Comments on "A New Theory for the Generation of the Equatorial Subsurface Countercurrents"

FRÉDÉRIC MARIN,* BACH LIEN HUA, AND RICHARD SCHOPP

Laboratoire de Physique des Océans, CNRS-IFREMER-UBO, Plouzané, France

(Manuscript received 15 June 2004, in final form 29 October 2004)

ABSTRACT

From a numerical simulation of the Atlantic Ocean, Jochum and Malanotte-Rizzoli provide evidence that the equatorial subsurface countercurrents can be triggered by tropical instability waves through eddy–mean flow interactions in a low-Rossby-number regime. Adapting the transformed Eulerian mean formalism to a shoaling jet, they propose eddy heat fluxes to be the driving mechanism for the subsurface countercurrents. Here it is shown that such a formalism relying on the existence of a residual meridional streamfunction cannot be applied to a shoaling jet, so that the eddy heat fluxes term in the zonal momentum equation cannot be rigorously justified. Moreover, the role of the zonal pressure gradient that was dropped in their study needs to be reassessed. Despite this mathematical questioning of Jochum and Malanotte-Rizzoli's framework, the authors agree with them that eddy heat fluxes may contribute to the dynamics of the subsurface countercurrents.

1. Introduction

Equatorial subsurface countercurrents (SCCs) are common features to the Pacific and Atlantic Oceans. Observations (Rowe et al. 2000) suggest that SCCs are both diapycnal and inertial, with a high-Rossby-number regime of order 0.7. Three theories have been proposed recently to explain their existence. First, McCreary et al. (2002) presented a nonlocal basin-scale theory in which SCCs are forced by eastern boundary upwellings in off-equatorial regions. Second, Marin et al. (2000, 2003) and Hua et al. (2003) invoked a local mechanism in which SCCs result from the redistribution of the mean angular momentum by secondary meridional circulation cells forced by the large-scale equatorward shoaling of the ventilated thermocline. Third, Jochum and Malanotte-Rizzoli (2004, JM04 hereinafter) provided numerical evidence for a local formation process in which SCCs are triggered by tropical instability waves through eddy–mean flow interaction for a low-Rossby-number regime (of order 0.05).

To understand the dynamical origin of the subsurface countercurrents requires the evaluation of both time-averaged zonal momentum budget

$$\overline{u}\,\overline{u}_x + \overline{v}\,\overline{u}_y + \overline{w}\,\overline{u}_z - f\overline{v} = -\frac{\overline{p}_x}{\rho_0} - (\overline{u'u'})_x - (\overline{u'v'})_y - (\overline{u'w'})_z + \mathcal{F} \qquad (1)$$

and heat budget

$$\overline{u}\,\overline{T}_x + \overline{v}\,\overline{T}_y + \overline{w}\,\overline{T}_z = -(\overline{u'T'})_x - (\overline{v'T'})_y - (\overline{w'T'})_z + \mathcal{D}, \qquad (2)$$

where (u, v, w) denote the components of the velocity field, f is the Coriolis parameter, p is the pressure, T is the temperature, and $\mathcal{F}$ and $\mathcal{D}$ are the frictional and diffusive terms. Overbars and primes refer respectively to temporal average and deviations from it.

In an attempt to combine Eqs. (1) and (2) into a single equation, JM04 focus on the SCC core and reformulate the transformed Eulerian mean (TEM) equations for a shoaling jet. Within this modified

* Current affiliation: Centre IRD de Bretagne, Plouzané, France.

Corresponding author address: Bach Lien Hua, Laboratoire de Physique des Océans, UMR6523, CNRS-IFREMER-UBO, BP 70, 29280 Plouzané, France.
E-mail: lien@ifremer.fr

framework, they propose eddy heat fluxes to be the driving mechanism maintaining the zonal momentum balance of the subsurface countercurrents. Two questions arise from such considerations: is such a formalism appropriate for a shoaling SCC, and does the eddy heat flux play a dominating role in the zonal momentum budget?

2. Existence of a residual meridional streamfunction?

We show hereinafter that a residual meridional streamfunction cannot be defined in the case of a shoaling SCC.

Defining the SCC core as the location of the time-averaged zonal velocity maximum, JM04 noted that $\overline{u}_x = 0$ along the jet core given the eastward shoaling of SCCs. The continuity equation thus reduces locally, right along the jet core, to

$$\overline{v}_y + \overline{w}_z = 0. \tag{3}$$

Contrary to what JM04 state in their Eq. (11), it is the full meridional velocity $\overline{v}$, and not only its ageostrophic component $\overline{v}_a$, that must appear in Eq. (3). Let us insist that Eq. (3) is valid only at the pointwise location of the jet core in a given meridional plane.

Moreover, JM04 assume that the heat budget in Eq. (2) amounts at order 0 to

$$\overline{w}\,\overline{T}_z = -(\overline{v'T'})_y, \tag{4}$$

that is, a balance between the mean vertical advection of temperature and the meridional eddy heat flux. This assumption requires the zonal and vertical eddy heat fluxes to be negligible, which is verified in their simulation (see their Fig. 11). It also requires that the mean horizontal advection of temperature is negligible, which has not been checked in JM04.

Using the apparent 2D character of Eq. (3) and defining an eddy-induced residual vertical velocity w^* following Eq. (4), JM04 then apply a modified version of the TEM framework to define a residual meridional circulation as

$$w^* = \overline{w} + \frac{1}{\overline{T}_z}(\overline{v'T'})_y \quad \text{and} \quad v^* = \overline{v} - \left[\frac{1}{\overline{T}_z}(\overline{v'T'})\right]_z. \tag{5}$$

Here v^* is defined such that the eddy-induced contribution to Eq. (5) derives from the streamfunction $\Psi = (1/\overline{T}_z)(\overline{v'T'})$, under the restrictive assumption that the meridional gradient of $\overline{T}_z$ is small enough for the eddy term $(1/\overline{T}_z)_y(\overline{v'T'})$ to be negligible. This is true in the quasigeostrophic assumption, which is the cornerstone of the original TEM framework. It, however, needs to be verified at the SCC core, which is the location of a meridional front where $\overline{T}_z$ is not expected to be constant.

The key issue is that the particular choice of v^* in Eq. (5) is meaningful only if (v^*, w^*) also derive from a streamfunction. Is this the case? The global continuity equation associated with this velocity field writes at each point of the flow:

$$v^*_y + w^*_z = -\overline{u}_x. \tag{6}$$

The meridional field (v^*, w^*) is not a residual meridional streamfunction, because $\overline{u}_x$ vanishes only locally at the location of the jet core and not outside it. At each point of the flow, continuity [Eq. (6)] implies that the meridional velocity field verifies

$$v^*(y) = v^*(y_0) - \int_{y_0}^{y} w^*_z\,dy - \int_{y_0}^{y} \overline{u}_x\,dy, \tag{7}$$

where y_0 has to be a latitudinal point away from the jet core. The second integral in the right-hand side of Eq. (7) will not be zero in general, even if $\overline{u}_x = 0$ right at the jet core: for this integral to be zero, as implicitly assumed by JM04, a necessary local condition that $\overline{u}_{xy} = 0$ is at least required.

For example, the velocity field

$$\overline{u} = u_0 \exp\{-[y - b(t)x]^2 - [z - a(t)x]^2\},$$
$$\overline{v} = b(t)\overline{u}, \quad \text{and} \quad \overline{w} = a(t)\overline{u},$$

where $a(t)$ and $b(t)$ are scalar functions of time, satisfies Eq. (3) along the jet core defined as $y - bx = 0$ and $z - ax = 0$. However, such a flow is meridionally divergent ($\overline{u}_x \neq 0$ globally) and thus does not derive from a streamfunction. In other words, verifying Eq. (3) along a one-dimensional *curve* (the jet core) does not imply that the *fields* (v^*, w^*) locally derive from a streamfunction.

In summary, JM04 do not provide a sound rationale for the definition of a residual meridional streamfunction that would describe the effects of eddies in both zonal momentum and heat budgets. The TEM framework as modified by JM04 cannot therefore apply to a shoaling jet.

3. Eddy heat fluxes in the zonal momentum budget?

To highlight the dominant role of the horizontal eddy heat fluxes in the zonal momentum budget at the core of the subsurface countercurrents (where $\overline{u}_x = \overline{u}_y = \overline{u}_z$

= 0 by definition), JM04 rewrite Eq. (1) in terms of residual meridional velocity υ^* [see their Eq. (12)]:

$$-f\upsilon^* = -f\overline{\upsilon} + \frac{f}{\overline{T}_z}\left(\overline{\upsilon' T'}\right)_z = -\frac{\overline{p}_x}{\rho_0} - \left(\overline{u'u'}\right)_x - \left(\overline{u'\upsilon'}\right)_y + \mathcal{F} + \left[\frac{f}{\overline{T}_z}\left(\overline{\upsilon' T'}\right)\right]_z, \tag{8}$$

where the vertical eddy momentum flux $-\left(\overline{u'w'}\right)_z$ has been neglected.

Our main concern is that the eddy heat flux convergence $[(f/\overline{T}_z)(\overline{\upsilon' T'})]_z$ appears in Eq. (8) only because JM04 have chosen $-f\upsilon^*$ as the relevant quantity instead of $-f\overline{\upsilon}$. The eddy heat flux contribution to the zonal momentum budget is thus a consequence of JM04's particular choice for υ^*, which was shown in section 1 to be unjustified.

Another issue is that the zonal pressure gradient that is present in Eq. (8) was dropped in JM04's Eq. (12), although υ^* must include both the geostrophic and ageostrophic components of the meridional velocity, as shown in section 1. This term has no reason to vanish at the jet core and may contribute significantly to the zonal momentum budget. It has to be evaluated before concluding that SCCs are maintained against friction by the convergence of Eliassen–Palm fluxes.

Moreover, remember that the zonal momentum budget reduces to Eq. (8) only at the precise pointwise location of the time-averaged SCC core, so that mean advection terms do vanish by definition. This does not imply that mean advection terms play no role for the generation and maintenance of the whole jet (outside its pointwise core).

4. Summary and discussion

The main physical question addressed by JM04 concerns the role of eddy heat fluxes for the SCCs. For this purpose, they modify the TEM framework for application to a shoaling jet. This formalism uses the property $\overline{\upsilon}_y + \overline{w}_z = 0$, because $\overline{u}_x = 0$ at the precise location of the jet core, and relies on the existence of a residual meridional streamfunction. In this comment, we have shown that such a streamfunction cannot be defined for a SCC because of the globally meridionally divergent character of a shoaling jet. Also, the neglect of the zonal pressure gradient is one of some additional assumptions that would need to be verified not only in JM04's low-Rossby-number simulations, but also for higher-Rossby-number regimes comparable to observations. Furthermore, JM04 focus on the SCC core to compute the zonal momentum budget, thus discarding by construction the potential role of mean advection terms for the global dynamics of the jet (outside of the jet core). Even if the balance were between friction and Eliassen–Palm fluxes along the jet core, this would not imply that it corresponds to the first-order mechanism for the generation of the entire jet. In other words, eddy fluxes may contribute to the maintenance of the jet, but are they able to generate an inertial jet that otherwise would not exist?

Despite our mathematical questioning of the TEM framework as modified by JM04, we agree that the potential role of eddy heat fluxes for SCCs should not be discarded. More work is needed to quantify eddy heat flux contributions to the dynamics of the subsurface countercurrents.

Acknowledgments. The authors thank G. Lapeyre and A. M. Tréguier for helpful discussions during the course of this work. Valuable comments by J. P. McCreary, acting as a reviewer, have been helpful in revising this comment.

REFERENCES

Hua, B. L., F. Marin, and R. Schopp, 2003: Three-dimensional dynamics of the subsurface countercurrents and equatorial thermostad. Part I: Formulation of the problem and generic properties. *J. Phys. Oceanogr.,* **33,** 2588–2609.

Jochum, M., and P. Malanotte-Rizzoli, 2004: A new theory for the generation of the equatorial subsurface countercurrents. *J. Phys. Oceanogr.,* **34,** 755–771.

Marin, F., B. L. Hua, and S. Wacongne, 2000: The equatorial thermostad and subsurface countercurrents in the light of the dynamics of atmospheric Hadley cells. *J. Mar. Res.,* **58,** 405–437.

——, R. Schopp, and B. L. Hua, 2003: Three-dimensional dynamics of the subsurface countercurrents and equatorial thermostad. Part II: Influence of the large-scale ventilation and of equatorial winds. *J. Phys. Oceanogr.,* **33,** 2610–2626.

McCreary, J. P., P. Lu, and Z. Yu, 2002: Dynamics of the Pacific subsurface countercurrents. *J. Phys. Oceanogr.,* **32,** 2379–2404.

Rowe, D. G., E. Firing, and G. C. Johnson, 2000: Pacific equatorial subsurface countercurrent velocity, transport, and potential vorticity. *J. Phys. Oceanogr.,* **20,** 1172–1187.