Eulerian versus Lagrangian Approaches to the Wave-Induced Transport in the Upper Ocean

JAN ERIK H. WEBER

Department of Geosciences, University of Oslo, Oslo, Norway

GÖRAN BROSTRÖM

Department of Meteorology, Stockholm University, Stockholm, Sweden

ØYVIND SAETRA

Norwegian Meteorological Institute, Oslo, Norway

(Manuscript received 31 March 2005, in final form 22 December 2005)

ABSTRACT

It is demonstrated that the Eulerian and the Lagrangian descriptions of fluid motion yield the same form for the mean wave-induced volume fluxes in the surface layer of a viscous rotating ocean. In the Eulerian case, the volume fluxes are obtained in the familiar way by integrating the horizontal components of the Navier–Stokes equation in the vertical direction, as seen, for example, in the book by Phillips. In the direct Lagrangian approach, the perturbation equations for the second-order mean drift are integrated in the vertical direction. This yields the advantage that the form drag, which is a source term for the wave-induced transports, can be related to the virtual wave stress that acts to transfer dissipated mean wave momentum into mean currents. In particular, for waves that are periodic in space and time, comparisons between empirical and theoretical relations for the form drag yield an estimate for the wave-induced bulk turbulent eddy viscosity in the surface layer. A simplistic approach extends this analysis to account for wave breaking. By a generalization from a wave component to a wave spectrum, a set of equations for the wave-induced transport in the surface layer is derived for a fully developed sea. Solutions are discussed for an idealized spectral formulation. The problem is formulated such that a numerical wave prediction model can be used to generate the wave-forcing terms in a numerical barotropic ocean surge model. Results from the numerical simulations with a wave-influenced surge model are discussed and compared with similar results from forcing the surge model only by the traditional mean horizontal wind stress computed from the 10-m wind speed. For the simulations presented here, the wave-induced stress constitutes about 50% of the total atmospheric stress for moderate to strong winds.

1. Introduction

It is a well-known fact that surface waves carry mean momentum (Stokes 1847). For monochromatic waves in a viscous nonrotating fluid, the pioneering paper on this subject is Longuet-Higgins (1953). He applied an Eulerian fluid description with curvilinear coordinates to solve this problem. For a direct Lagrangian approach to wave drift in a rotating ocean, earlier treatments are found in Chang (1969), Ünlüata and Mei (1970), and Weber (1983). Also, the generalized Lagrangian mean formulation of Andrews and McIntyre (1978) can be applied to this problem.

Numerical general ocean circulation models (GCMs) are widely used to predict oceanic motions caused by the wind. Such models are usually based on an Eulerian description of motion. Furthermore, they often assume hydrostatic balance in the vertical, and accordingly they do not capture wind-induced surface waves. Even nonhydrostatic models do not resolve surface waves, or they lack the forcing conditions that allow the generation of wind waves. Thus, it is not expected that such models would include the mean drift resulting from pe-

Corresponding author address: Prof. Jan Erik H. Weber, Department of Geosciences, University of Oslo, Blindern, NO-0315 Oslo, Norway.
E-mail: j.e.weber@geo.uio.no

riodic wave motion. As pointed out by McWilliams and Restrepo (1999), GCMs may underestimate the currents by taking only the mean horizontal wind stress into account, yielding the traditional Ekman current. One often asks if the Eulerian approach is capable of determining the total mean wave-induced current, because the Stokes drift in this case is confined between the wave crests and the wave troughs at the surface (Phillips 1977). The aim of the present paper is to compare results for the wave-induced drift obtained by an Eulerian and a Lagrangian analysis. To determine the wave-drift current in the entire fluid column from an Eulerian approach is rather laborious (e.g., Longuet-Higgins 1953), while it is fairly simple to do so from a direct Lagrangian starting point (Weber 1983). We shall therefore be content with making this comparison for the mass or volume fluxes in the oceanic surface layer. In this case the Eulerian approach is the simplest. Here we extend Phillips' (1977) analysis to a viscous rotating ocean. For a simplified ocean with constant (eddy) viscosity we can use earlier results (e.g., Weber and Melsom 1993a) to obtain the mean Lagrangian fluxes. This approach yields the additional bonus that an explicit expression for the form drag is obtained. For simplicity, we consider periodic waves with amplitudes that vary slowly in time and space separately. Earlier, Jenkins (1986, 1987) treated these cases simultaneously.

This paper is organized as follows: First we compare the results for the mass fluxes derived by Eulerian and Lagrangian starting points in a viscous rotating ocean when the waves are 1) spatially periodic with amplitudes that may vary slowly in time, and 2) temporally periodic with amplitudes that may vary slowly in space. Then we derive a general set of equations that includes both these cases for a single wave component along the x axis. This set of equations is generalized to a spectral formulation, where the wave-forcing terms are evaluated for a theoretical one-dimensional frequency spectrum for a saturated sea. Last, we formulate our wave-influenced mass flux problem for a two-dimensional wave spectrum that can be obtained from an operational wave prediction model. The wave-induced forcing terms computed from this model drive a numerical barotropic ocean surge model. Results from this model are compared with results from similar simulations using the wind stress computed from the 10-m wind speed (Large and Pond 1981).

2. The Eulerian approach

We consider plane waves that propagate along the x axis in a Cartesian coordinate system, where the x and y axis are situated at the undisturbed sea surface (see

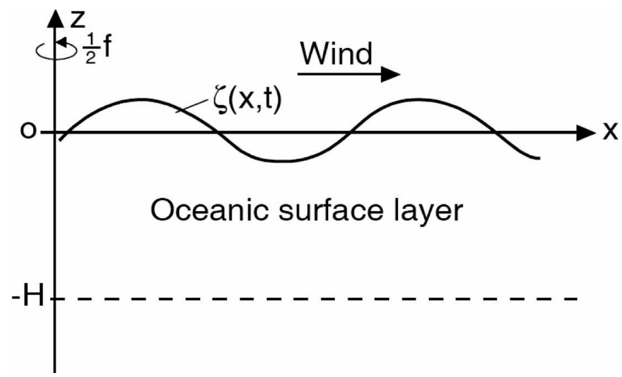

FIG. 1. Sketch of the ocean layer.

Fig. 1). The z axis is vertical and directed upward. The corresponding unit vectors are $(\mathbf{i}, \mathbf{j}, \mathbf{k})$, respectively. Our system rotates with constant angular velocity $f/2$ about the z axis, where f is the Coriolis parameter. We first consider a traditional Eulerian description where the velocity $\mathbf{v} = (u, v, w)$ and the pressure p are functions of time t and the spatial coordinates (x, y, z). The Navier–Stokes equation and the continuity equation for a viscous, rotating fluid of constant density can be written as

$$(\rho\mathbf{v})_t + \mathbf{v}\cdot\nabla(\rho\mathbf{v}) + f\mathbf{k}\times(\rho\mathbf{v}) = -\nabla p - \rho g\mathbf{k} + \mu\nabla^2\mathbf{v} \quad \text{and} \tag{1}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{2}$$

where ρ is the density, μ is the dynamic viscosity, and g is the acceleration due to gravity. Furthermore, ∇ and ∇^2 are the gradient and Laplacian operators, respectively. In the real ocean the presence of turbulence, and its generation, maintenance, and interaction with the mean flow are not known in detail. There are elaborate models for some of these problems, but they are often highly speculative. However, we know that laminar waves propagating in a turbulent fluid suffer attenuation (Ölmez and Milgram 1992). To model this effect in the simplest possible way, we make the traditional Boussinesq approximation of isotropic turbulence with a constant eddy viscosity, that is, all of our variables in (1) and (2) are averages and do not contain turbulent fluctuations. Hence, the entire effect of turbulence is embedded in the value of the viscosity coefficient in (1), hereinafter referred to as the dynamic eddy viscosity.

The wind and waves in our problem are directed along the x axis, and we assume that the variables are independent of y. We follow Phillips' (1977) approach, and integrate the horizontal components of (1) between a constant depth $z = -H$, where the viscous stresses are assumed to vanish, and the material surface $z = \zeta(x, t)$;

see Fig. 1. The surface layer is assumed to be so deep that it encompasses the Ekman layer as well as the deep-water wave field. In practice H will be comparable to the Ekman depth $D_E = \pi(2\nu/f)^{1/2}$ in the open ocean, where $\nu = \mu/\rho$ is the kinematic eddy viscosity.

Utilizing the kinematic boundary condition at the free surface (e.g., Phillips 1977), we find

$$Q_t^{(x)} - fQ^{(y)} - \nu Q_{xx}^{(x)} = -\left[\int_{-H}^{\zeta} (\rho uu + p)\, dz\right]_x + p(\zeta)\zeta_x, + \mu(u_z - u\zeta_{xx} - 2u_x\zeta_x)_{z=\zeta} \quad (3)$$

and

$$Q_t^{(y)} + fQ^{(x)} - \nu Q_{xx}^{(y)} = -\left(\int_{-H}^{\zeta} \rho u\nu\, dz\right)_x + \mu(\nu_z - \nu\zeta_{xx} - 2\nu_x\zeta_x)_{z=\zeta}. \quad (4)$$

Here subscripts denote partial differentiation. The mass transport components are defined as

$$Q^{(x)} \equiv \int_{-H}^{\zeta} \rho u\, dz \quad \text{and} \quad Q^{(y)} \equiv \int_{-H}^{\zeta} \rho v\, dz. \quad (5)$$

It is worth pointing out that the mass transport components in (5) include the Stokes drift, because the integration is carried out to the undulating surface. Accordingly, $Q^{(x)}$ and $Q^{(y)}$ represent the Lagrangian mass transport.

The dynamical condition at the surface becomes second order in the wave steepness (neglecting the effect of surface films),

$$\tau^{(t)} - \tau^{(n)}\zeta_x = \mu(u_z + w_x) + p\zeta_x - 2\mu u_x\zeta_x \quad \text{and} \quad z = \zeta(x, t). \quad (6)$$

Here $\tau^{(t)}$ is the tangential wind stress and $\tau^{(n)}$ is the normal wind stress along the sea surface. Utilizing (6), (3) and (4) reduce to

$$Q_t^{(x)} - fQ^{(y)} - \nu Q_{xx}^{(x)} = -\left[\int_{-H}^{\zeta} (\rho uu + p)\, dz\right]_x + \tau^{(t)} - \tau^{(n)}\zeta_x - \mu(w_x + u\zeta_{xx})_{z=\zeta} \quad (7)$$

and

$$Q_t^{(y)} + fQ^{(x)} - \nu Q_{xx}^{(y)} = -\left(\int_{-H}^{\zeta} \rho u\nu\, dz\right)_x - \mu[(\nu\zeta_x)_x]_{z=\zeta}. \quad (8)$$

To second order in wave amplitude, utilizing (2), the last term in (7) becomes

$$(w_x + u\zeta_{xx})_{z=\zeta} = (w_x)_{z=0} + (w_{xz})_{z=0}\zeta + (u\zeta_{xx})_{z=0} = (w_x)_{z=0} + (-u_{xx}\zeta + u\zeta_{xx})_{z=0}. \quad (9)$$

Here the last term vanishes identically for a wave component. For high-frequency wind waves along the x axis, the velocity component in the y direction is very small, which means that the friction term in (8) can be neglected. Similarly, the friction term $\mu w_x(z = \zeta)$ in (7) is small. It practically vanishes when averaged over the wave cycle. Hence, to $O(\zeta^2)$,

$$Q_t^{(x)} - fQ^{(y)} - \nu Q_{xx}^{(x)} = -\left[\int_{-H}^{\zeta} (\rho uu + p)\, dz\right]_x + \tau^{(t)} - \tau^{(n)}\zeta_x \quad \text{and} \quad (10)$$

$$Q_t^{(y)} + fQ^{(x)} - \nu Q_{xx}^{(y)} = -\left(\int_{-H}^{\zeta} \rho u\nu\, dz\right)_x. \quad (11)$$

The mean mass transport will be obtained from these equations by averaging over one wave cycle.

In a recent paper Ardhuin et al. (2004) discuss the mean mass transport by integrating the Eulerian equations in the vertical from the ocean bottom to the free surface. Like Hasselmann (1971), they define their mean flow by integrating the horizontal mean velocity to the position of the mean sea level, and the wave part (the Stokes mass transport) by the averaged integral of the velocity from the mean sea level to the position of the free surface. In this way the correlation between the variable air pressure and the free surface slope becomes a part of the forcing of the Stokes mass transport. In addition, Ardhuin et al. do not apply the dynamic surface boundary conditions, which introduce the normal and tangential surface stresses into the problem. In this way the role of the form drag [averaged last term on the right-hand side of (10)] as the main source term for the total wave-induced mean mass transport is obscured. We think that Phillips' (1977) formalism, as used here, is the most convenient method for describing the total mean mass transport resulting from wind and waves in an Eulerian context.

We consider first waves that are periodic in x. The amplitude is spatially homogeneous, but may grow or decay slowly in time. By averaging over one wavelength, (10) and (11) reduce to

$$\frac{\partial \overline{Q}}{\partial t} + if\overline{Q} = \overline{\tau^{(t)}} - \overline{\tau^{(n)}\zeta_x}. \quad (12)$$

Here we have defined the complex mass transport as $Q \equiv Q^{(x)} + iQ^{(y)}$, where i is the imaginary unit. In (12) the mean tangential wind stress is essentially responsible for the traditional Ekman transport. We are particularly interested in the wave-induced part of the mean transport. This is basically related to the action of the form drag τ_D, defined as

$$\tau_D \equiv -\overline{\tau^{(n)}\zeta_x}. \quad (13)$$

Here the fluctuating air pressure dominates in the expression for $\tau^{(n)}$ (Phillips 1977). Also, a fluctuating tangential viscous stress in the air (skin friction) may give rise to wave growth (Lamb 1932). However, the skin friction in phase with the surface elevation appears to be a minor factor in transferring energy from the wind to the wave field under realistic conditions (Chalikov and Makin 1991). The Ekman transport forced by $\overline{\tau^{(t)}}$ is well discussed in the literature and will not be treated here. Separating the fluxes in the linear Eq. (12) as $Q = Q_{\text{Ekman}} + Q_{\text{wave}}$, we may introduce wave-induced volume fluxes (U, V) such that

$$\rho U \equiv \overline{Q_{\text{wave}}^{(x)}} \quad \text{and} \quad \rho V \equiv \overline{Q_{\text{wave}}^{(y)}}. \tag{14}$$

From (12) the total flux resulting from spatially periodic waves in a viscous rotating ocean forced by τ_D can then be written, with $W \equiv U + iV$, as

$$\frac{\partial W}{\partial t} + ifW = \tau_D/\rho. \tag{15}$$

The wave-induced fluxes in (15) include both the Stokes drift and the wave-induced Eulerian mean motion. Accordingly, (15) expresses the Lagrangian volume transport derived from the Eulerian equations. We realize from (15) that for no wind ($\tau_D = 0$), that is, decaying waves, the mass transport is zero when averaged over one inertial cycle; see Weber (1983), who derived this result from a direct Lagrangian approach. For waves in the absence of friction, this was first shown by Hasselmann (1970). In a nonrotating viscous ocean ($f = 0$) with no wind, we find that the total mean wave momentum must be conserved. This means that the decaying Stokes drift, which in an Eulerian description is confined between the wave crests and the wave troughs, induces a compensating mean Eulerian current in the fluid.

The mean Eulerian and Stokes drift contributions are recognized straight away in the present approximation by writing the integrals in (15) for the wave-induced drift as

$$\int_{-H}^{\zeta} u\,dz = \int_{-H}^{0} u\,dz + \int_{0}^{\zeta} u\,dz \approx \int_{-H}^{0} u\,dz + (u\zeta)_{z=0}$$

and

$$\int_{-H}^{\zeta} v\,dz = \int_{-H}^{0} v\,dz + \int_{0}^{\zeta} v\,dz \approx \int_{-H}^{0} v\,dz + (v\zeta)_{z=0}. \tag{16}$$

When averaged, the first term on the right-hand side yields the mean Eulerian volume flux. The average of the second term is the volume transport due to the Stokes drift (U_S, V_S), as easily can be seen by inserting for a linear wave component (e.g., Longuet-Higgins 1953). The averaged procedure (16) corresponds basically to the subdivision made by Hasselmann (1971). In the present problem the Stokes transport becomes

$$U_S = \omega\hat{\zeta}^2/2 \quad \text{and} \quad V_S = 0, \tag{17}$$

where ω is the angular wave frequency and $\hat{\zeta}$ is a wave amplitude that is allowed to vary slowly with time (slow relative to the wave period $2\pi/\omega$). Accordingly, if we write the averaged integrals in (16) as

$$U = U_E + U_S \quad \text{and} \quad V = V_E, \tag{18}$$

we find from (15) that

$$\frac{\partial W_E}{\partial t} + ifW_E = \frac{\tau_D}{\rho} - \frac{\partial U_S}{\partial t} - ifU_S. \tag{19}$$

Here, $W_E \equiv U_E + iV_E$. We realize that the Stokes transport terms $-\partial U_S/\partial t$ and $-fU_S$ appear as forcing terms in the x and y directions, respectively, in this equation for the wave-induced Eulerian transport. However, it is somewhat deceptive to regard the problem in this way. The important fluxes in the oceanic surface layer affecting the mass balance are the Lagrangian fluxes, that is, in general $(U_E + U_S, V_E + V_S)$. As shown by (15), the only forcing term in the spatially periodic wave-drift problem is the form drag.

3. Lagrangian approach

In the Lagrangian description a fluid particle is associated with its initial coordinates (a, b, c). The particle position at later times (X, Y, Z) and the pressure P will then be functions of a, b, c, and time t. Velocity components and acceleration are given by $(X_t, Y_t, Z_t,)$ and (X_{tt}, Y_{tt}, Z_{tt}), respectively. For plane waves along the x axis, the deviations (x, y, z, p) from the initial state will not depend on b. We then may write

$$X = a + x(a, c, t),\ Y = b + y(a, c, t),$$
$$Z = c + z(a, c, t) \quad \text{and} \quad P = -\rho gc + p(a, c, t). \tag{20}$$

For convenience, we introduce a complex horizontal velocity $q \equiv x_t + iy_t$. By including the effect of the earth's rotation, the equations for the conservation of momentum and mass can be obtained from Lamb (1932). With the present notation, the momentum equations become, to second order in wave steepness,

$$q_t + ifq - \nu\nabla_L^2 q = -\frac{1}{\rho}(p + \rho gz)_a - \frac{1}{\rho}J(p, z)$$
$$+ \nu[J(q_a, z) + J(x, q_c) + J(q, z)_a + J(x, q)_c] \quad \text{and} \tag{21}$$

$$z_{tt} - \nu\nabla_L^2 z_t = -\frac{1}{\rho}(p + \rho g z)_c - \frac{1}{\rho}J(x, p) - gJ(x, z) + \nu[J(z_{ta}, z) + J(x, z_{tc}) + J(z_t, z)_a + J(x, z_t)_c], \tag{22}$$

where $\nabla_L^2 \equiv \partial^2/\partial a^2 + \partial^2/\partial c^2$ is the Laplacian operator in Lagrangian coordinates, and $J(A, B) \equiv A_a B_c - A_c B_a$ is the Jacobian. The conservation of mass (here volume) leads to

$$x_a + z_c = -J(x, z). \tag{23}$$

In the Lagrangian formulation the free material surface is given by $c = 0$. When averaging in time or space, we then obtain

$$\overline{\int_{-H}^{0} q\, dc} = \int_{-H}^{0} \overline{q}\, dc. \tag{24}$$

We can therefore integrate the mean drift equations to second order in the wave steepness to obtain the desired equations for the volume fluxes, that is, we can use the results of Weber and Melsom (1993a) [their (3.8) with constant eddy viscosity]. Disregarding the traditional Ekman flow (i.e., the flow driven directly by frictional wind stress), we find for spatially periodic waves that

$$\frac{\partial}{\partial t}\int_{-H}^{0} \overline{q}\, dc + if\int_{-H}^{0} \overline{q}\, dc = (B_r + \beta/\omega)\omega^2\zeta_0^2 \exp(2\beta t), \tag{25}$$

which is (7.5) of Weber and Melsom (1993a). Here β is the wave growth/decay rate, and ζ_0 is the initial wave amplitude. The real coefficient B_r is related to the vorticity part of the primary wave field. Neglecting the effect of the tangential fluctuating wind stress in phase with the surface elevation, which is of minor influence in comparison with the fluctuating wind stress in phase with the surface slope in the wave generation process, we find that

$$B_r = k^2/\gamma^2. \tag{26}$$

Here k is the wavenumber, and $\gamma = (\omega/2\nu)^{1/2}$ is the inverse viscous boundary layer thickness at the surface. The form drag in this problem can be written

$$\tau_D = -\overline{\tilde{\sigma}\tilde{z}_a}, \quad c = 0, \tag{27}$$

where $\tilde{\sigma}$ is the real part of the fluctuating wind stress component normal to the sea surface, and $\tilde{z}$ is the real part of the vertical displacement of the primary wave field. Inserting for the primary wave field from Weber and Melsom (1993a), we find that

$$\tau_D = \rho\left(\frac{\beta}{\omega} + \frac{k^2}{\gamma^2}\right)\omega^2\zeta_0^2 \exp(2\beta t). \tag{28}$$

From (26) and (28) we realize that (25) can be written

$$\frac{\partial}{\partial t}\int_{-H}^{0} \overline{q}\, dc + if\int_{-H}^{0} \overline{q}\, dc = \tau_D/\rho. \tag{29}$$

To second order in wave amplitude, (29), for the mean Lagrangian volume fluxes resulting from spatially periodic wave motion, is identical to (15), which was obtained from an Eulerian analysis.

In the present Lagrangian approach, we have solved the linear wave problem in the ocean, derived the equations for the mean drift, and integrated these equations in the vertical. This yields the advantage that we have obtained an expression (28) for the form drag that contains wave parameters. For example, for vanishing form drag (purely damped waves), we obtain from (28) the well-known result for the attenuation rate, $\beta = -2\nu k^2$. The form drag (28), with $\hat{\zeta} = \zeta_0 \exp(\beta t)$, can also be written

$$\tau_D = \frac{\partial}{\partial t}(\rho U_S) + \tau_w, \tag{30}$$

where $\rho U_S = \rho\omega\hat{\zeta}^2/2$ is the total horizontal wave momentum, and τ_w is the virtual wave stress originally introduced by Longuet-Higgins (1969). It can be written [Weber 1997, his (77)] as

$$\tau_w = 2\rho\nu k^2\omega\hat{\zeta}^2. \tag{31}$$

Here we interpret ν as the bulk turbulent eddy viscosity. We see right away from (30) that for purely decaying waves ($\tau_D = 0$)

$$\int_0^\infty \tau_w\, dt = \rho U_S(t = 0). \tag{32}$$

In this case the virtual wave stress acts to transfer the entire lost mean wave momentum into an Eulerian current. For a steady sea state, the form drag and the virtual wave stress have equilibrium values that balance; that is, from (30),

$$\tau_D^{(\mathrm{eq})} = \tau_w^{(\mathrm{eq})} = 2\rho\nu k^2\omega\zeta_0^2. \tag{33}$$

Although the computations involved up to now are based on the assumption of a well-defined material wavy sea surface, we allow that the eddy viscosity in (31) may contain a contribution from breaking or white capping. This means that the virtual wave stress τ_w transfers the lost wave momentum from all kind of dissipative processes.

From experimental data Phillips (1977) suggests that the amplitude of the air pressure variation p_{am} over a smooth wave component can be approximated by

$$p_{\text{am}} = 0.05\rho_a u_{10}^2 k\zeta_0, \tag{34}$$

where ρ_a is the density of the air and u_{10} is the wind speed at 10-m height. Hence, we can write the form drag (27) over a wave component as

$$\tau_D \approx \overline{\tilde{p}\tilde{z}_a} = 2.5 \times 10^{-2}\rho_a u_{10}^2 k^2 \zeta_0^2. \tag{35}$$

Combining (33) and (35) we obtain

$$\nu = 1.25 \times 10^{-2} s \frac{u_{10}^2}{\omega}, \tag{36}$$

for the bulk eddy viscosity in the surface layer, where $s = \rho_a/\rho$. Associating the wave component in question with the peak of the wind–wave spectrum, we have approximately that $\omega/k = u_{10}$ for this peak. Applying the deep-water dispersion relation $\omega^2 = gk$, (36) can be written as

$$\nu = 1.25 \times 10^{-2} s \frac{u_{10}^3}{g}. \tag{37}$$

This relation yields reasonable values for the bulk eddy viscosity associated with wave motion in a turbulent ocean, for example, $\nu = 15\ \text{cm}^2\ \text{s}^{-1}$ for $u_{10} = 10\ \text{m s}^{-1}$ and $\nu = 122\ \text{cm}^2\ \text{s}^{-1}$ for $u_{10} = 20\ \text{m s}^{-1}$. For rougher sea, where breaking occurs, the form drag is larger than over a smooth wave (Banner 1990). Then (35) underestimates the value of the form drag. We shall return to the effect of wave breaking on the eddy viscosity in section 5.

4. Temporally periodic waves

For waves that are periodic in time, we go back to the Eulerian formulations (10) and (11). We now allow the wave amplitude to vary slowly in space (slow relative to the wavelength), and we average the integrated equations over the wave period. The horizontal divergence terms may now be calculated to second order in the wave amplitude by utilizing the results from linear deep-water waves and the pressure from integrating the vertical component of (1) (e.g., Phillips 1977). We consider a horizontally unlimited ocean and take the mean surface gradient to be zero ($\overline{\zeta}_x = 0$). Disregarding the mean tangential wind stress, and considering only the mean volume fluxes in (14) induced by temporally periodic wave motion, (10) and (11) finally reduce to

$$\frac{\partial W}{\partial t} + ifW = \frac{\tau_D}{\rho} - \frac{1}{\rho}\frac{\partial(C_g \rho U_S)}{\partial x}. \tag{38}$$

This equation is valid for an ocean that is much deeper than the wavelength. In (38) the form drag τ_D is defined by (13) and $C_g = \omega/(2k)$ is the group velocity. In the derivation of (38), we have neglected the small friction terms $\overline{\nu Q_{xx}^{(x)}}$ and $\overline{\nu Q_{xx}^{(y)}}$.

The last term on the right-hand side of (38) is the horizontal divergence of the wave momentum flux. It is just an alternative expression for the radiation stress introduced by Longuet-Higgins and Stewart (1960). This can be seen as follows: for deep-water waves the radiation stress tensor $\Re = [R^{(mn)}\mathbf{i}_m\mathbf{i}_n]$ of Longuet-Higgins and Stewart (1960) can be written for a single wave component as

$$R^{(mn)} = \frac{E l_m l_n}{2(l_1^2 + l_2^2)}, \tag{39}$$

where l_1 and l_2 are the horizontal wavenumber components, and $E = MC$ is the wave energy density (Starr 1959). For waves along the x axis ($l_1 = k, l_2 = 0$) we find that $M = \rho U_S$, and hence $E = \omega\rho U_S/k$. By insertion into (39), it is then seen that the last term of the wave-induced stress in (38) is just the remaining nonzero component of the divergence of the radiation stress tensor per unit density.

Equation (38) for the volume fluxes, induced by temporally periodic waves, has also been derived from a direct Lagrangian analyses by Weber [2003, his (4.11)],

$$\frac{\partial}{\partial t}\int_{-H}^{0} \overline{q}\, dc + if\int_{-H}^{0} \overline{q}\, dc = \frac{\tau_D}{\rho} - \frac{1}{\rho}\frac{\partial(C_g \rho U_S)}{\partial a}, \tag{40}$$

which is equivalent to (38). Because (40) was based on vertical integration of the mean drift equations to second order in the wave steepness, it gave the added bonus that the form drag could be expressed in terms of the wave parameters. For waves with no spatial decay, τ_D becomes equal to the virtual wave stress τ_w, and the relation (33) reappears.

For waves that are periodic in time and space, there must be a balance between the energy input from the wind and the viscous dissipation in the fluid. The rate of energy input from the wind can be written as

$$-(\overline{\tilde{p}\tilde{z}_t})_{c=0} = \frac{\omega}{k}(\overline{\tilde{p}\tilde{z}_a})_{c=0} = C\tau_D. \tag{41}$$

The numerical value of rate of energy dissipation D for deep-water gravity waves with an uncontaminated surface is determined by the irrotational part of the wave field (Phillips 1977), and is given by

$$D = 2\rho\nu\omega^2 k\zeta_0^2. \tag{42}$$

In a balanced state we obtain from (41) and (42)

$$C\tau_D = D, \tag{43}$$

which yields exactly the same form drag as in (33). The work done by the form drag over one wave period $T = \lambda/C$ in this case is thus given by

$$C\tau_D T = \lambda\tau_D = TD. \tag{44}$$

The physics behind this relation will be utilized in the next section, where we consider breaking waves in a fully developed sea.

Last, the cases of temporally and spatially modulated wave amplitudes can be combined. This was first done by Jenkins (1986), who calculated the mean Lagrangian drift resulting from such waves. From his (3.3) and (3.7), the form drag associated with temporally and spatially modulated waves can be written in our notation as

$$\tau_D = \tau_w + \frac{\partial}{\partial t}(\rho U_S) + \frac{\partial}{\partial a}(C_g \rho U_S). \tag{45}$$

We then realize that the equation

$$\frac{\partial W}{\partial t} + ifW = \frac{\tau_w}{\rho} + \frac{\partial U_S}{\partial t}, \tag{46}$$

for the mean Lagrangian volume fluxes, covers the combined cases of temporally and spatially varying wave amplitudes. We note that (29) and (39) follow directly from (45) and (46) for the two different cases of amplitude modulation. Equivalently, from (45), the right-hand side of (46) can be expressed by the form drag minus the radiation stress.

5. Effect of wave breaking: A simplistic approach

In a saturated sea state, the energy input from the wind must essentially be balanced by dissipation in the form of wave breaking. If the wave amplitudes before and after breaking are ζ_{cr} and ζ_0, respectively, the energy lost per wavelength for an infinitely long wave train during the breaking event is

$$\Delta E = \frac{1}{2}\rho g(\zeta_{cr}^2 - \zeta_0^2). \tag{47}$$

Let the average time between breaking events be T_B. Then, by analogy with (44), this energy loss must be compensated by the work done by the equilibrium form drag, that is,

$$C\tau_D^{(eq)} T_B = \Delta E. \tag{48}$$

By assuming that the wave amplitude grows exponentially in the time interval from post- to the next breaking, the relative energy lost during one breaking event can be written (Melsom 1996) as

$$\Delta e = \frac{\zeta_{cr}^2 - \zeta_0^2}{\zeta_0^2} = \exp(2\beta T_B) - 1, \tag{49}$$

where β is the growth rate of the fastest growing waves. From experimental data we find that

$$\frac{\beta}{\omega} = K\frac{U_*^2}{C^2}, \tag{50}$$

where U_* is the friction velocity in the air. A typical value for K is 1×10^{-2} (Plant 1982). Furthermore, experimental observations indicate that Δe lies between 10^{-2} and 10^{-1} (Melville and Rapp 1985). We then obtain approximately from (49)

$$T_B = \frac{\Delta e}{2\beta}. \tag{51}$$

By inserting the results above into (48), we obtain the equilibrium form drag in the presence of white capping. According to our previous results, this must equal the equilibrium virtual wave stress that transfers this momentum to Eulerian flows. We then find

$$\tau_D^{(eq)} = \tau_w^{(eq)} = \rho c_D K u_{10}^2 k^2 \zeta_0^2. \tag{52}$$

For a saturated sea state we typically have $c_D \approx 2 \times 10^{-3}$ for the drag coefficient. We note the interesting fact that this expression for the form drag has the same functional dependence of the wind velocity and the wave steepness as (35) for nonbreaking waves.

A crude estimate for the bulk eddy viscosity ν_B associated with a breaking wave component can now be obtained by combining (33) and (52). For the spectral peak component this leads to

$$\nu_B = c_D K\left(\frac{u_{10}^3}{g}\right). \tag{53}$$

With $K = 1 \times 10^{-2}$ and $c_D = 2 \times 10^{-3}$, as suggested above, the coefficient in (53) becomes 2×10^{-5}. This is about 25% higher than the corresponding value estimated for smooth waves in (37), and appears to be a reasonable result.

In fact, from purely dimensional grounds one would expect the bulk eddy viscosity associated with wind-generated gravity waves to scale with the friction velocity U_* in the air and g as U_*^3/g, or equivalently as u_{10}^3/g, because $U_*^2 = c_D u_{10}^2$. This also follows from applying the law-of-the-wall distribution $\nu = -\kappa u_* c$ for the eddy viscosity in the ocean (Madsen 1977). Here κ is von Kármán's constant and u_* is the friction velocity in the ocean ($u_* = s^{1/2}U_*$). When integrating this distribution in the vertical over one wavelength, we find for the vertically averaged eddy viscosity that $\overline{\nu} \approx u_{10}^3/g$. Based on analogy with grid-induced turbulence, Kitaigorodskii [1996, his (25)] obtained for the bulk eddy viscosity in breaking waves in our notation

$$\nu = 2 \times 10^{-4}\,\frac{u_{10}^3}{g}. \qquad (54)$$

However, this formula overestimates the bulk eddy viscosity in the ocean; that is, a wind of 20 m s^{-1} would yield $\nu \sim 1600$ cm^2 s^{-1}. Breaking or white capping in the open sea will increase the eddy viscosity, but not that dramatically. We therefore argue that our (53) is much closer to a realistic modeling of the magnitude of the wave-induced eddy viscosity than (54). It should be noted that the viscosity discussed here is relevant for the transport of mean momentum, that is, for ocean currents. For the wave field itself the appropriate eddy viscosity should be considerably less (Jenkins 1989; Weber and Melsom 1993a).

6. Spectral considerations for a fully developed sea

We now apply the general form (46) of the transport equations, valid for sinusoidal waves with temporally and spatially modulated amplitudes. In generalizing, going from a single wave component to a fully developed sea, our wave amplitude must be associated with the spectral distribution of the wave energy. Because we here have considered plane waves along the x axis, we shall be content with considering the one-dimensional wave spectrum for illustrative purposes. However, an extension to a two-dimensional spectrum is straightforward. This is left for the next section where we apply a wave prediction model.

In a fully developed sea the energy spectrum varies only slowly in time and space around a mean saturated state. Letting $\zeta_0^2 = 2\varphi(\omega)\Delta\omega$, where φ is the frequency spectrum (Bye 1967), the equilibrium virtual wave stress in (52) may be written in spectral form as

$$\{\tau_w\} = \frac{2\rho c_D K u_{10}^2}{g^2}\int \omega^4 \varphi(\omega)\, d\omega, \qquad (55)$$

where {} denotes spectral average. The explicit slow variations in time and space of the forcing terms are given by the time derivative of the total wave momentum and the negative horizontal divergence of the wave momentum flux expressed in terms of the surface wave spectrum. From (46) the equations for the wave-induced average volume fluxes in a fully developed sea can then be written as

$$\frac{\partial W}{\partial t} + ifW = \frac{\{\tau_w\}}{\rho} + \frac{1}{\rho}\frac{\partial\{\rho U_s\}}{\partial t}. \qquad (56)$$

In (56) the generalized wave momentum is given by

$$\{\rho U_S\} = \rho\int \omega\varphi(\omega)\, d\omega. \qquad (57)$$

The frequency spectrum, and its time and space dependence, may be obtained from an ocean wave prediction model (e.g., Komen et al. 1994). It is the wave-induced fluxes from (56) that should be added to the mean wind stress–driven fluxes from a general ocean circulation model in order to obtain the total transports in the oceanic surface layer.

For illustrative purposes we introduce the frequency spectrum suggested by Toba (1973). It can be stated as

$$\varphi = \frac{\alpha_T g U_*}{\omega^4}. \qquad (58)$$

Field measurements yield a value of Toba's constant α_T between 6×10^{-2} and 11×10^{-2} (Phillips 1985). The expressions (55) and (57) will be integrated from the peak spectral frequency ω_p to a high-frequency limit ω_h, representing the upper tail of the spectrum ($\omega_h \gg \omega_p$). Utilizing that $\omega_p^2 = gk_p$, and $\omega_h^2 = gk_h = rg^2/U_*^2$, where $r^{1/2}$ is a constant of order unity (Phillips 1985), we find from (55) that

$$\{\tau_w\} = 2\rho\alpha_T c_D r^{1/2} K u_{10}^2. \qquad (59)$$

By assuming that the spectrum locally adjusts to the changes in the wind, we obtain from (57)

$$\frac{\partial\{\rho U_s\}}{\partial t} = \frac{3\rho\alpha_T c_D^{1/2} u_{10}^2}{2g}\left(\frac{\partial u_{10}}{\partial t}\right). \qquad (60)$$

First we check the ratio of the virtual wave stress (59) to the total momentum flux at the sea surface, that is,

$$\frac{\{\tau_w\}}{\rho_a U_*^2} = \frac{2\alpha_T r^{1/2} K}{s}. \qquad (61)$$

Taking "middle of the road" values from Phillips (1985) and Plant (1982), that is, $\alpha_T = 8 \times 10^{-2}$, $r^{1/2} = 5 \times 10^{-1}$, $K = 1 \times 10^{-2}$, and $s = 1.25 \times 10^{-3}$, we find that the ratio (61) is about 0.6. This is the order of magnitude one would expect for an ocean where the dissipation is dominated by wave breaking (Mitsuyasu 1985; Melville and Rapp 1985; Weber and Melsom 1993b).

To verify our hypothesis that the time-dependent term in the generalized flux equation in (56) introduces only minor deviations from the forcing by the virtual wave stress, we consider an oceanic example where the wind typically varies over a time scale T_s of about 12 h ($T_s = 0.5 \times 10^5$ s), while the length scale L_s of the wind field is 500 km. The maximum wind speed u_{10} is taken to be 20 $\mathrm{m\,s^{-1}}$. Utilizing our adopted values for the parameters, we find that

$$\frac{|\partial\{\rho U_s\}/\partial t|}{\{\tau_w\}} \sim \frac{3}{4c_D^{1/2}r^{1/2}K}\left(\frac{u_{10}}{gT_s}\right) \sim 0.1. \qquad (62)$$

According to this, we could expect contributions of the order of 10% from the time derivative of the total wave momentum as compared with the action of the virtual wave stress in the flux equations. A similar small contribution comes from the radiation stresses if we use the alternative description in (45) with the form drag minus the radiation stress on the right-hand side of (56).

7. Model results

Here we apply an operational wave prediction model to compute the various source terms in (56), while the Lagrangian fluxes U and V are obtained by a numerical simulation. First we introduce the two-dimensional frequency spectrum $F(\varpi, \theta)$. Here $\varpi = \omega/2\pi$, and θ is the angle of the wavenumber vector $\boldsymbol{\kappa}$ with respect to the y axis, measured positive in the clockwise direction, that is, $\boldsymbol{\kappa} = (\kappa \sin\theta, \kappa \cos\theta)$. For deep water, the equation for the evolution of the wave spectrum becomes (Komen et al. 1994)

$$\frac{\partial F(\varpi, \theta)}{\partial t} + \mathbf{C}_g \cdot \nabla F(\varpi, \theta) = S_{\mathrm{in}}(\varpi, \theta) + S_{\mathrm{nl}}(\varpi, \theta) + S_{\mathrm{ds}}(\varpi, \theta), \qquad (63)$$

where the group velocity vector for deep-water waves is defined by $\mathbf{C}_g = [C_g^{(x)}, C_g^{(y)}] = (\pi\varpi \sin\theta/\kappa, \pi\varpi \cos\theta/\kappa)$. Furthermore, S_{in} is the rate of energy input from the atmosphere, S_{nl} is the contribution from components of different wavenumbers by nonlinear wave–wave interaction, and S_{ds} is the wave energy dissipation.

We intend to apply (63) to generate the forcing terms from real weather situations in a numerical barotropic storm surge model. In that case we must include a mean surface elevation $h(x, y, t)$ and a variable air surface pressure $P_0(x, y, t)$ into the model. We must also include some sort of bottom friction to dampen inertial oscillations. Here we choose a linear Rayleigh friction with a constant friction coefficient R_f. We take the depth of integration to be considerably larger than the wavelength of the most energetic surface waves, implying that we can apply deep-water wave theory. Extending our wave-forcing formulation in (56) to two dimensions, the ocean surge model for a wave-influenced surface stress can be written in vector form as

$$\frac{\partial \mathbf{V}}{\partial t} + f\mathbf{k} \times \mathbf{V} + R_f\mathbf{V} + H\nabla(gh - P_0/\rho) = \frac{1}{\rho}(\boldsymbol{\tau}_{\mathrm{wind}} + \boldsymbol{\tau}_{\mathrm{wave}})$$

and

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{V} = 0, \qquad (64)$$

where $\mathbf{V} = (U, V)$. Furthermore $\boldsymbol{\tau}_{\mathrm{wind}} = [\tau_{\mathrm{wind}}^{(x)}, \tau_{\mathrm{wind}}^{(y)}]$ is the horizontal wind stress. Its form will be specified later. According to (46), the wave-induced stress component is given by

$$\frac{\boldsymbol{\tau}_{\mathrm{wave}}}{\rho} = \frac{\{\boldsymbol{\tau}_w\}}{\rho} + \frac{\partial\{\mathbf{V}_S\}}{\partial t}. \qquad (65)$$

Here {} denotes the two-dimensional spectral average, and $\mathbf{V}_S = (U_S, V_S)$ is the Stokes transport. For a single wave component, we have in the present notation that $U_S = (\omega\zeta_0^2 \sin\theta)/2$ and $V_S = (\omega\zeta_0^2 \cos\theta)/2$. Associating now the wave amplitude for a single component in the spectrum by $\zeta_0^2 = 2F(\varpi, \theta)\Delta\varpi\Delta\theta$, we obtain for the spectral distribution

$$\{\mathbf{V}_S\} = 2\pi \int_0^{2\pi} \left[\int_0^{\infty} \varpi F(\varpi, \theta)\{\mathbf{i} \sin\theta + \mathbf{j} \cos\theta\}\, d\varpi \right] d\theta; \qquad (66)$$

see also Jenkins (1989).

The virtual wave stress components $\{\tau_w^{(x)}\}$ and $\{\tau_w^{(y)}\}$ in (65) are related to the wave energy dissipation S_{ds} in (63) caused by turbulent dissipation and breaking [e.g., (33) and (52)]. In our spectral formulation, we can write

$$\{\boldsymbol{\tau}_w\} = -2\pi\rho \int_0^{2\pi} \left[\int_0^{\infty} \varpi S_{\mathrm{ds}}(\varpi, \theta)\{\mathbf{i} \sin\theta + \mathbf{j} \cos\theta\}\, d\varpi \right] d\theta. \qquad (67)$$

A related expression for the terms that redistribute the momentum lost from the waves by dissipation is proposed by Jenkins (1989).

It is interesting to note that if we multiply the wave equation in (63) by $2\pi\varpi \sin\theta\Delta\varpi\Delta\theta\mathbf{i}$, and $2\pi\varpi \cos\theta\Delta\varpi\Delta\theta\mathbf{j}$, respectively, and integrate over the spectrum, we obtain

$$\frac{\partial\{\mathbf{V}_S\}}{\partial t} + \nabla \cdot \{\mathbf{C}_g\mathbf{V}_S\} = \{\boldsymbol{\tau}_D\}/\rho - \{\boldsymbol{\tau}_w\}/\rho, \qquad (68)$$

where $\{\boldsymbol{\tau}_w\}$ is given by (67), and the spectral form drag is

$$\{\boldsymbol{\tau}_D\} = 2\pi\rho \int_0^{2\pi} \left[\int_0^{\infty} \varpi S_{\mathrm{in}}(\varpi, \theta)\{\mathbf{i} \sin\theta + \mathbf{j} \cos\theta\}\, d\varpi \right] d\theta. \qquad (69)$$

Here we have assumed that the integral over the spectrum of the nonlinear interaction terms is zero. Accordingly, by rearranging (68), we find

$$\{\boldsymbol{\tau}_D\} = \{\boldsymbol{\tau}_w\} + \frac{\partial\{\rho\mathbf{V}_S\}}{\partial t} + \nabla \cdot \{\mathbf{C}_g\rho\mathbf{V}_S\}. \qquad (70)$$

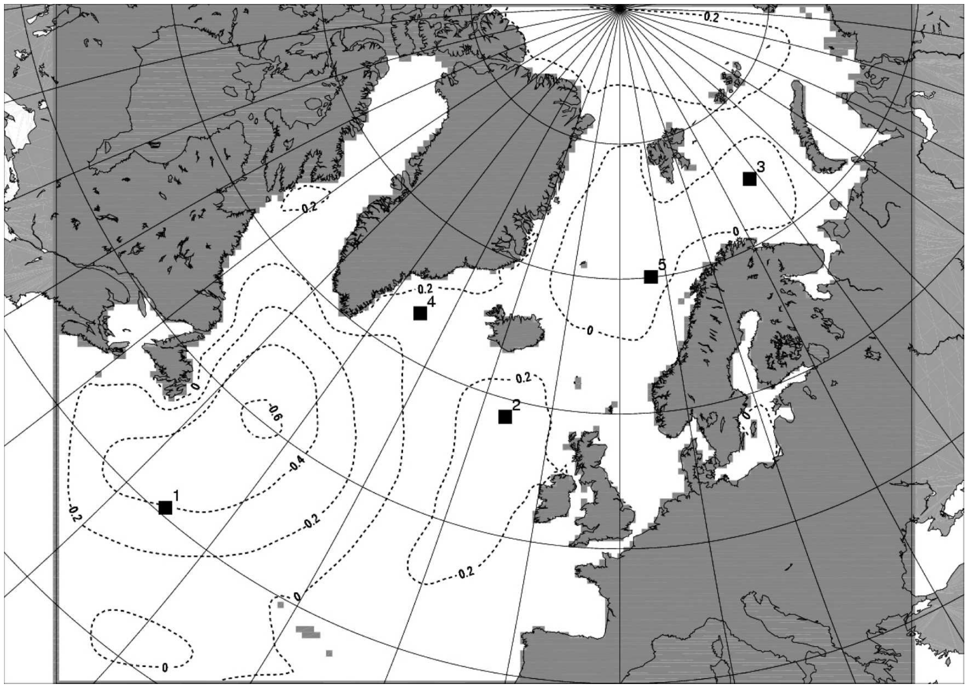

Fig. 2. Model domain for the wave and storm surge model runs. The dashed line is the monthly (February 2004) mean difference in surface elevation (m) between the control run and the experiment (positive values when the experiment elevation is larger than the surface elevation of the control run). The black squares are the locations of the stations, together with the station numbers, where the modeled wind- and wave-induced forcing has been compared with the surface forcing from the 10-m wind speed (Table 1).

This is the two-dimensional spectral analogy to (45), which was derived entirely from the momentum equation and the dynamical boundary conditions. This means that the wave-forcing terms (65) in the storm surge equation alternatively can be written

$$\boldsymbol{\tau}_{\text{wave}} = \{\boldsymbol{\tau}_D\} - \nabla \cdot \{\mathbf{C}_g \rho \mathbf{V}_S\} = \{\boldsymbol{\tau}_D\} - \nabla \cdot \{\Re\}, \tag{71}$$

where $\Re$ is the radiation stress tensor (39).

To produce forcing data for the storm surge model, the numerical ocean wave model (WAM; Komen et al. 1994) was run for a period of 2 months, starting at 1 January 2004. As input to the wave model we used analyzed winds from the European Centre for Medium-Range Weather Forecasts (ECMWF). The model domain, which is the same for both the wave model and the storm surge model, is shown in Fig. 2. The grid is rotated spherical with the equator located at 60°N. The horizontal resolution is 0.45° in both directions. Every third hour, the wave model calculates the forcing terms on the right-hand side of (64). The storm surge equations in (64) were discretized on a C grid with centered differences in both time and space. To remove any possible spurious modes, an Eulerian (forward) time step was applied every 20 time steps. At each time step, the surface elevation was updated first and then each of the two horizontal components was updated. In this procedure, the Sielecki method (Sielecki 1968) was used. This means that all the updated variables are used immediately in the subsequent equations. The bathymetry for the computational domain is in some places more than 4000 m deep. For such deep waters, the Courant–Friederichs–Lewy (CFL) criterion imposes an extremely short time step on the storm surge model. Therefore, the water depth is limited everywhere to 200 m in this study. This of course, makes the experiments rather unrealistic as storm surge simulations for the real world. However, because the main scope of this experiment is not to forecast the surge as realistically as possible but to quantify the relative effects of the wave-forcing terms on the right-hand side of (64), we believe that this simplification can be justified. Another simplification is the removal of all the open boundaries for the storm surge model. This makes it easier to handle the boundary conditions in the model. The introduction of these artificial walls may introduce spurious reflections of long barotropic waves. The justification for this is again the fact that the main purpose of this investigation is to quantify the effects of the wave-induced forcing. Reflected barotropic waves will probably be present in both the experiment and the control runs. Our assumption is that this will have little effect on the difference between the two runs.

The model was run twice for the experiment period. First, we performed a control run using the traditional method of calculating the surface stress from the 10-m wind speed (Large and Pond 1981). Here, the stress components are given by

$$\tau_{10}^{(x)} = c_{DM}\rho_a|\mathbf{v}_{10}|u_{10} \quad \text{and}$$
$$\tau_{10}^{(y)} = c_{DM}\rho_a|\mathbf{v}_{10}|\upsilon_{10}, \tag{72}$$

where u_{10} and υ_{10} are the x and y components of the 10-m wind vector $\mathbf{v}_{10}$. The model drag coefficient c_{DM} is independent of the wind when the wind speed is below the threshold value of 11 m s^{-1} and is linearly dependent on the wind for stronger wind speeds, that is,

$$c_{DM} = \begin{cases} 0.0012 & \text{if} \quad |\mathbf{v}_{10}| < 11 \text{ m s}^{-1}, \\ (0.49 + 0.065|\mathbf{v}_{10}|)10^{-3} & \text{if} \quad |\mathbf{v}_{10}| > 11 \text{ m s}^{-1}. \end{cases} \tag{73}$$

The larger drag coefficient for $|\mathbf{v}_{10}| > 11$ m s^{-1} is introduced to model, in a crude way, the increasing effect of the sea state on the momentum transfer from the atmosphere to the ocean at higher wind speeds. For the control run, the stresses calculated from (72) and (73) were used instead of $\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}}$ on the right-hand side of (64). Second, an experiment was run for the same period, but with the wave-forcing term $\boldsymbol{\tau}_{\text{wave}}$, defined by (65), calculated from the WAM. For simplicity, the turbulent wind-forcing term $\boldsymbol{\tau}_{\text{wind}}$ in (64) for this case was parameterized as in (72), but with a constant drag coefficient ($c_D = 1.2 \times 10^{-3}$). As already mentioned, the simulation period was the first two months of 2004. January was basically used as a period for spinning up the models. Accordingly, all of the results presented here will be for February 2004, which is taken to be the experiment period.

In Fig. 2, the monthly mean differences in sea surface elevation are depicted. Here, the mean differences in surface elevation are in some areas larger than 0.6 m. The largest differences are found in the North Atlantic. For substantial parts of the computational domain, the mean differences between the experiment and the control run are about 0.2 m. Clearly, the introduction of a sea-state-dependent forcing has a significant impact on storm surge modeling. However, because our experimental model runs gave consistently higher values for the surface elevation than the control run, it is obvious that the flat drag coefficient $c_D = 1.2 \times 10^{-3}$ from Large and Pond (1981), which we used in our runs for the wind-induced stress, contains some wave effects as well. To remedy this, we calculated the average flat drag coefficient from five selected stations depicted in Fig. 2 (black squares) such that the average stress of the control run with (72) and (73) was equal to the mean value of $\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}}$ at these stations. This indicated that our wind stress could be approximated by

$$\boldsymbol{\tau}_{\text{wind}} = \rho_a c_D|\mathbf{v}_{10}|\mathbf{v}_{10}, \quad c_D = 0.95 \times 10^{-3}. \tag{74}$$

TABLE 1. Forcing terms (m^2 s^{-2}) on the right-hand side of (64) at five selected stations. The table shows the monthly average for February 2004.

	$\lvert\partial\{\mathbf{V}_S\}/\partial t\rvert$	$\lvert\{\boldsymbol{\tau}_w\}/\rho\rvert$	$\lvert(\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}})/\rho\rvert$	$\lvert\boldsymbol{\tau}_{10}/\rho\rvert$
Station 1	0.010	0.095	0.258	0.230
Station 2	0.007	0.069	0.207	0.185
Station 3	0.003	0.018	0.097	0.098
Station 4	0.007	0.057	0.183	0.168
Station 5	0.010	0.090	0.248	0.222

To investigate the relative importance of the various terms on the right-hand side of (64), a time series of separate terms has been written to file at the five selected stations inside the model domain (see Fig. 2). In Table 1, the monthly mean values for February 2004 of these forcing terms are given for all five stations.

We realize from Table 1 that on these five stations the magnitude of the average virtual wave stress alone is nearly 50% of mean stress from the 10-m wind speed. This is in good accordance with our previous estimates in section 6. All terms involving the Stokes drift in (65) are, on the average, one to two orders of magnitude smaller than the total stress. This also agrees well with our findings in section 6. If we used the alternative expression in (71) for the wave-induced stress, the magnitudes of the radiations stress terms are found to be equally small. To illustrate further the effect of the sea-state-dependent surface stresses, the time series for the wind-induced stress in (74) and the wave-induced stress in (65) is plotted in Fig. 3 together with the mean stress in (72) and (73) calculated from the 10-m wind speed. Here, we have depicted the results from stations 1 and 5. From Fig. 3 we note the interesting fact that for small-to-moderate winds, the wind-induced stress is larger than the wave-induced stress. However, for stronger winds (large peaks in the plot) the wave-induced part is the largest. At such winds our computed value of $|(\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}})/\rho|$ exceeds the traditional value of $|\boldsymbol{\tau}_{10}/\rho|$ in the forcing terms for the surge. The quantification of the enhanced influence of the wave part of the stress for stronger winds is important, because the assessment of damage caused by the surge is particularly relevant in the case of storm events.

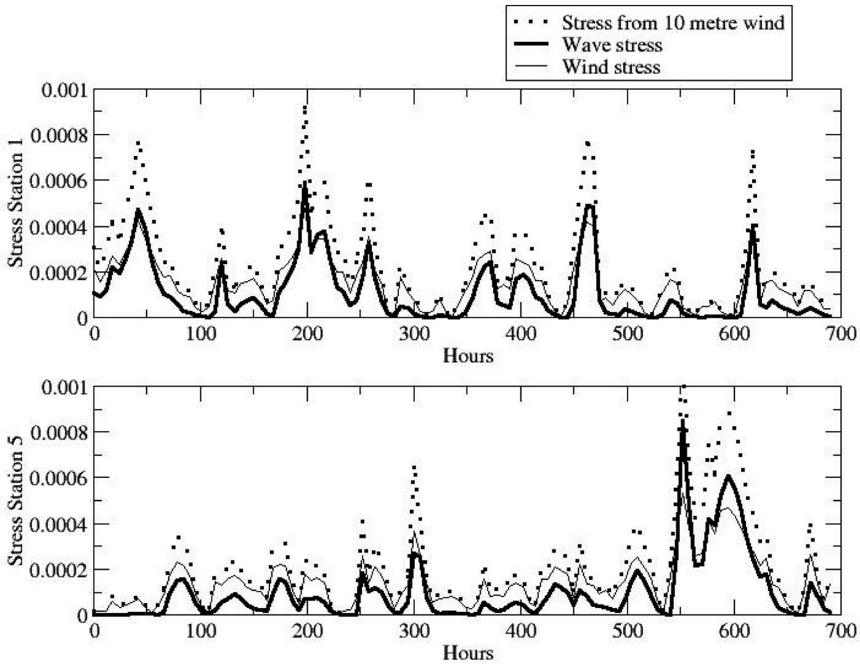

FIG. 3. Time series of the surface stresses ($m^2\ s^{-2}$) at two selected stations (1 and 5) for February 2004. The locations of these stations are depicted in Fig. 2. The dotted lines are the stresses calculated from the 10-m wind speed. The solid thick lines are the wave-induced stress (65) and the solid thin lines are the wind-induced stress (74).

8. Summary and concluding remarks

We have demonstrated that integration of the Eulerian momentum equation from a constant depth of vanishing motion to the oscillating surface yields the same equations for the volume transport in periodic waves as that obtained from a direct Lagrangian analysis to second order in the wave steepness. It is found that the form drag associated with the action of the fluctuating wind stress over the wave slopes is the only source term in the equation for the integrated Lagrangian volume transport induced by spatially periodic waves. Accordingly, waves that decay in time in the absence of external forcing do not induce any net transport in a rotating ocean. This is valid whether the decay is due to viscous dissipation or wave breaking. In the case of temporally periodic waves, where the wave amplitude may grow or decay slowly in space, we show that the horizontal divergence of the total wave momentum flux is an additional source term in the equations for the wave-induced volume transports. Alternatively, this term can be written in terms of the radiation stress, as shown by Phillips (1977). Comparison between analytical and empirical expressions for the form drag over smooth waves in a balanced state (statistically steady waves) leads to a simple estimate for the bulk eddy viscosity in the surface layer associated with wind waves. By modeling wave breaking in a simple way, a similar formula for the eddy viscosity in a saturated sea, where breaking dominates the dissipation process, is obtained. On the basis of the results for a single wave component, we derive equations for the wave-induced volume transports in a fully developed sea where the wave spectrum may change slowly in space and time. For an idealized one-dimensional frequency spectrum (Toba 1973), and for reasonable estimates for the time and space variation of the wind, our equations for the wave-induced volume fluxes appear to have realistic forcing terms. For the precise form of the wave spectrum in a real ocean, these equations need input from an ocean wave prediction model (e.g., Komen et al. 1994).

The wave-forcing terms for a storm surge model have been calculated for a 2-month period in 2004 by running the WAM (Komen et al. 1994) over a model domain covering the northern North Atlantic and the Nordic Seas. These terms were then used to force a storm surge model for the same period. The calculated surface elevations were compared with the results from a control run where the surface stresses were obtained in the traditional way, using the 10-m wind speed. In this way it was found that the non-wave-dependent drag coefficient for the wind stress part of the forcing could be approximated as $c_D = 0.95 \times 10^{-3}$. Time series of each individual forcing term for selected locations revealed that the contribution from the virtual wave stress amounted to about 50% of the total forcing for moderate to strong winds. The terms involving the time rate of change of the Stokes transport and the radiation stress in the alternative formulation were at least one order of magnitude smaller than the total stress. In the case of storm events with rough seas, the wave-induced part of the stress is larger than the wind stress. This is an important finding, because it is particularly in connection with strong winds that reliable surge simulations are needed.

The order-of-magnitude estimates from the idealized spectral formulation in section 6 for the wave-induced forcing were remarkably close to those obtained from the numerical ocean simulations in section 7, using the WAM. This lends support to the robustness of the present formulation for the wave-influenced transport in the oceanic surface layer. For shallow waters, where the influence of bottom friction and wave breaking are more prominent, the effect of the radiation stresses will also be larger. This may change the balance between the forcing terms in the surge equation. A wave-influenced storm surge model for a shallow coastal region is clearly the next step in line for this type of investigation.

Acknowledgments. This study has been performed under the Strategic University Programme "Modelling of Currents and Waves for Sea Structures" funded by The Research Council of Norway.

REFERENCES

Andrews, D. G., and M. E. McIntyre, 1978: An exact theory of nonlinear waves on a Lagrangian-mean flow. *J. Fluid Mech.,* **89,** 607–646.

Ardhuin, F., B. Chapron, and T. Elfouhaily, 2004: Waves and the air–sea momentum budget: Implications for ocean circulation modeling. *J. Phys. Oceanogr.,* **34,** 1741–1755.

Banner, M. L., 1990: The influence of wave breaking on the surface pressure distribution in wind–wave interactions. *J. Fluid Mech.,* **211,** 463–495.

Bye, J. A. T., 1967: The wave drift current. *J. Mar. Res.,* **25,** 85–102.

Chalikov, D. V., and V. K. Makin, 1991: Models of the wave boundary layer. *Bound.-Layer Meteor.,* **56,** 83–99.

Chang, M.-S., 1969: Mass transport in deep-water long-crested random gravity waves. *J. Geophys. Res.,* **74,** 1515–1536.

Hasselmann, K., 1970: Wave-driven inertial oscillations. *Geophys. Astrophys. Fluid Dyn.,* **1,** 463–502.

——, 1971: On the mass and momentum transfer between short gravity waves and larger-scale motions. Part 1. *J. Fluid Mech.,* **50,** 189–205.

Jenkins, A. D., 1986: A theory for steady and variable wind- and wave-induced currents. *J. Phys. Oceanogr.,* **16,** 1370–1377.

——, 1987: Wind and wave induced currents in a rotating sea with depth-varying eddy viscosity. *J. Phys. Oceanogr.,* **17,** 938–951.

——, 1989: The use of a wave prediction model for driving a near-surface current model. *Dtsch. Hydrogr. Z.,* **42,** 133–149.

Kitaigorodskii, S. A., 1996: The influence of breaking wind waves on the aerodynamic roughness of the sea surface as seen from below. *The Air–Sea Interface: Radio and Acoustic Sensing, Turbulence and Wave Dynamics,* M. A. Donelan, W. H. Hui, and W. J. Plant., Eds., The Rosenstiel School of Marine and Atmospheric Science, 177–187.

Komen, G. J., L. Cavaleri, M. Doneland, K. Hasselmann, S. Hasselman, and P. A. E. M. Janssen, Eds., 1994: *Dynamics and Modelling of Ocean Waves.* Cambridge University Press, 533 pp.

Lamb, H., 1932: *Hydrodynamics.* 6th ed. Cambridge University Press, 738 pp.

Large, W. G., and S. Pond, 1981: Open ocean momentum flux measurements in moderate to strong winds. *J. Phys. Oceanogr.,* **11,** 324–336.

Longuet-Higgins, M. S., 1953: Mass transport in water waves. *Philos. Trans. Roy. Soc. London,* **245A,** 535–581.

——, 1969: A nonlinear mechanism for the generation of sea waves. *Proc. Roy. Soc. London Soc. London,* **311A,** 371–389.

——, and R. W. Stewart, 1960: Changes in the form of short gravity waves on long waves and tidal currents. *J. Fluid Mech.,* **8,** 565–583.

Madsen, O. S., 1977: A realistic model for the wind-induced Ekman boundary layer. *J. Phys. Oceanogr.,* **7,** 248–255.

McWilliams, J. M., and J. M. Restrepo, 1999: The wave-driven ocean circulation. *J. Phys. Oceanogr.,* **29,** 2523–2540.

Melsom, A., 1996: Effects of wave breaking on the surface drift. *J. Geophys. Res.,* **101,** 12 071–12 078.

Melville, W. K., and R. J. Rapp, 1985: Momentum flux in breaking waves. *Nature,* **317,** 514–516.

Mitsuyasu, H., 1985: A note on the momentum transfer from wind to waves. *J. Geophys. Res.,* **90,** 3342–3345.

Ölmez, H. S., and J. H. Milgram, 1992: An experimental study of attenuation of short water waves by turbulence. *J. Fluid Mech.,* **239,** 133–156.

Phillips, O. M., 1977: *The Dynamics of the Upper Ocean.* 2d ed. Cambridge University Press, 336 pp.

——, 1985: Spectral and statistical properties of the equilibrium range in wind-generated gravity waves. *J. Fluid Mech.,* **156,** 505–531.

Plant, W. J., 1982: A relationship between wind stress and wave slope. *J. Geophys. Res.,* **87,** 1961–1967.

Sielecki, A., 1968: An energy-conserving difference scheme for the storm surge equations. *Mon. Wea. Rev.,* **96,** 150–156.

Starr, V. P., 1959: Hydrodynamical analogy to $E = mc^2$. *Tellus,* **11,** 135–138.

Stokes, G. G., 1847: On the theory of oscillatory waves. *Trans. Camb. Philos. Soc.,* **8,** 441–455.

Toba, Y., 1973: Local balance in the air-sea boundary processes. III: On the spectrum of wind waves. *J. Oceanogr. Soc. Japan,* **29,** 209–220.

Ünlüata, Ü., and C. C. Mei, 1970: Mass transport in water waves. *J. Geophys. Res.,* **75,** 7611–7618.

Weber, J. E., 1983: Attenuated wave-induced drift in a viscous rotating ocean. *J. Fluid Mech.,* **137,** 115–129.

——, 1997: Mass transport induced by surface waves in a viscous rotating fluid. *Free Surface Flows with Viscosity,* P. A. Tyvand, Ed., Computational Mechanics Publications, 37–67.

——, 2003: Wave-induced mass transport in the oceanic surface layer. *J. Phys. Oceanogr.,* **33,** 2527–2533.

——, and A. Melsom, 1993a: Transient ocean currents induced by wind and growing waves. *J. Phys. Oceanogr.,* **23,** 193–206.

——, and ——, 1993b: Volume fluxes induced by wind and waves in a saturated sea. *J. Geophys. Res.,* **98,** 4739–4745.