1.03, 1.10, 1.30)f$. Here $f$ is evaluated locally at each geographic point in Fig. 10, and normally incident scattering is assumed. (a) The mean amplification of forward-scattered waves is significant on both the lower and upper slopes. This average is dominated by the few critical locations where amplification approaches infinity. (b) For low-frequency waves ($\sigma < 1.01f$), a significant fraction of the upper slope becomes back reflective. (c) With the exception of $\sigma < 1.03f$ waves, most of the western Arctic slope is forward reflective, with moderate amplification. (d) Low-frequency waves can experience extreme amplification and presumably contribute to deep local mixing.
5%–20% of the incident flux will be at frequencies sufficiently low to interact strongly with topography (Fig. A1). Even in the Arctic, the $\beta$ effect is significant in the propagation of equatorward traveling near-inertial ($\sigma < 1.1f$) waves. Both vertical group speed and propagation slope increase along the propagation path. This improves the chance for forward reflection on continental slopes and reduces the contact time with the dissipative underice boundary layer. However, the distance traveled before reencountering the surface is significantly shortened. Zonally propagating near-inertial waves experience these effects only following $\beta$-induced equatorward refraction.

The process of interpreting the complex 3D scattering problem (suggested by Figs. 2 and 10) in terms of simplified 2D models is, of course, suspect. The approach has proven worthwhile, in part because the energy enhancement associated with the up-ice reflection process is proportional to the spatial compaction of the reflected wave groups. The greater the enhancement of the reflected group, the less likely that it will be passing through the measurement volume at any given time. This principle applies independent of the azimuth of the incident wave relative to the topographic slope. Spectral changes are primarily associated with changes in the scale and bandwidth of the reflected waves, as well as the overall increase in proximity of independent wave groups as the water shoals.

Although the total energy of a wave group is unchanged on reflection from a slope, the increase in energy density within the group combines with the change in scale to alter the shear variance by the factor $R \mathbf{k}$. Gregg (1989) has found empirical evidence that mean turbulent dissipation levels are proportional to the square of shear variance, or $R \mathbf{\kappa}^2$. Thus turbulent mixing in the near field of up-ice reflection regions might be greatly enhanced, particularly in more energetic temperate oceans.

Given an established vertical mixing rate, Young et al. (1982) have suggested that lateral diffusion in the ocean is significantly enhanced in the presence of inter-

![Figure 12](image_url)
nal wave shears. They express this enhancement [their Eq. (45)] in terms of an eddy diffusivity, $\Delta K_H$, where

$$\Delta K_H = 1/4K_v(S^2_1)(f^2).$$  \hspace{1cm} (5)

Here $K_v$ is the vertical eddy diffusivity and $(S^2_1)$ is the wave field shear variance, considering motions down to a vertical scale of order 1 m. For an rms (1 m) shear of order 0.005 s$^{-1}$, $K_H \sim 300K_v$ in the polar ocean.

If the effective vertical diffusivity is associated with wave breaking, the combination of Gregg’s observation with the Young et al. (1982) model suggests that lateral diffusivities vary as the sixth power of rms wave field shear, further emphasizing the role of wave field variability. Armi (1979) and more recently Samelson (1998) emphasize the significant effect of a spatially variable diffusivity on the large-scale circulation. Observations in the near field of likely slope-mixing sites are called for to establish the reality of these conjectures.

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APPENDIX

Modeling Near-Inertial Propagation and Attenuation in the Western Arctic

In Figs. 5–9 we find no trend in the magnitude of the downward-propagating energy/shear over the course of the drift. A weak (inverse) dependence of variance on seafloor depth is found for upward-propagating waves. This is consistent with the propagation scenario illustrated in Fig. 2d, where underice absorption limits the propagation of surface-generated waves to a single round trip. In this appendix, a simple linear propagation/dissipation model is introduced to assess the cumulative effect of both volume and surface (underice) dissipation. The model is described in detail in the second section of this appendix. In the first section, the model is briefly introduced and the scientific findings are summarized.

a. Application of a linear near-inertial propagation model to the Arctic

Key assumptions of the model are the following.

- The underice boundary layer is the generation region for downward-propagating near-inertial waves.
- Near-inertial wave dissipation at the flat seafloor of the Canada Basin is modest.
- For simplicity, strictly meridional propagation is considered.
- The cross-scale transfer of energy by nonlinear processes is neglected. It is presumed that nonlinear transfer rates are low in the Arctic, because of the low overall energy level of the wave field.
- Refraction caused by variable background currents, a potentially significant process in the Arctic, is not considered.
- The lateral divergence of wave energy with decreasing latitude, associated with the sphericity of the earth, is neglected as well. The model can be easily extended to include this effect. For purely meridional propagation, $E \cos(\Phi)$ is the appropriate conserved quantity. Over the latitude range of the SHEBA drift, 75°–82°, the signature of relative divergence is small in comparison with the observed scatter in the data.

For propagation from depths $z_0$ to $z_1$, the modeled variation in wave energy density takes the form

$$E(z_1) = E(z_0)N(z_1)/N(z_0)e^{-q(z_0,z_1)}.$$  \hspace{1cm} (A1)

Here

$$q(z_0, z_1) = (2\nu k^3_0 \sigma / \Delta^2) \int_{z_0}^{z_1} N^2 dz$$  \hspace{1cm} (A2)

describes the cumulative effects of viscous decay, with $\nu$ being a coefficient of viscosity, $k_0$ the horizontal wavenumber, and $\sigma$ the wave frequency, and $\Delta = (\sigma^2 - f^2)^{1/2}$ (see following section).

To estimate surface and bottom reflection losses, the turbulent boundary layer model of Morison et al. (1985) is applied (again, see following section). The central feature of the model is the “no-slip” condition that is applied at the ice–water interface. Internal wave horizontal velocities are constrained to decay as the interface is approached. Momentum diffuses down the resulting velocity gradients at a rate established by the background near-surface flow, which is assumed to have established a turbulent underice boundary layer.$^A1$ Morison et al. assert that the rate of wave energy loss during reflection is

$$\varepsilon_{BL} = \rho C_d U_{\text{mean}} u_{\text{wave}}^2 = 2C_d U_{\text{mean}} E_{\text{wave}} \text{(W m}^{-2}),$$  \hspace{1cm} (A3)

where $C_d$ is a characteristic drag coefficient ($C_d = 0.0034$ for the underice surface), $U_{\text{mean}}$ is the mean, low-frequency flow that establishes the underice boundary layer, and $u_{\text{wave}}$ is the perturbation velocity associated with the waves.

$^A1$ In the open sea, “preexisting” wind driven turbulence is typically much greater than the turbulence found under arctic sea ice. However, there is no analog of the “no slip” condition in the open sea. The predictions of extreme energy loss obtained from this model are thus not relevant to open-ocean conditions.
At the seafloor, both $U_{\text{mean}}$ and $u_{\text{wave}}$ are small. Morison et al. (1985) and D’Asaro (1982) conclude that seafloor attenuation is negligible. At the underice surface, we write

$$\varepsilon_{\text{BL}} = c_{\text{gz}}(E_{\text{up}} - E_{\text{dn}}) = 2C_d U_{\text{mean}}(E_{\text{up}} + E_{\text{dn}}).$$

(A4)

For a single reflection, $E_{\text{dn}} = \alpha_i E_{\text{up}}$, where $\alpha_i$ is the underice reflection coefficient, which, from (A4), is

$$\alpha_i = \frac{c_{\text{gz}} - 2C_d U_{\text{mean}}}{c_{\text{gz}} + 2C_d U_{\text{mean}}}. \quad (A5)$$

An attractive aspect of the Morison et al. model is that it is linear in wave energy. Thus, the expression for $\alpha_i$ derived for a single reflection also applies to a superposition of wave groups (of identical group velocity) arriving at the reflection site from a variety of distant sources. A weakness in the model is that the rate at which a wave packet loses energy in the boundary layer, $\varepsilon_{\text{BL}}$, is not constrained by the rate of energy supply to the boundary, $c_{\text{gz}} E_{\text{up}}$. This formally enables $\alpha_i$ to assume nonphysical negative values. However, over most of the wavenumber–frequency domain, and for typical ice speeds of $U_{\text{mean}} < 0.3 \text{ m s}^{-1}$, $\alpha_i$ is positive.

To examine the interplay of the factors that affect propagation, we evaluate (A1)–(A5) assuming a range of values for viscosity ($1.8 \times 10^{-6} \text{ m}^2 \text{s}^{-1}$, molecular, representative of the deep Arctic Ocean, $5 \times 10^{-6}$, typical of the midlatitude open ocean, and $10^{-4}$, representing a canonical region of great mixing), for $U_{\text{mean}} = (0.03, 0.1, \text{and } 0.3 \text{ m s}^{-1})$ and for a variety of vertical wavenumbers and frequencies. A double exponential buoyancy profile (Fig. A2a) is used in this calculation.

Initially examining a single round-trip cycle through the exponential waveguide, Figure A1a presents the ratio of viscous to surface reflection losses. In the intermediate–wavenumber, low-frequency section of the plot, absorption in the underice boundary layer is total. Throughout the regime, underice absorption equals or exceeds volume dissipation by up to 5 orders of magnitude. Viscosity only becomes a significant influence at scales so small and frequencies so low that the waves are unable to complete a single round trip.

Attention is thus focused on the discretization of the propagation into distinct surface encounters. In Fig. A1b, the horizontal propagation distance between successive surface reflections is presented as a function of frequency for ocean depths of 500 and 4000 m. The low-frequency limit to the calculation is $\sigma = 1.005 f$, at which point propagation distances of $1000 \text{ km}$ separate successive surface reflections in the deep basins.

Given the fractional energy loss per reflection, $\alpha_i$, how many reflections occur before $E/E_0 \leq e^{-1}$, that is, $\alpha^N_i = e^{-1}$. Figure 10c presents this value as a function of vertical wavenumber and frequency. Apparently, a large vertical component of group velocity is needed to survive attenuation in the underice boundary layer. Using this $e$-folding criterion, waves in the near-inertial band and short, higher-frequency waves will not survive a single encounter. Only the low-mode high-frequency waves [reported to have significant vertical displacement and coherence by Levine (1990)] can survive the reflection process.

Representative maps of “propagation distance” are given as a function of frequency and vertical wavenumber in Figs. A1d–f, for assumed mean ice speeds of 0.03 (Fig. A1d), 0.10 (Fig. A1e), and 0.30 (Fig. A1f) m s$^{-1}$ and viscosities of $1.8 \times 10^{-6}$ (Fig. A1d), $5 \times 10^{-6}$ (Fig. A1e), and $10^{-4}$ (Fig. A1f) m$^2$ s$^{-1}$. Propagation distance is defined as the path traveled before the wave packet attenuates to $e^{-1}$ of its original energy. The highest-wavenumber, low-frequency waves (area 1 in Fig. A1f) are attenuated before completing a single round trip. The influence of the varying levels of viscosity is seen in the region of horizontal contour lines at high $k_z$. Low-wavenumber near-inertial motions can travel in excess of 500 km, by virtue of their shallow propagation angle. These waves do not survive a single surface encounter even at low ice speeds. Hence their propagation distance is unaffected by changes in ice speed. They occupy the region of $\Delta k_z$ space characterized by vertical contours (area 2).

To the right of the curved boundary are the waves that can survive one (area 3) or more (area 4) encounters with the surface. In this region, the expected propagation distance is highly sensitive to ice speed. Periods of rapid ice motion might sweep the established high-frequency wave field from the upper halocline, perhaps replacing it with freshly generated waves. The “saw tooth” pattern seen at intermediate vertical wavenumbers reflects the degree of attenuation associated with each incremental surface bounce. The slope of the individual “teeth” reflects the fact that the lowest frequency waves, for a given number of surface reflections, will propagate farthest, because of their shallow propagation angle. The lateral separation between the teeth indicates the increment in frequency necessary for a wave of fixed $k_z$ to survive an additional reflection.

These simple results require modification if the earth’s curvature is considered. Specifically, equatorward-propagating near-inertial waves return to the surface (Figs. A2a, b) with a vertical component of group velocity ($-\Delta^2$) that can be 2–3 times that at the generation site (Fig. A2c). Underice attenuation will be reduced, given the correspondingly faster transit times through the boundary layer. Zonally propagating waves do not share this benefit. Curvature effects depend on the latitude and $\Delta$ at initial generation. They are generally inconsequential in the Arctic for $\sigma > 1.1 f$ at generation.

The central finding of the propagation study is that, for typical ice speeds, underice dissipation is a dominant process. Finescale near-inertial wave propagation in the Arctic is essentially a “one bounce” phenomenon. Surface energy input greater than $\sim 500$ km from
topography should not contribute significantly to boundary mixing.

b. A linear model of near-inertial propagation

In the steady state, the two-dimensional energy conservation equation for wave groups takes the form

\[ c_g \cdot \nabla E + E \nabla \cdot c_g = \text{sources} - \text{sinks}. \quad (A6) \]

Here, \( E \) is the local energy per unit mass in \((\text{m s}^{-1})^2\), \( c_g \) is the vector wave group velocity, and “sources and sinks” account for wave generation and dissipation.

The application of \((A6)\) is a trivial simplification of Garrett (2001), who has recently discussed the long-range propagation of near-inertial waves at mid- and low latitudes. Garrett was primarily concerned with the refractive geometry of the waves on a spherical earth. Here, for initial simplicity, various propagation properties are derived under the assumption that \( \beta = df/dy \) is negligibly small. Comparison with a more accurate model that includes a changing inertial frequency over the propagation path is presented in Fig. \( \Delta 2 \).

The aspect ratio of the surface-generated waves is specified by \( N/\Delta \) where

\[ \Delta = (\sigma^2 - f^2)^{1/2} \quad (A7) \]

and \( \sigma \) is the wave frequency. For near-inertial waves,

\[ \Delta = k_h N/k_z, \quad \sigma \ll N, \quad (A8) \]

where \( k_h \) and \( k_z \) are horizontal and vertical wavenumbers, respectively.

Fig. A1. (a) The ratio of viscous energy loss, over a round-trip path from the sea surface to seafloor, to loss in the underice boundary layer, as a function of vertical wavenumber and frequency. (b) The propagation distance per round-trip surface to seafloor cycle, as a function of wave frequency and ocean depth. For this calculation, the inertial frequency is held constant along the propagation path. (c) The number of surface encounters required before wave packet energy drops to \( e^{-1} \) of its initial value. An ice speed of 0.1 m s\(^{-1}\) and viscosity of \( 5 \times 10^{-6} \) are assumed for (a) and (c). (d)–(f) The horizontal propagation distance attainable before wave energy drops to \( e^{-1} \) of its initial value, for mean ice speeds of 0.03, 0.10, and 0.30 m s\(^{-1}\).
To evaluate the consequences of the conservation model, a canonical (Garrett and Munk 1972, 1975) exponential buoyancy frequency profile, modified to include an arctic halocline, can be applied: \[ N(z) = N(z)_{\text{basin}} + N(z)_{\text{halo}} = N_0 e^{-z/b_0} + N_1 e^{-z/b_1} \] with \( N_0 = 3.6 \) cph, \( b_0 = 1000 \) m, \( N_1 = 6 \) cph, and \( b_1 = 100 \) m (Fig. A2a). For \( \sigma \ll N \), the vertical component of wave group velocity is \[ c_{gz} = -\Delta^2/(\sigma k_z) = -\Delta^2/(\sigma k_b N). \] The propagation time from depth \( z_0 \) to depth \( z_1 \) is \[ T(z_0, z_1) = \int_{z_0}^{z_1} \frac{1}{c_{gz}} \, dz = \int_{z_0}^{z_1} \left[ \frac{\sigma k_b}{\Delta^2} \left[ b_0 N_0 \left( 1 - e^{-z/b_0} \right) \right] + b_1 N_1 \left( 1 - e^{-z/b_1} \right) \right] \, dz. \] (A11)

Note that the waves spend much of their lives in the upper ocean. With a traditional Garrett–Munk (GM) buoyancy profile (\( N_1 = 0 \)), the travel time from the surface to 700 m is one-half of the time required for propagation to infinite depth. Adding the model halocline, the “residence depth” shrinks to 520 m. The interaction of the waves with baroclinic eddies and other mesoscale denizens of the halocline is thus a significant...
aspect of Arctic propagation. The factor of $\Delta^{-3}$ in the travel time emphasizes the extreme sensitivity of the process to the horizontal scale of the waves.

The horizontal component of group velocity is

$$c_{gh} = N^2(k_c \sigma) = \Delta^2/(k_b \sigma),$$ (A12)

corresponding to a round-trip bounce distance of

$$D(H, \Delta) = 2c_{gh} T(0, H) = 2\Delta^{-1}[b_1 N_0 (1 - e^{-H/b_0})]
+ b_1 N_1 (1 - e^{-H/b_1})$$ (A13)

for a locally flat ocean of depth $H$.

Dissipation is modeled with a constant viscosity, such that

$$\text{Sink} = -\nu (U^2 U/dz^2) = 2\nu k_c^2 E.$$ (A14)

In this situation, (A6) takes the form

$$E^{-1} dE/dz = -1/\epsilon c_{gh} (dE/dz + 2\nu k_c^2),$$

with solution

$$E(z_1) = E(z_0) N(z_1)/N(z_0) e^{-q(z_0-z_1)}.$$ (A16)

The initial factor $N(z_i)/N(z_0)$ accounts for the refractive variation in group velocity with depth. Viscous effects are described by the exponential, where

$$q(z_0-z_1) = (2\nu k_c^2 / \Delta^3) \left| \int_{z_0}^{z_1} N^3 \, dz \right|.$$ (A17)

If the initial downward-propagating waves reflect off the seafloor with attenuation $\alpha_o$ (i.e., $E_{z0} = \alpha_o E_{z0}$), and subsequently reflect off the surface with attenuation $\alpha_s$, joining freshly generated, downward-propagating waves, and this process is repeated indefinitely, a steady state energy profile is achieved. For the downward-propagating waves,

$$E_{z0}(z) = E_{z0}(z)/N(0) e^{-q(0,z)} \sum_{n=0}^\infty (\alpha_o \alpha_s)^n e^{-2\nu q(0,H)}.$$ (A18)

For upward-propagating energy,

$$E_{z0}(z) = \alpha_o E_{z0}(z)/N(0) e^{-q(H,z)} \sum_{n=0}^\infty (\alpha_o \alpha_s)^n e^{(2n+1)q(0,H)}.$$ (A19)

The summation index indicates the contribution to the overall wave field energy of waves that have made $n$ “round trips.” The standing up/down energy ratio, $R(z) = \alpha_o e^{-q(z,H)}$ is independent of both the efficiency of surface reflection $\alpha_s$ and the characteristic number of reflections before attenuation.

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