Structure, Propagation, and Mixing of Energetic Baroclinic Tides in Mamala Bay, Oahu, Hawaii

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ABSTRACT

Large semidiurnal vertical displacements (~100 m) and strong baroclinic currents (~0.5 m s⁻¹; several times as large as barotropic currents) dominate motions in Mamala Bay, outside the mouth of Pearl Harbor, Hawaii. During September 2002, the authors sought to characterize them with a 2-month McLane moored profiler deployment and a 4-day intensive survey with a towed CTD/ADCP and the Research Vessel (R/V) Revelle hydrographic sonar. Spatial maps and time series of turbulent dissipation rate ε, diapycnal diffusivity Kᵌ, isopycnal displacement η, velocity u, energy E, and energy flux F are presented. Dissipation rate peaks in the lower 150 m during rising isopycnals and high strain and shows a factor-of-50 spring–neap modulation. The largest Kᵌ values, in the western bay near a submarine ridge, exceed 10⁻³ m² s⁻¹. The M₂ phases of η and u increase toward the west, implying a westward phase velocity c_p ~ 1 m s⁻¹ and horizontal wavelength ~60 km, consistent with theoretical mode-1 values. These phases vary strongly (~±45°) in time relative to astronomical forcing, implying remotely generated signals. Energy and energy flux peak 1–3 days after spring tide, supporting this interpretation. The group velocity, computed as the ratio F/E, is near ~1 m s⁻¹, also in agreement with theoretical mode-1 values. Spatial maps of energy flux agree well with results from the Princeton Ocean Model, indicating converging fluxes in the western bay from waves generated to the east and west. The observations indicate a time-varying interference pattern between these waves that is modulated by background stratification between their sources and Mamala Bay.

1. Introduction

Internal tides have been observed for years in many locations, often dominating kinetic and potential energy signals. The importance of internal tides in modulating ocean currents (Wunsch 1975), dispersing pollutants (Petrenko et al. 2000), acoustic propagation (Dushaw et al. 1995), and even sea level (Colosi and Munk 2006) has long been appreciated. In the past few years their importance in mixing the deep ocean has become recognized.

Unlike their near-deterministic surface counterparts, internal tides are a much more broadband and intermittent phenomenon, with highly temporally variable energy and phase. Varying stratification at the generation site can modulate their energy (Mitchum and Chiswell 2000). Phase variations are thought to arise from fluctuations in stratification, and hence propagation speed, between their source and the observation site (Munk and Cartwright 1966; Colosi and Munk 2006). Refraction by mesoscale currents can also play a role (Rainville 2004).

This article describes intensive observations of energetic semidiurnal internal tides in Mamala Bay, on the south shore of the island of Oahu, Hawaii (Fig. 1). Much of Mamala Bay is between 400 and 500 m deep, displaying gentle slopes (s ~ 0.03) until the very steep inshore embankments (s ~ 0.2) are encountered. A prominent ridge is evident in the west.

From an array of moorings deployed for 1.5 yr by Science Applications International Corporation (SAIC), Hamilton et al. (1995) reported M₂ barotropic currents >0.5 m s⁻¹ at the headlands near Barbers Point (A4) and Diamond Head (E4), with opposing phases. Near
the bay center (B3), barotropic currents were much weaker, but peak–peak vertical displacements inferred from temperature approached 150 m. Hamilton et al. (1995) attributed the large displacements to convergent barotropic currents, noting a progression of phase in recorded sea level around Oahu. However, from analysis of phase and energy flux from the same data, together with Princeton Ocean Model (POM) runs, Eich et al. (2004) suggest remote generation at sites eastward and westward of the bay (Fig. 1, arrows). They interpreted the opposing baroclinic velocities at the headlands and the enhanced displacements near the bay center as a standing-wave pattern between eastward- and westward-propagating waves. The westward fluxes in the eastern portion of the bay imply a partially standing pattern resulting from a stronger westbound wave.

Here we build upon this previous work, revisiting Hamilton’s moored data and also taking advantage of new data (Fig. 1)—namely, an intensive 4-day towed CTD/ADCP survey, two 25-h microstructure measurements, and a 2-month deployment of a McLane moored profiler, which crawled up and down a mooring wire delivering hourly profiles of temperature $T$, salinity $S$, and velocity. Nesting the high-resolution space–time towed survey within the context provided by the longer moored records, we have constructed a much more complete picture of the wave’s structure, energy flux, and dissipation than was previously possible.

Three aspects of the motions (the latter two previously unobserved) make them of particular interest, as follows.

1) Their amplitude: As will be shown, vertical displacements are large (>100 m). Baroclinic currents ($\approx 0.5$ m s$^{-1}$) are strong, often several times in excess of the depth-averaged flow.

2) Their resultant mixing: The motions support large, strongly spatially and temporally variable diapycnal diffusivities, $K_d$ (Fig. 1, colored circles). These exceed $10^{-3}$ m$^2$ s$^{-1}$ in the western part of the bay, potentially large enough to significantly impact pollutant dispersal, biological processes, and scalar budgets on seasonal time scales.

3) Their strong space and time dependence: The magnitude, mixing, and the interference pattern between the east- and westbound waves all vary substantially in space and time. We argue that slowly varying background stratification between the eastbound and westbound waves’ sources and Mamala Bay modulates the arrival time, and thus the interference pattern, within the bay. The strong time dependence of internal tides and their mixing has implications for similar embayments worldwide, as well as for the global distribution of deep mixing (Alford 2003a).

The paper is organized as follows: Descriptions of the data and techniques employed are presented in sections 2 and 3. Our observations are described next (section 4). We then investigate the signal’s time dependence in the context of Hamilton’s long time series (section 4d). Conclusions follow.

2. Data and methods

This paper employs data from various sources, which are all described here. First the synoptic data collected
during fall 2002 are reported; then microstructure and moored data collected at other times are described.

a. Shipboard survey

1) Operations

From 23 to 26 September 2002, we repeatedly occupied the four legs shown in Fig. 1 for 24 h each (nearly two \( M_2 \) cycles). The legs were designed so that they could be completed in 1.5 h while towing at 4 kt (1 kt \( \approx 0.5144 \) m s\(^{-1}\)), thus providing about four points per tidal cycle (12.4 h) at the ends and about eight nearer the middle of the legs. The legs were occupied in succession, as numbered in Fig. 1. The intention was to rely upon the quasi-deterministic nature of the internal tide to treat the successive days as if they were occupations of the same \( M_2 \) cycle.

2) Shallow Water Instrumented Mapping System

Temperature, salinity, and velocity were measured by repeatedly cycling our towed body, the Shallow Water Instrumented Mapping System (SWIMS II), behind the ship in a sawtooth pattern (Fig. 2a). Falling near 2 m s\(^{-1}\) and rising at 1 m s\(^{-1}\), profiles were taken every 700 m or so while steaming at 4 kt. A gimballed altimeter allows profiling to within several meters of the bottom; however, we avoided profiling the bottom 20 m owing to the large number of submerged objects in Mmala Bay. Short-baseline acoustic tracking of the instrument allows determination of the instrument’s absolute position to an accuracy of tens of meters.

SWIMS II’s in situ sensors include two pumped Sea-Bird Electronics (SBE) CTDs to measure temperature and salinity. Additional sensors include dissolved oxygen, pH, chlorophyll, and optical backscatter. Of these, only the CTD data are reported here. All data are sampled at 24 Hz, telemetered up the sea cable for real-time monitoring, and recorded.

In a recent enhancement to SWIMS II, we added upward- and downward-looking 300-kHz RD Instruments (RDI) Workhorse ADCPs. These complement shipboard ADCP velocity measurements in that they are valid closer to the surface and the bottom, avoiding the blanking and surface-hit-contaminated regions, respectively. In addition, since measurements are taken much closer to the transducer, higher frequency (hence higher resolution) can be used, and degradation of horizontal and vertical resolution due to beam-spreading effects (e.g., Alford and Pinkel 2000) are reduced.

Rotation of the raw ADCP velocities from the rolling, pitching, and depth-cycling body into earth coordinates is nontrivial, and the techniques will be reported...
elsewhere. To demonstrate the success of the method, a sample of zonal velocities from SWIMS II and the *Revelle* hydrographic sonar (described below) is shown in Fig. 2. The improved coverage near the surface and bottom is clear.

3) *Revelle* sonar

Shipboard velocities were from the *Revelle* hydrographic sonar, a two-frequency (50/140 kHz), high-precision Doppler sonar system installed on the *Revelle* by R. Pinkel (Scripps Institution of Oceanography). Specialized data-processing techniques for this system have been developed by G. Carter (2002, personal communication). Briefly, data from each unit are despiked and averages computed over useful time and depth intervals for analysis (10 min and 10 m). To minimize introduction of navigational noise, data taken during ship turns are not used (Fig. 2b, white vertical lines). A gradual ramp is used to blend data from the higher-resolution 140-kHz unit at depths <140 m smoothly into 50-kHz data in the deeper ranges beyond the reach of the 140-kHz unit.

During the shipboard survey, two several-hour gaps resulted when the *Revelle* system crashed unnoticed. To achieve maximal coverage, a hybrid velocity product is used that combines data from four ADCPs: both *Revelle* sonars as well as the SWIMS II system.

b. McLane moored profiler

To provide temporal context for the spatial surveys, a McLane moored profiler (MP) was deployed on the 395-m isobath (Fig. 1). These devices (Doherty et al. 1999) climb up and down a standard mooring wire using a traction drive, carrying sensors including a Falmouth Scientific CTD and a Neil Brown acoustic current meter (ACM). Deployed on 11 August 2002, the MP profiled from 50- to 350-m depth each hour for 2 months bracketing the shipboard survey, until a spring failed on 6 October 2002 and the device could no longer climb.

c. Other data

1) Microstructure data

Microstructure was measured near Barber’s Point (A4) and at B3 in the bay (Fig. 1, colored circles) during spring tide in September 2000 as part of the Hawaii Ocean Mixing Experiment (HOME) survey cruise. The advanced microstructure profiler (AMP) was used at Barber’s Point, and the modular microstructure profiler (MMP) was used at B3. Both are loosely tethered profilers that measure kinetic energy dissipation rate $\epsilon$ by detecting centimeter-scale shear with airfoil probes (Gregg 1987). Each also carries a CTD. Each station was occupied for 25 h ($2 M_2$ cycles), with 1–3 profiles per hour, depending on the water depth.

2) Hamilton’s moorings

Moorings were deployed for over 1 yr during 1994–95 at locations indicated in Fig. 1. Some had bottom-mount ADCPs, while some had current meters and temperature sensors at several discrete depths. All were sampled at least hourly. These data are reported fully in Hamilton et al. (1995) and in several subsequent investigations (Petrenko et al. 2000; Eich et al. 2004). While the horizontal and vertical coverage of these data are limited, we employ them in a later section to examine long-term variability. Our primary interest is computing energy and energy flux; consequently, only those moorings with full-column temperature and current data are used, namely, A4 (Barber’s Point; measurements at 70, 125, 240, and 450 m), B3 (central bay; measurements at 70, 125, and 240 m), and E4 (Diamond Head; measurements at 70, 125, 240, and 450 m).

3) Honolulu tide gauge

As an index of astronomical tidal forcing, sea level data are used from the tide gauge in Honolulu harbor, within Mamala Bay. Hourly sea level estimates were obtained for the years 1994 (Hamilton’s moorings) and 2002 (present work) from the University of Hawaii Sea Level Center. These are a small subset of a very much longer record starting in 1905, which is described in Colosi and Munk (2006). Since our focus here is the $M_2$ frequency, we semi-diurnally bandpass the records to gain an index of the strength of the barotropic semi-diurnal tide.

3. Processing and techniques

a. Baroclinic velocities

Near sloping topography, barotropic and baroclinic velocities are not easily separated. Here, we simply define $\mathbf{u}_{BC} = \mathbf{u} - \mathbf{u}_{BT}$, where $\mathbf{u}_{BT}$ is the depth-average flow, and $\mathbf{u}$ is velocity.

b. Isopycnal displacements

Isopycnal displacements $\eta(z, t)$ are computed from potential density data for both the SWIMS II survey and the MP following standard procedures (e.g., Alford and Pinkel 2000). Briefly, the depth of an evenly spaced set of isopycnal surfaces $z(\rho, t)$ is computed at all times via linear interpolation. Then, a mean density profile
$p(z)$ is computed. (In the case of the survey data, the 4-day cruise mean is used. Here, horizontal spatial gradients in the mean density profile do exist in the upper 100 m, introducing small errors of several meters. In the case of the MP, long-term cooling trends are removed by using a 5-day sliding boxcar window.) Then, the displacement of each isopycnal relative to its mean position is simply $\eta(p, t) = z(p, t) - \bar{z}(p)$. The corresponding Eulerian quantity is then given by interpolation by $\xi(z, t) = \eta[z(p, t), t]$.

Barotropic tidal flow over sloping topography introduces vertical displacements with a linear depth profile (Baines 1982), $\eta_{BT} = s \cdot x_{BT}(t)(z/H)$, where $s$ is the vector slope, $x_{BT} = \int_0^H u_{BT}(t) \, dH$ is the barotropic tidal displacement, and $H$ is the water depth. In Mamala Bay, flow is typically along isobath (Hamilton et al. 1995), so $\eta_{BT}$ is negligible. At the location of the moored profiler, the flow does have a cross-isobath component at times, but the resultant displacements are $<10$ m.

c. Energy flux

The baroclinic energy flux is given by the velocity-pressure covariance, $F = \langle u_{BC} p' \rangle$, where $\langle \cdot \rangle$ indicates a mean over the wave period. The baroclinic pressure anomaly $p'$ is obtained by depth-integrating $N^2 \eta(z)$ and subtracting the mean, following the hydrostatic assumption (Kunze et al. 2002). The method is becoming standard (Nash et al. 2004; Alford 2003a), but the calculation and interpretation of energy flux still require care (Nash et al. 2005).

d. Modal decomposition

It is useful to decompose the displacement and velocity signals into a set of orthogonal modes. For low-frequency waves such as the tides ($\omega \ll N$), the modes over a flat bottom of depth $H$ are the solutions to

$$\frac{\partial^2}{\partial z^2} \eta(z) + c_i^{-2} N^2(z) \eta(z) = 0,$$

subject to the boundary conditions $\eta(0) = \eta(H) = 0$, where $c_i$ are the eigenspeeds. The modal shapes are computed numerically by shooting (Fig. 3), and then modal amplitudes are computed from data at each time via weighted least squares fitting.

The eigenspeeds are well approximated by assuming constant vertically averaged $N$,

$$c_i \approx \frac{NH}{\pi^2},$$

These are related to the wave frequency $\omega = M_2$, and horizontal wavenumber $\alpha$, via the dispersion relation,

$$\omega^2 = c_i^2 \alpha^2 + f^2.$$  \hspace{1cm} (3)

The horizontal group velocity is then

$$c_p = \frac{\partial \omega}{\partial \alpha} = c_i^2 \alpha/\omega,$$  \hspace{1cm} (4)

and the phase velocity is

$$c_p = \omega/\alpha.$$  \hspace{1cm} (5)

Combining Eqs. (4) and (5), it is evident that the eigenspeed is the geometric mean of the phase speed and the group speed, $c_i^2 = c_p c_e$.

Since $M_2/f \approx 3$ in Mamala Bay, all three speeds are fairly close for both modes (Table 1), indicating that semidiurnal tides at this latitude are nearly nondispersive. These will be compared with observed values in section 4a(3).

Over a flat bottom, these modes have dynamical significance; over a slope, the so-called slope modes (Wunsch 1969) more correctly describe subinertial motions. Superinertial solutions near a sloping bottom are, surprisingly, only very recently being investigated (Dale and Sherwin 1996; Dale et al. 2001; Llewellyn-Smith 2004). We suspect that these dynamics may be appropriate for describing the waves propagating into the bay from the east and west, and will revisit this in a future work. Since most of our survey displays gently sloping bottom ($s < 0.03$), we employ the flat-bottom modes as a qualitative measure of wavenumber content. They are clearly not appropriate near the steeply banked inshore shoals.

e. Dissipation rate and diapycnal diffusivity

The dissipation rate was directly measured with shear-probe measurements during the HOME survey.
Theoretical and observed wave speeds, wavelength, and Rossby radius, for modes 1 and 2. Theoretical speeds are computed using measured $N$ at the mooring, in 400-m depth, as in section 3d. Observational estimates of $c_p$ are from spatial gradients of phase [section 4a(2)]. Group velocity is determined from the ratio of flux to energy as in section 4d(2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenspeed $c_n$ (m s$^{-1}$)</td>
<td>1.27</td>
<td>0.66</td>
</tr>
<tr>
<td>Phase speed $c_p$ (m s$^{-1}$)</td>
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<td>0.70</td>
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<tr>
<td>$\eta$, leg 4-3</td>
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<tr>
<td>$\eta$, leg 4-3</td>
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</tr>
<tr>
<td>$u$, leg 4-3</td>
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<td>0.7</td>
</tr>
<tr>
<td>$u$, leg 2</td>
<td></td>
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</tr>
<tr>
<td>Group speed $c_g$ (m s$^{-1}$)</td>
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<td>0.61</td>
</tr>
<tr>
<td>Theory</td>
<td>1.41</td>
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</tr>
<tr>
<td>E4</td>
<td>0.50</td>
<td>0.22</td>
</tr>
<tr>
<td>MP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td></td>
<td></td>
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<tr>
<td>B3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wavelength, $\lambda = \alpha^{-1}$ (km)</td>
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<td>31</td>
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<tr>
<td>$\eta$, leg 4-3</td>
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<td>$\eta$, leg 4-3</td>
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<td></td>
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<tr>
<td>$u$, leg 4-3</td>
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<td>A4</td>
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<td>25</td>
</tr>
<tr>
<td>B3</td>
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<td>10</td>
</tr>
<tr>
<td>Rossby radius = $c_o/f$ (km)</td>
<td>25</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 1. Theoretical and observed wave speeds, wavelength, and Rossby radius, for modes 1 and 2. Theoretical speeds are computed using measured $N$ at the mooring, in 400-m depth, as in section 3d. Observational estimates of $c_p$ are from spatial gradients of phase [section 4a(2)]. Group velocity is determined from the ratio of flux to energy as in section 4d(2).

In Fig. 4 we have constructed 3D plots of $\eta$ (top two rows) and $u_{bc}$ (bottom) at eight phases spanning one $M_2$ cycle. Here, each panel represents the occupation of

Several “techniques” papers have emerged (Galbraith and Kelley 1996; Stansfield et al. 2001; Johnson and Garrett 2004). The success or failure of the method depends entirely upon 1) the noise, $\Delta \rho$, of the CTD relative to the expected overturning density signatures and 2) the size of the overturns relative to the sample interval, $\Delta z$. The impact of both of these on $\epsilon$ depends on $N_{ov}$, which varies greatly over a typical profile. The first yields a spurious $\epsilon_p = 4(0.8)^2 g^2 (\Delta \rho)^{-1} N_{ov}^{-1}$, and the second yields a detection limit of $\epsilon_c = 0.64 n^2 \Delta z^2 N^2_{ov}$. Consequently, we choose to monitor the magnitude of each of these error sources at each overturn using the in situ value of $N_{ov}$.

Whether $\epsilon$ is determined from overturns or shear probes, we estimate the diapycnal diffusivity according to the standard formula, $K_p(z) \leq 0.2 \epsilon(z) N(z)^{-2}$ (Osbourn 1980).

4. Observations

a. Spatial structure

In this section we discuss the spatial structure of the wave as observed from the SWIMS II surveys. By assuming that the wave’s structure was constant over the four days of the cruise, the simultaneous space/time structure of the motions are examined by plotting sections collected at like $M_2$ phase. We will begin by plotting the full three-dimensional-plus-time structure of the wave and will then take successive cuts: first along the $x$ axis and then along a constant-depth surface.

1) Velocity and displacement

In Fig. 5 we have constructed 3D plots of $\eta$ (top two rows) and $u_{bc}$ (bottom) at eight phases spanning one $M_2$ cycle. Here, each panel represents the occupation of...
each of the four legs closest to the phase indicated. Data are plotted along each survey track, atop a rendering of the bathymetry.

Considering the displacement signals, several observations can be made: 1) The vertical structure is primarily mode 1, with significant higher-mode structure superimposed in some times/places. 2) Signals are coherent from leg to leg; however, the east side of the bay leads the west (e.g., upward displacements in the east at phase 337°, not appearing in the west until 72°, by which time the eastern displacements have begun to fall again). These are the signatures of a westward-propagating wave with phase speed about 1 m s⁻¹, as is shown more clearly later.
The vertical structure of baroclinic velocity appears more complex than that of displacement, but mode 1 can still be clearly identified. Zonal velocities on the east and west legs are nearly out of phase, as found by Hamilton et al. (1995) (e.g., phase 289°: east/west shallow/deep flow in the east, and the reverse in the west, with the transition seen along the cross-bay leg). Note that the phase of \( u_{\text{BC}} \) is such that when isopycnals are elevated, deep flow is westward and shallow flow is eastward: another signature of westward energy flux.

Meridional velocity signals (not shown) are generally weaker and more complicated, consistent with the finding (Hamilton et al. 1995; Eich et al. 2004) that both barotropic and baroclinic signals are oriented along isobaths.

Note that in these short time series, \( M_2 \) signals cannot be isolated from other frequencies, most notably \( K_1 \). These plots are successful because \( K_1 \) signals are much weaker than \( M_2 \) for all of Mamala Bay (Hamilton et al. 1995), particularly for displacement. The dominance of \( M_2 \) will be further evident in later displays.

2) Phase

We next consider the spatial patterns of \( M_2 \) phase of the displacement signals, \( \Phi_n \), and zonal velocity, \( \Phi_u \). Only zonal velocity is considered, both because the signals are stronger and because we wish to identify zonal propagation. Our goals are to distinguish between purely propagating signals (increasing phase toward east or west, \( \Phi_n - \Phi_u = 0° \) or 180°), and standing patterns resulting from their superposition (constant phase for a pure standing wave, \( \Phi_n - \Phi_u = 90° \) or 270°). In addition, a purely propagating wave can be identified via increasing phase in the direction of propagation. In section 4a(3), these measurements will be used to infer phase speeds and wavelength.

First we focus on leg 2, taking an E–W cut through the 3D fields just presented. In Fig. 6, baroclinic zonal velocity is plotted as time series at successively eastern longitudes, with isopycnal depths overlain. As with all of the legs, the leg was occupied for 24 h, or nearly two full \( M_2 \) cycles. The top panel represents the measured (red) and semidiurnally filtered relative Honolulu sea level (black).

In this view, it is clear that the displacement signals are dominated by a single mode-1 wave of \( M_2 \) frequency, with a peak–peak amplitude of nearly 100 m. By eye, it is clear that successive maxima occur later the farther one moves west, indicating westward phase propagation.

In velocity, both mode-1 and mode-2 signatures are evident in the vertical structure. The temporal content is overwhelmingly \( M_2 \), as for displacement. Phase is constant relative to that of displacement, again with deep westward velocities accompanying upward displacements. This is a signature of westward energy flux, in line with the visible phase propagation.

Taking advantage of the \( M_2 \) dominance of both signals, we next simplify further to a time series view, in which propagation can be clearly seen. In Fig. 7, the displacement at 240 m (heavy line in Fig. 6) is plotted on the same axes as the mean baroclinic zonal velocity below 300 m, and best-fit \( M_2 \) sine waves (thin) are overlaid. The sine fits are clearly a good representation of the data, indicating the dominance of \( M_2 \) and low modes. The phase of both displacement and zonal deep baroclinic velocity increases smoothly toward the west.

This analysis is performed for all four legs, and the results are synthesized in Fig. 8. The \( M_2 \) phase lag relative to passage of the tide-generating force over Greenwich is computed for the sine-wave fits at each location. The phase lags for displacement and deep zonal velocity, plotted as red and blue clocks, respectively, present a coherent picture. The phase of displacement (red clocks and numbers) increases steadily from a mean of 239° on the eastern leg 4 to 280° on leg 3 (5 km to the west) to 357° on leg 1 (15 km to the west).

The phase of velocity (blue numbers, clocks) also increases by 25° from leg 4 to leg 3, and across leg 2 from 65° to 114°, to 229° at the eastern leg and the mooring. A more rapid phase change occurs in the western bay, reproducing the nearly 180° phase difference across the bay that was observed by Hamilton et al. (1995) and Eich et al. (2004).

Increasing phase toward the west is interpreted as direct observation of westward propagation, as anticipated by spatial flux patterns from the POM (Fig. 1) (Eich et al. 2004) and observations [section 4a(5)]. The more rapid phase change of \( u_{\text{BC}} \) in the western bay is consistent with the presence of eastbound signals there, as indicated in Fig. 1. (The eastbound waves have the same \( \eta \) phase but opposing \( u \) phase.)
The phase angle between $\eta$ and $u_{BC}$ (evident as the angle between the red and blue “clock” hands) is another indicator of propagation, with $\Phi_{\eta} - \Phi_{u} = 0^\circ$ or $180^\circ$ indicating an eastward-/westward-propagating free wave, and $90^\circ$ or $270^\circ$ indicating a standing wave and no propagation. Displacement leads $u$ by about $220^\circ$ on the eastern legs, consistent with westward propagation. On the western leg, $\Phi_{\eta} - \Phi_{u} = 120^\circ$, also consistent with interference from eastbound signals there.

In summary, the spatial patterns of $u_{BC}$, $\eta$ phase, and their difference imply dominant western propagation in the central and eastern portions of the bay. More rapid phase shifts in the western bay imply the presence of eastbound signals there, consistent with the general picture of converging, remotely generated waves presented by Eich et al. (2004), Fig. 1, and below.

3) WAVELENGTH AND PHASE SPEED

For a single freely propagating wave, the cyclic phase differences $\Delta \Phi$ and separations $\Delta x$ from section 4a(2) can be used to compute horizontal wavelength $\lambda = \alpha^{-1} = \Delta x/\Delta \Phi$ and associated phase velocities. The estimates from the various legs fall around 60 km and 1 m s$^{-1}$ (Table 1), consistent with the speeds [Eq. (5)] appropriate for mode-1 and mode-2 waves propagating in 400-m depth with the observed stratification profile. The wavelengths are also consistent with those estimated from group velocity [section 4d(2)].

The presence of eastbound signals would lead to overestimated wavelength by this method (in the extreme case of a standing wave, the method would yield infinite wavelength). The agreement of the inferred and theoretical wavelengths supports our earlier conclusion that westbound signals dominate at the time of the survey.

4) ENERGY

Depth-integrated baroclinic kinetic energy

$$ KE = \frac{1}{2} \rho \int_{-H}^{0} \overline{u_{BC}} \, dz $$

and potential energy

$$ PE = \frac{1}{2} \rho \int_{-H}^{0} N^2 \eta \, dz $$

are indicated in Fig. 8 by the size of the blue and red clocks, respectively (the overbar indicates a time average). Total energy, indicated by the color of the circle, exceeds 15 kJ m$^{-2}$ near the bay center [as compared to the 10 kJ m$^{-2}$ modeled by Eich et al. (2004)], decaying steadily by a factor of 3–4 over the 10-km length of leg 3. Kinetic energy (KE) is slightly greater near the headlands than in the bay center, as seen qualitatively by Hamilton et al. (1995). Potential energy (PE) is greatest in the bay center, exceeding that at the headlands by a factor of 3.

For a single free internal wave, $KE > PE$ (e.g., Phillips 1977). The observation that $PE > KE$ implies influence of the northern boundary and/or a partly standing pattern resulting from superimposed east- and west-bound waves.
5) Energy Flux

The general picture of wave structure and propagation presented by the POM model (Fig. 1) and the phase analysis is upheld by direct energy-flux calculations from the baroclinic velocity and displacement signals. At each location in Fig. 8, time series of displacement and baroclinic velocity are used to compute the flux over two $M_2$ cycles. The depth-integrated flux vectors are shown in Fig. 9 (red arrows). The 2-month mean flux from the mooring, bracketing the time of the cruise, is also shown. Model fluxes (Eich et al. 2004) (thin black) are in general agreement.

As suggested by the phase progression and $\eta$-$u$ phases, measured baroclinic flux is westward at all SWIMS II locations. Strong northwestward (along isobath) fluxes are seen near Diamond Head, which propagate around eastward, and weaken, inside the bay on leg 2. At the mooring and along leg 1, flux is still westward, developing an offshore component at the southern end of the leg.

The red flux vectors represent a synoptic snapshot of the Mamala Bay fluxes. For completeness, flux estimates from other times are plotted in blue arrows. Flux estimates from Hamilton’s moorings A4, B3, and E4, and from the 25-h MMP survey at B3 (blue arrows) all agree well with the picture just presented, even though they were collected in other years. However, the fluxes from the autumn 2000 AMP station at Barber’s Point, taken immediately before the MMP B3 station, are westward. Apparently, at times, westward propagation can extend westward all the way past Barber’s Point. Long time series of energy flux at Hamilton’s A4 mooring (section 4d) confirms that this is the case. These observations underscore the strong time dependence of the system.

b. Mixing

The colored dots in Fig. 9 represent depth-integrated dissipation rates inferred from overturns, which are discussed in the context of the bay’s energy budget in section 4b(3). We first present profiles (section 4b(1)) at different locations, then discuss mechanisms in section 4b(2).
1) PROFILES

Depth profiles of station-average dissipation rate $\varepsilon$ (left) and diapycnal diffusivity $K_p$ (right) are shown in Fig. 10. Diffusivity varies from $\approx 10^{-3}$ m$^2$ s$^{-1}$ in the upper water column near the mooring (middle) to $K_p > 10^{-1}$ m$^2$ s$^{-1}$ toward the west of the bay (SWIMS II line 1).

The microstructure and SWIMS II profiles (top and bottom) represent spring-tide conditions. As will be seen from time series, below, mixing displays a strong spring–neap cycle. This is indicated in Fig. 10 (middle) by computing separate profiles from the moored time series during neap, spring, and mean. Spring-tide enhancement below 250 m is clear.

2) MECHANISMS AND CASE STUDIES

Most of the $K_p$ profiles display a middepth maximum (e.g., A4, leg 1) and/or bottom enhancement (e.g., B3, MP, legs 2–4). The former case is illustrated in Fig. 11 for SWIMS II leg 1 (which is similar to the A4 station). Meridional sections of $u_{BC}$ (top) and $\varepsilon$ (bottom) are plotted spanning two $M_2$ cycles. Only every other section is plotted. The station-mean profile just presented is dominated by the spatially coherent layers of high dissipation rate extending out from the slope near 250 m. These generally are collocated with high zonal shear (top panels) and low 20-m Richardson number $R_i = N^2 S^{-2}$ (not shown), implicating shear instability (Miles 1961; Howard 1961). The combined effects of the ridge and the convergent east- and westbound waves act to increase the low-mode shear, leading to the instability. Dissipation rate is also high in bottom-boundary layers during strong near-bottom flows (e.g., lower-right panel).

At the mooring, whose profiles are strongly bottom enhanced, a different mechanism appears to be active. Figure 12 displays quantities measured at the mooring over the eight $M_2$ cycles spanned by the Revelle survey. Dissipation-rate signals (bottom panel) display a clear semidiurnal modulation. Elevated values as high as $10^{-3}$ W kg$^{-1}$ are observed as far as 200 off the bottom, with maxima while isopycnals are rising (gray lines). These coincide with periods of high strain $\gamma = \frac{[\nabla^2(z)]}{N^2(t, z)}$ (Fig. 12c), which modulates the 20-m Ri ($R_i^{-1}$...
plotted in Fig. 12e). Strain in the lower 100 m is dominantly $M_2$ and is sufficient to modulate $N^2$ by a factor of 10. Shear modulation (Fig. 12d) plays a lesser role, as found by Alford and Pinkel (2000) and Alford and Gregg (2001). Values of $\text{Ri} < 1$ are observed, in phase with the high-strain periods. Though $\text{Ri}$ is too coarsely resolved for quantitative statements, these results suggest that shear instability, triggered primarily by the strain of the wave, is responsible for the elevated dissipation rates.

Similar mixing signals seen at the Kaena Point Ridge appear to result from a new mixing mechanism being investigated numerically by Slinn and Levine (2003). In their simulations, an oscillatory, barotropic flow on a slope decreases to zero at the bottom boundary because of frictional effects. Consequently, the faster interior flows advect heavier water above lighter during up-slope flow, resulting in gravitational instability when isopycnals are rising, as observed here. In our case, however, observed velocity differences (not shown) are too small to lead to this instability.

3) Geography and energy budget

We now return to Fig. 9 to compare observed depth-integrated dissipation rate (colored circles) with the model and observed flux convergences. Not surprisingly, the spatial pattern of depth-integrated mixing mimics that of diffusivity (Fig. 1), with the highest values over the ridge in the western bay. It decreases smoothly along line 2 toward the east and along all legs toward the south away from the shoals.

In the absence of advection or secular time trends, an energy-flux convergence must balance turbulent dissipation of mechanical energy. The observed and modeled fluxes show the same pattern; hence, we use the better-resolved model fluxes to estimate divergence. Even with the model fluxes, coarse resolution leads to substantial uncertainty. Still, two primary convergence regions are identifiable (Eich et al. 2004, their Fig. 8): $\approx 0.3 \text{ W m}^{-2}$ in the western bay near the north end of leg 1 and $\approx 0.2 \text{ W m}^{-2}$ near the eastern end of leg 2.

Observed mixing in the western portion of the bay is sufficient to balance the flux convergences ($\approx 0.3 \text{ W m}^{-2}$, yellow-orange circles). Further east, mixing is a factor of 4–5 too weak to support the eastern convergence. Given that mixing on all legs increases in the shoals, a likely explanation is that elevated but unmeasured mixing north of our observations accounts for the difference.

The integral of the model flux divergence over a square area of the bay bounded by latitudes $21.17°–21.3°$ and longitudes $158.05°–157.8°$ is 19 MW. This figure is only a tiny percentage (0.1%) of the total amount of energy (20 GW) extracted from the barotropic tide by the entire Hawaiian Ridge (Egbert and Ray 2000, 2001). Thus, the region does not appear to significantly affect the energetics of the tide at the Hawaiian Ridge. However, the strong diapycnal diffusivity $>10^{-3} \text{ m}^2 \text{s}^{-1}$ will tend to smooth over 100-m scales in $\tau = L^2 K_p^{-1} = 10^7 \text{ s} = 4$ months. Thus, we expect that tidally driven mixing in the western bay to be an important factor in determining water-column properties (temperature, salinity, and biology, as well as pollutants) on seasonal time scales.

c. Time dependence

To place the spatial snapshot discussed in the last section in temporal context, observations from the 2-month moored profiler deployment (Fig. 1) are discussed in this section. The modal content, phase, en-
ergy, energy flux, and dissipation rate are presented for the entire 2-month deployment in Fig. 13. To index the spring–neap cycle, we plot Honolulu sea level in Fig. 13b, which has been semidiurnally bandpassed to remove diurnal contributions. The spring–neap envelope is indicated in red. Several spring tides are spanned by the deployment, which are indicated in successive panels with vertical dashed lines.

1) Modal Amplitude

The total amplitude, and the partition between the modes, of both zonal velocity (Fig. 13c) and displacement (Fig. 13d), estimated as in section 3d, vary substantially with time. For velocity, the baroclinic signals (red, black) generally exceed the barotropic signals. (Barotropic and baroclinic $v$ velocities, not shown, are much weaker.) Mode 2 (red) generally exceeds mode 1 (black). The reverse is true for displacement, with mode 1 generally dominating. The errors (green) in each case are smaller than the modal amplitudes, indicating that modes 1 and 2 explain most of the variance.

2) Phase

Standard harmonic analysis (Foreman 1996) is used to determine the $M_2$ amplitude and UTC phase lag of the mode-1 signals using 7-day windows. The amplitudes closely follow those just presented, so they are not replotted. The phase signals are plotted in Fig. 13e. Mode-1 displacement (red) is nearly in phase with that of the surface tide ($60^\circ$) in the early portion of the record, leading it in the later portion (consistent with Figs. 6, 7, and 12). The differences are easily discernible by eye from time series (not shown).

Overall, the phase of both displacement and velocity varies substantially, with somewhat more variability seen in velocity. Part of this variability can be interpreted in terms of travel-time fluctuations from the remote source (next section). In addition, the mooring is situated in a region of rapidly evolving phase in the velocity signals (Fig. 8), which likely explains the greater variability observed in $u$ than in $\eta$. Increasing the demodulate window size from 7 to 27 days yields fewer points but does not change the results.

Since $u$’s phase varies more than that of $\eta$, the phase difference between them (black) is variable. Near the beginning of the record, during the cruise, and at the very end, the phase difference is nearly $180^\circ$, consistent with westward propagation as seen in the spatial survey. However, during the middle of the deployment the
phase is nearly 90°, implying a nearly standing component for the first mode. We interpret these differences as a shifting of the interference pattern between the westbound and eastbound waves.

3) **HODOGRAPHS**

Another way of viewing the phase modulation is in hodographs of the mode-1 amplitudes, shown at the top of the diagram.
Fig. 13. Time series plots from the moored profiler in the central bay. (a) Hodographs of mode-1 amplitudes. The orientation of the isobaths is indicated in gray. (b) Semidiurnal Honolulu sea level fluctuations (black) and an envelope (red), highlighting the spring–neap cycle. Dotted lines in all successive panels indicate peak springs. (c) Zonal velocity amplitude for modes 0 (barotropic, blue), 1 (black) and 2 (red), and the error (green). (d) Displacement amplitude for modes 1 (black) and 2 (red), and the error (green). (e) $M_2$ phase lag of mode-1 displacement (red), zonal velocity (blue), and their difference (black). (f) Eastward energy flux in modes 1 (black) and 2 (red), and the total (blue). (g) Kinetic energy (black), potential energy (red), and their total (blue). (h) $\log_{10} \sigma$. The cruise time is indicated in white. (i) The $\tau$ from 250–350 (black), 50–350 (red), and 50–250 m (blue), and a sine fit (green).
of Fig. 13. For each 7-day period used in the phase estimates, conventional hodographs of mode-1 \( u \) and \( v \) amplitudes are plotted at top. Mode-1 flow is predominantly along isobath (gray line) near the middle of the deployment (yearday 250), developing cross-isobath components at most other times. Below these, “\( u - \eta \) hodographs” are plotted by normalizing the mode-1 modal amplitudes of velocity and \( \eta \) by their maximum value over each 7-day window, and plotting them as a hodograph. In these views, in-phase variables appear as lines oriented along a line \( y = x \), while out-of-phase variables appear as circles. The relative phase modulation between \( u \) and \( \eta \) is seen clearly in the middle row, where the 180° phase differences near the beginning appear as hodographs aligned along the line \( y = -x \) (westward propagation). Near the middle, they are more circular (near standing).

4) ENERGY FLUX

Energy flux (Fig. 13f) is overwhelmingly westward at all times, with peaks of 2–4 kW m\(^{-2}\) occurring 1–3 days after each spring tide. The magnitude of the flux is not constant between spring tides; nor is the proportion of the flux carried by the first and second modes. For example, during the period near yearday 252, when the mode-1 velocities and displacements are nearly in quadrature, nearly one-half of the flux is carried by the second mode, whose signals (not shown) are more nearly 180° out of phase. At other times (e.g., the very beginning), flux is carried nearly entirely by mode 1, as expected and usually observed (Kunze et al. 2002). The variable modal content of the flux is expected given the time-varying modal partition of both displacement and velocity.

5) ENERGY

Kinetic, potential, and total energy (Fig. 13g) rise and fall in tandem with the energy flux, with peaks 1–3 days after each spring tide, bracketing the 2-day lag reported by Eich et al. (2004). As with the flux, the peak is variable between spring tides. Its mean is near 10 kJ m\(^{-2}\), comparable to model values (Eich et al. 2004) and those measured from the SWIMS II survey.

The spring–neap modulation of PE is much stronger than that of KE. Consequently, PE approximately equals KE during neaps, exceeding KE by about a factor of 2 during springs.

6) DISSIPATION

The depth–time series of dissipation rate detected from overturning signatures (Fig. 13i) also peaks just after each spring tide, with largest values in the bottom 150 m of the water column, as indicated in Fig. 10. (Recall that the water depth was 400 m and the bottom 50 m was not sampled.) The high shallow values near yeardays 252 and 260 occur when the wind (not plotted) deepened the mixed layer to nearly 80 m. Since these are weak relative to the deeper signals and are unrelated to the tides, they are not discussed further.

Depth-averaged dissipation rate from 250 to 350 m (black) and over the entire column (red) demonstrate a spring–neap modulation by about a factor of 50 (sine fit, green). The shallower signals (<250 m, blue) show a much weaker modulation. When compared with the observed and modeled energy-flux divergence (Fig. 9) the depth-integrated dissipation (right-hand axis) is insufficient at all times to balance the divergence of about 0.3 W m\(^{-2}\). It is likely that dissipation rates are much higher in the unsampled 50 m, as is apparent from the bottom-enhanced mean profiles (Fig. 10), accounting for the difference.

d. Remote generation: Hamilton’s moorings revisited

The modeled and observed spatial patterns of phase, energy, and energy flux appear consistent with the superposition of an east-going wave generated at Kaena Ridge and a stronger west-going wave generated at Makapuu Point. However, our observations of 1) temporally variable energy, flux, and phase at the mooring and 2) westward fluxes during HOME 2000 at Barber’s Point indicate that the resulting interference pattern is strongly temporally variable. Aside from spring–neap modulation, we hypothesize that variability results from modulation of the 1) strength and 2) propagation speed of the two waves. The former can lead to times at which a stronger westbound wave results in westward flux at all locations, and the latter determines the arrival time and hence phase of each wave, affecting the interference pattern resulting from the superposed signals.

1) ENERGY AND ENERGY FLUX

To examine these hypotheses, we revisit Hamilton’s moorings, giving up spatial resolution in favor of year-long time series. Vertical resolution at moorings A4 (Barber’s Point), B3 (central bay), and E4 (Diamond Head) is sufficient to compute energy, energy flux, and phase for the first two modes, as in section 3d. In Fig. 14, semi-diurnal sea level measured at Honolulu (a proxy for the barotropic forcing) is plotted in Fig. 14a, with maxima (spring tides) indicated in successive panels with dotted lines. In successive panels, total en-
ergy KE + PE (Fig. 14b) and eastward flux (Fig. 14c) are plotted for A4 (black), B3 (thick), and E4 (gray).

Several observations are apparent:
1) Energy shows a clear spring–neap cycle, but the magnitude of individual maxima is variable, as seen at the moored profiler.
2) Energy generally rises and falls together at all three moorings, but the relative strength of the energy at A4 (eastbound waves) and at E4 (westbound waves) varies substantially.
3) Energy generally peaks somewhat after the surface tide maximum [as found in the MP and by Eich et al. (2004)].
4) Energy flux (Fig. 14c) generally rises and falls in tandem with energy (Fig. 14b) at all three moorings.
5) Energy flux is overwhelmingly westward at E4, westward at B3, and eastward at A4. However, opposing bursts do occur at all locations. The westward fluxes observed at A4 during the HOME 2000 AMP station apparently occurred during one of these times.

Observations support the notion of convergent fluxes (observation 5) into Mamala Bay by remote sources (observation 3), but observation 2 indicates that the strength of each wave is strongly time variable. Modulation of the generation strength at each site by the slowly varying thermocline depth, as suggested by Mitchum and Chiswell (2000), is one explanation.

2) GROUP VELOCITY

Modeling indicates that the stronger westbound signals are generated at Makapuu Point, 50 km to the east (Fig. 1). We wish to determine whether the observed phase variability can result from variable travel times from this location. First, we require an observational estimate of the group speed of the wave. In this section, we employ a new technique that exploits the strong correlation between energy and energy flux to estimate the group velocity of the wave at each mooring site by employing the relation $F = c_E E$. To compute $c_E$, energy flux is binned and the mean energy computed for each bin. Flux is then plotted versus binned energy for each of the three Hamilton moorings and the moored profiler (Fig. 15). Modes 1 (dots) and 2 (crosses) are examined separately for both eastward (black) and westward (gray) fluxes. It is clear that westward fluxes dominate within the bay (MP and B3). Eastward fluxes are dominant at A4 and westward fluxes at E4, but in both of these cases there are many occurrences of opposite-sign fluxes, as seen in the time series (Fig. 14) and in the Barber’s Point fluxes from AMP (Fig. 9).

In all cases, points fall on a straight line whose slope is the group velocity for each mode. Mode-1 values range from 0.5 to 1.5 m s$^{-1}$, mode-2 values from 0.2 to 0.7, bracketing theoretical values (Table 1) in both cases. Except for MP, where the eastward fluxes are
very weak, there is excellent agreement between group velocity estimates for east- and westward fluxes.

3) TRAVEL TIME AND PHASE

We now investigate the travel-time hypothesis, by examining the $M_2$ phase of displacement and velocity signals at each mooring, using 27-day overlapping demodulate windows. In Fig. 16, amplitude [(a), (b)] and phase (c) of the mode-1 velocity and displacement signals from A4, B3, and E4 are plotted. The UTC phases (Fig. 16c) of displacement (means of 340°, 320°, and 340° at A4, B3, and E4) and velocity (means of 300°, 170° and 80°) are consistent with those inferred from the SWIMS II and MP data and Eich et al. (2004), with displacement near 330°, and velocity 0°–90° at E4 and 300° at A4. Displacement signals at B3 consistently lag those at A4 and E4 by about 20°, again indicative of two remotely generated waves to the east and west. The mean $\eta-u$ phase differences of 20°, 170°, and 240° imply eastward, westward, and westward propagation, respectively, consistent with other measures.

To highlight phase changes rather than absolute phase, we subtract the time-mean phase from each series and replot (Fig. 16e). Now it is clear that phase of displacement and velocity rise and fall in tandem at all three moorings, with phase differences of 90° possible over several months, underscoring the notion that tidal “constants” vary with time for internal tides. Indeed, the observed spring–neap cycle and the “pulse”-like signals call for caution in using harmonic analysis to seek deterministic signals at all.

Our hypothesis is that these phase differences result from travel-time modulation due to variations in stratification between the source and the observation location. We examine this possibility for the westbound signals by computing the time-dependent group speed from stratification measured at the nearly monthly profiles taken at station ALOHA as part of the Hawaiian Ocean Time-Series (information available online at http://hahana.soest.hawaii.edu/hot/). Depth-mean buoyancy frequency $N(t)$ (Fig. 16d) is estimated from measured density differences between 15 and 500 m for each profile during the period 1994–95. A seasonal thermocline near 70 m, associated with $\sim$3°C warming, peaks during autumn.

The associated group speed in 400 m of water [Eq. (4)] is indicated at the right of Fig. 16d. A signal generated at Makapuu Point arrives between 11.5 and 14 h after generation, bracketing one wave period.

We see that $N(t)$ and $c_g(t)$ are modulated at these time scales by 10%–20%. Is this sufficient to produce the observed phase differences? Over a trip distance $x_o$, the resulting travel-time difference is $\Delta t = x_o c_g^{-1}$, and the phase difference $\Delta \Phi = M_2 \Delta t$. 

Fig. 15. Scatterplots of binned eastward (black) and westward (gray) flux vs binned energy for moorings A4, B3, E4, and the MP. Dots indicate mode 1; pluses indicate mode 2. The best-fit line for each is plotted and their slopes (the group velocity; m s$^{-1}$) are indicated. In E4, the fit is for energies < 6 kJ m$^{-2}$. 

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The resultant phase shift is overplotted with the observed phase differences in Fig. 16e. Agreement is far from perfect, but the predicted phase is of the correct order of magnitude and does exhibit some correspondence with the observed values. In addition, the model and observed phase trends generally agree. Given the other factors that influence phase [e.g., spatial variability in $N(t)$, modulation of generation location, and refraction by mesoscale currents (Rainville 2004)], this is taken as evidence that travel time due to $N(t)$ modulation is at least partly responsible for the observed phase variability. We are not aware of other attempts to test this hypothesis directly, but Colosi and Munk (2006) have presented compelling data that suggest it holds true for time scales longer than 1 yr. In addition, M. Merrifield (2004, personal communication) observes comparable phase differences in POM runs using spring and autumn stratification.

5. Summary and conclusions

Simultaneous high-resolution moored and survey data in Mamala Bay, Oahu, Hawaii, present a synoptic 3D-plus-time view of an energetic internal tide. Together with a model and reanalyzed longer historical records, they offer a sharpened picture of the motions’ mixing, spatial structure, and temporal variability. We have concluded the following, which were previously unobserved:

- Directly measured energy flux agrees well with model estimates (Fig. 9). The observed pattern indicates a convergence resulting from the superposition of strong westbound signals generated 50 km away at Makapuu Point and weaker east-going waves from Kauai Channel. At the time of our survey, the westbound wave dominated, resulting in net westward energy flux. At other times, a more nearly standing pattern occurs.
- Directly observed $c_p$ (estimated from a new technique as $F/E$; Fig. 15), $c_m$, and $\lambda$ (estimated from spatial phase gradients; Fig. 8 and Table 1) agree with expected values for mode-1 and -2 signals in the observed stratification.
- Mixing, measured in Mamala Bay for the first time from overturns and then verified with microstructure measurements, is intensified in the western bay near a submarine ridge, apparently from shear instability (Figs. 9–11). It displays both an $M_2$ and a spring–neap cycle (Figs. 12 and 13). The total energy dissipated ($\sim$19 MW) is too small to significantly impact the Hawaiian Ridge’s energy budget, but the depth-averaged diffusivity ($10^{-3} \text{ m}^2 \text{s}^{-1}$ in the western bay) is sufficient to smooth properties over 100 vertical meters in 4 months. Tidally driven mixing may thus...
be expected to be of comparable importance as the seasonal heating/cooling cycle in determining the thermal structure of the water column.

- Energy and phase are variable on time scales of months at all locations studied (A4, B3, MP, and E4; Figs. 14 and 16). We have attempted to interpret the large ($\pm 45^\circ$) month-to-month phase differences in terms of travel-time fluctuations from a remote source. Assuming a primary source 50 km to the east, observed phase changes show some similarities with those estimated from travel time estimated from observed time-dependent stratification. We speculate that the energy changes result from varying stratification near the source modulating the amplitude of the generated waves, as observed by Mitchum and Chiswell (2000).

Taken together, these observations indicate an interference pattern between two remotely generated waves. Modulation of the waves’ group velocity by variable ambient stratification affects their arrival times, causing the pattern to be temporally variable. Given the ability of internal tides to travel thousands of kilometers before dissipating (Ray and Mitchum 1997; Alford 2003a), we expect similar time-dependent signals at open-ocean sites, as observed (Alford 2003b).

Superinertial waves propagating along a side boundary such as the waves impinging on Mamala Bay are common at many coastlines and embayments. Vexingly, their dynamics are still poorly understood (Llewellyn-Smith 2004). Ongoing higher-resolution modeling studies and model–data comparisons should yield progress.

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